Focus on the charge pump BC PhL 3.20.14

Review 5.15.14.

The very first part below seems OK and I refined it and put it into Section 6.5 (d). It is the idea that the function $E_z(a,\theta)$ tracks with $n(\theta)$, at least at large ω in Section 6.5. The tracking argument is not rigorous at low ω. But I think the cpbc is rigorous at all ω, so I think App D is OK down to $\omega = 0$.

The remainder of what is below is wrong because I confuse together two problems:

[In the rest of this doc, I seem to be confusing two different physical problems. One has a shorted end to the transmission line (where I expect to have symmetric J_z at $\omega = 0$) and the other has $Z = \infty$ at the end at $\omega = 0$, since it is after all supposed to be an infinite transmission line, and that is what Appendix D treats. For this reason, nothing said below seems too useful.]

I am still working on the low frequency limit anomaly of Appendix D. Here I will focus directly on the charge pump boundary condition which, together with divE = 0, seems to be causing the anomaly.

Here is the BC in its two forms, where E_r is "just under the surface"

$$E_r(a,\theta) = (j\omega/\sigma)\, n(\theta) \; . \qquad \text{(D.2.24)}$$

$$E_r(a,m) = (j\omega/\sigma)\, N_m \; . \qquad \text{(D.2.25)}$$

Here is the div E = 0 equation at an arbitrary interior location, again in two forms.

$$\partial_r (r\, E_r) + \partial_\theta E_\theta + r\, \partial_z E_z = 0 \; .$$

$$[1 + r\partial_r]\, E_r(r,m) + jmE_\theta(r,m) + r\,(-j\beta_d)E_z(r,m) = 0 \; . \qquad \text{(D.1.19)}$$

If we evaluate the second form at the surface, it says

$$[1 + r\partial_r]\, E_r(a,m) + jmE_\theta(a,m) + a\,(-j\beta_d)E_z(a,m) = 0 \qquad . \qquad \text{(D.1.19)}$$

In the electrostatic capacitor model to which I "cling" strongly, $E_\theta(a,m) = 0$ and div E=0 says

$$[1 + r\partial_r]\, E_r(a,m) + a\,(-j\beta_d)E_z(a,m) = 0$$

$$E_z(a,m) = (1/ja\beta_d)\,[1 + r\partial_r]\, E_r(a,m) \qquad \text{where} \quad E_r(a,m) = (j\omega/\sigma)\, N_m$$

This seems to provide a "mechanism" by which $n(\theta)$ propagates itself into $E_z(a,m)$, but I only know the value of $E_r(a,m) = (j\omega/\sigma)\, N_m$ and not $[\partial_r E_r(r,m)]|^{r=a}$, so not clear what to do next.

[the above logic now appears in lines doc Section 6.5 (d).]

Now here is a problem I have. Write the following, where now "m" implies m > 0 always,

$$E_z(a,m) = (1/jr\beta_d)\,\{\, E_r(a,m) + [\partial_r E_r(r,m)]|^{r=a}\,\} \qquad \text{" non-symmetric part"}$$

$E_z(a,0) = (1/jr\beta_d)\ \{\ E_r(a,0) + [\partial_r E_r(r,0)]|^{r=a}\ \}$ " symmetric part"

Then look at the ratio of non-sym / sym :

$$\frac{E_z(a,m)}{E_z(a,0)} = \frac{E_r(a,m) + [\partial_r E_r(r,m)]|^{r=a}}{E_r(a,0) + [\partial_r E_r(r,0)]|^{r=a}}$$

$E_r(a,m) = (j\omega/\sigma)\ N_m$ => $\frac{E_r(a,m)}{E_r(a,0)} = \eta_m$

$E_r(a,0) = (j\omega/\sigma)\ N_0$

My problem is that I want this ratio to become very small as ω becomes very small, so that only the symmetric part survives in this limit. The β_d factor cancels out in the ratio, so it cannot "help". It does seem to be a 0 / 0 type ratio as $\omega \to 0$.

