Repair of old ω_c loss model PhL 6.22.14

Here I review the old ω_c loss model. My focus here is on the usual complex averaging business, and I completely overlook the point that v_d is not the same at all ω which is the real reason this loss model is no good. The phase velocity is not always v_d. Somewhere else I cleared this up, but I don't offhand know where that is done. (10.8.14).

In Appendix D.11 I came up with a loss model that I now think is wrong. Here is that model

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(b) Derivation of the simple loss model

Start with this expression for power P in a wave,

$$P = IV = U v_d = [(1/2)L I^2 + (1/2)CV^2] v_d \ . \qquad \dim(Uv_d) = J/m*m/sec = watts \qquad (D.11.7)$$

Then

$$dP/dz = [L I\, dI/dz + C V\, dV/dz] v_d$$

$$= [L I (-\mathbf{y} V) + C V (-\mathbf{z} I)] v_d \qquad \text{// using transmission line equations (4.11.14b)}$$

$$= - IV [L\mathbf{y} + C\mathbf{z}] v_d = - P[L(G + j\omega C) + C(R + j\omega L)] v_d$$

$$= -2\alpha P \qquad (D.11.8)$$

where

$$\alpha = [L(G + j\omega C) + C(R + j\omega L)] (v_d/2) \ . \qquad (D.11.9)$$

So then

$$P(z) = e^{-2\alpha z} P(0)$$
$$|\mathbf{E}(z)| = e^{-\alpha z} |\mathbf{E}(0)| \ . \quad \text{// electric field and all other non-power quantities} \qquad (D.11.10)$$

Ignoring G and assuming low ω, we obtain the result quoted above,

$$\alpha = [L(G + j\omega C) + C(R + j\omega L)] (v_d/2) \approx (LG+RC) (v_d/2) \approx (R_{dc}C) (v_d/2)$$

$$\omega_c = v_d\alpha = (R_{dc}C) (v_d^2/2) \qquad f_c = (R_{dc}C) (v_d^2/4\pi) \qquad (D.11.11)$$

The main point is that in this simple model α and ω_c are constants, independent of ω.

According to the above model, since it says $|\mathbf{E}(z)| = e^{-\alpha z}\,|\mathbf{E}(0)|$, we can identify $\alpha = j\beta_d$ of Appendix D, and then we have

$$\beta_d = -j\alpha = -j[\,L(G + j\omega C) + C(R + j\omega L)]\,(v_d/2) = -j[\,L(G + j\omega C) + C(R + j\omega L)]\,(v_d/2)$$

I am not exactly sure what is wrong with this model, but I do know that it disagrees with the model developed in Chapter 4 which is this from (5.3.4)

$$\beta_d = k = -j\sqrt{\mathbf{zy}} = -j\sqrt{(R+j\omega L)(G+j\omega C)}$$

and which is *verified* in both HM and Matick.

One thing I notice is that the two models agree at large ω:

$$\beta_{d1} = -j[\,L(G + j\omega C) + C