I do know that

$$E_r(r,m) = a_m\ x^{-1}\ J_m(x) + \frac{K_m}{2}\ J_{m+1}(x)$$

but of course if I install my known a_m and K_m I will just reproduce the fact that this ratio does not get small. My problem is that when all the ingredients are mixed together (Helmholtz and divE = 0), I get a result which does not "make sense" for small ω. If instead I treat the coefficients as unknowns and do a small ω expansion, I will find that the ratio above is some function of K_m, K_0, a_m, a_0 each of which can in general be a function of ω, so that by itself proves nothing.

A big thing that does not "make sense" is the following. First consider

$$\text{curl } \mathbf{E} = \hat{\mathbf{r}}\ [\ r^{-1}\partial_\theta E_z - \partial_z E_\theta] + \hat{\boldsymbol{\theta}}\ [\partial_z E_r - \partial_r E_z] + \hat{\mathbf{z}}\ [\ r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r\ ]$$

so in partial waves

$$\text{curl } \mathbf{E} = \hat{\mathbf{r}}\ [\ r^{-1}jmE_z + j\beta_d E_\theta] + \hat{\boldsymbol{\theta}}\ [-j\beta_d E_r - \partial_r E_z] + \hat{\mathbf{z}}\ [\ r^{-1}\partial_r(rE_\theta) - r^{-1}jmE_r\ ]$$

and then

$$[\text{curl } \mathbf{E}]_\theta(r,m) = -j\beta_d E_r(r,m) - \partial_r E_z(r,m)$$

Then Maxwell $-j\omega B_\theta = [\text{curl } \mathbf{E}]_\theta$ says

$-j\omega B_\theta(r,m) = -j\beta_d E_r(r,m) - \partial_r E_z(r,m)$ $\beta_d \approx \omega/v_d$ ignoring loss

When I assume $\beta_d = (\omega/v_d)$ and $\beta = c\sqrt{\omega} = \beta'$, and when I install my known fields, I get for low ω,

$\omega\ B_\theta(r,m) = -(j/2a)\ \eta_m\ I\ R_{dc}\ m(m+1)(r/a)^{m-1}$ [back to the B→∞ paradox]

The problem is that for very small ω, and for fixed I, $B_\theta(r,m)$ gets large without limit, which seems very unphysical. With these same β assumptions, we can write

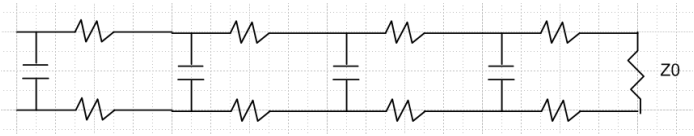
$$-j\omega B_\theta(r,m) \approx -\partial_r E_z(r,m) \qquad \text{very small } \omega$$

and then it becomes clear that the problem is that $\partial_r E_z(r,m)$ is failing to $\to 0$ as $\omega \to 0$. Since the small ω limit of E_z is $(1/2)\,(m+1)\,\eta_m\, I\, R_{dc}\,(r/a)^m$, $\partial_r E_z(r,m)$ is much the same and stays finite. Thus, the finiteness of $\partial_r E_z(r,m)$ results in the infiniteness of $B_\theta(r,m)$. So this is something I OUGHT to be able to understand and deal with.

[In the rest of this doc, I seem to be confusing two different physical problems. One has a shorted end to the transmission line (where I expect to have symmetric J_z at $\omega = 0$) and the other has $Z = \infty$ at the end at $\omega = 0$, since it is after all supposed to be an infinite transmission line, and that is what Appendix D treats. For this reason, nothing said below seems too useful.]

Now at the value f = 0.1 Hz, I have two infinite cylinders driven by $V_{ac} = 6\cos(\omega t)$ and I = very small due to the very large Z_0. The ± n(θ) is constant and quite finite on the conductor pair for a long way down the line from the driving end. All this surface charge came from the AC battery driving source. On every full cycle, this entire n(θ) is pulled off the conductor surface and pushed onto the other, so to speak.

So this brings me back to the "surface currents" issue. In the physical experiment just stated, one must ask: how does the n(θ) surface current "get" to where it is? It probably helps to have this little picture in mind,



This definitely brings to mind the notion that it is n(θ,z) and this is decreasing in z due to the resistance elements. When we write $n(\theta,z) = e^{j(\omega t - \beta_d z)}\, n(\theta,0)$, we will have |n| staying constant in z if we use our model that $\beta_d = (\omega/v_d)$ and then it looks like just a giant long capacitor with no resistance. But in this case we will have $Z_0 = 0$ and then the capacitor is shorted out and the source drives an infinite current I. So whatever we do, we have to allow for finite resistance!

I keep wanting to view the physical situation as two independent entities: [wrong!]

(1) two long resistors with some I flowing in them

(2) one long capacitor made from the surfaces of the two conductors.

In this "model", I guess the capacitor charge is fed along the surface by surface currents. At DC there is no radial pumping current (let's say). But this "model" is intrinsically wrong! The current charging a downstream capacitor has to pass through some resistance before reaching that capacitor, and that current flows through the interior, and then it really *is* radially pumped out to the surface. So we *must* include in β_d an imaginary part. I did the math on this in "including loss" doc and got

$$\beta_d = (\omega/v_d) - j\,[(R_{dc1} + R_{dc2})C\,]\,(v_d/2) = \beta_{d0} - j\alpha \qquad \alpha = [\,(R_{dc1} + R_{dc2})C\,]\,(v_d/2)$$

and there is the imaginary part of interest.

$$e^{-j\beta_d z} = e^{-j\beta_{d0} z}\, e^{-j(-j\alpha) z} = e^{-j\beta_{d0} z}\, e^{-\alpha z}$$

So here then is the physical situation. You have some R_{dc} and C for your line. You have n(θ) at full value at the driving end, but it and everything else decays with $e^{-\alpha z}$ which by the way is frequency independent! We can assume that α is small, but it is finite.

In "DC limit of App D" I wrote this as

$$\beta_d = (1/v_d)(\omega-\omega_c) \qquad \omega_c = R_{dc}\, C\, v_d^2/2$$

and for the Belden 8281 I found that f_c = 7.7 KHz.

How does this change the above analysis? Go back to

$$E_r(a,\theta) = (j\omega/\sigma)\, n(\theta)\ . \qquad \text{(D.2.24)}$$

$$E_r(a,m) = (j\omega/\sigma)\, N_m\ . \qquad \text{(D.2.25)}$$

$$N_m = \eta_m N_0$$

What is N_0 exactly? I do a little calculation below (D.2.30), but in this calculation I use my existing expression for J_z and E_z which may not be right if I am about to modify the theory. Maybe I should leave this as a TBD constant for a while. Let's just do that for the moment.

Now the div E = 0 says this:

$$\partial_r (r E_r) + \partial_\theta E_\theta + r\, \partial_z E_z = 0\ .$$

$$[1 + r\partial_r]\, E_r(r,m) + jmE_\theta(r,m) + r\,(-j\beta_d)E_z(r,m) = 0\ .$$

$$[1 + r\partial_r]\, E_r(a,m) + jmE_\theta(a,m) + r\,(-j\beta_d)E_z(a,m) = 0$$

$$[1 + r\partial_r]\, E_r(a,m) + r\,(-j\beta_d)E_z(a,m) = 0$$

$$E_z(a,m) = (1/ja\beta_d)\,[1 + r\partial_r]\, E_r(a,m) \qquad \text{where} \quad E_r(a,m) = (j\omega/\sigma)\, N_m$$

Now we DO have a difference because now I will insert

$$\beta_d = (1/v_d)(\omega-j\omega_c)$$

to get

$$E_z(a,m) = \frac{v_d}{a}\ \frac{1}{\omega-\omega_c}\,[1 + r\partial_r\,]\,E_r(a,m) \qquad\qquad E_r(a,m) = (j\omega/\sigma)\,N_m$$

So this shows a significant modification in the case that we account for loss. Now when all is done, we are going to find that $E_r(r,m)$ has some form $k(m,\omega)(r/a)^p$ near r = a, where p is some power like m or m±1 or something like that. Therefore I think we can say

$$E_r(r,m) = E_r(a,m)\,(r/a)^p \qquad \text{for r near a}$$

$$\partial_r\,E_r(r,m) = E_r(a,m)\,p(r/a)^{p-1}(1/a)$$

$$r\,\partial_r\,E_r(r,m) = E_r(a,m)\,p(r/a)^{p-1}$$

$$[r\,\partial_r\,E_r(r,m)]|^{r=a} = E_r(a,m)\,p$$

and then our equation above becomes

$$E_z(a,m) = \frac{v_d}{a}\ \frac{1}{\omega-\omega_c}\,(1+p)\,E_r(a,m) \qquad\qquad E_r(a,m) = (j\omega/\sigma)\,N_m$$

and then we insert to get

$$E_z(a,m) = \frac{v_d}{a}\ \frac{1}{\omega-\omega_c}\,(1+p(m))\,(j\omega/\sigma)\,\eta_m\,N_0 \qquad \text{// I is inside } N_0 \text{ here}$$

As stated, I regard N_0 as an unknown constant at this point, which I feel should be independent of ω.

This does achieve the goal of causing $E_z(a,m) \rightarrow 0$ as $\omega \rightarrow 0$, but unfortunately it achieves this goal both for m > 0 and m = 0. In fact we get

$$\frac{E_z(a,m)}{E_z(a,0)} = \frac{1+p(m)}{1+p(0)}\ \eta_m$$

This means then that you cannot have $E_z(a,m) \rightarrow 0$ while $E_z(a,m) \rightarrow$ constant.

For example, in the existing theory we have

$$E_z(a,m) = (1/2)\,\eta_m\,I\,R_{dc}\,(m+1)$$
$$E_z(a,0) = (1/2)\,I\,R_{dc}\,(1)$$

and then in this theory we get

$$\frac{E_z(a,m)}{E_z(a,0)} = \frac{1+m}{1+0}\ \eta_m$$

which agrees with the above.

Does this fix the B_θ problem?

$$-j\omega B_\theta(r,m) = -j\beta_d E_r(r,m) - \partial_r E_z(r,m)$$

$$j\omega B_\theta(r,m) = j\,(1/v_d)(\omega-\omega_c)E_r(r,m) + \partial_r E_z(r,m)$$

OK, now as $\omega \rightarrow 0$ the first term is proportional to $E_r(r,m)$ which in the current theory $\rightarrow 0$, never had an issue with that. The second term then gives

$$j\omega B_\theta(r,m) \approx \partial_r E_z(r,m)$$

If we assume that

$$E_z(r,m) = E_z(a,m)\,(r/a)^s$$

$$\partial_r E_z(r,m) = E_z(a,m)\,q\,(r/a)^{s-1}\,(1/a) \approx E_z(a,m)\,(s/a)$$

but from above,

$$E_z(a,m) = \frac{v_d}{a}\,\frac{1}{\omega-\omega_c}\,(1+p(m))\,(j\omega/\sigma)\,\eta_m N_0$$

so we then have

$$j\omega B_\theta(r,m) \approx \partial_r E_z(r,m) = E_z(a,m)\,(s/a)$$

$$= s\,\frac{v_d}{a^2}\,\frac{1}{\omega-\omega_c}\,(1+p(m))\,(j\omega/\sigma)\,\eta_m N_0$$

Then we can cancel the $j\omega$ to get

$$B_\theta(r,m) = s\,\frac{v_d}{a^2}\,\frac{1}{\omega-\omega_c}\,(1+p(m))\,(1/\sigma)\,\eta_m N_0$$

and this then approaches a finite value instead of an infinite one, so in some sense the B_θ problem is somewhat solved by including the losses.

dimensions: RHS $= \frac{v_d}{a^2}\,\frac{1}{\omega-\omega_c}\,(1/\sigma)\,N_0$

$= m/(\text{sec}\ m^2) * \text{sec} * (\text{ohm-m}) * (\text{Coul}/m^2) =$

= (ohm) * (Coul/m^2) = henry/sec * Cou/m^2 = amp-henry/m^2 = correct

For $B_\theta(r,m)$ I do expect to have an m = 0 contribution as well as m > 0 contributions since I expect the B field to be quite complicated even in the DC limit for two cylinders. I cannot find N_0 until I know the full extent of E_z.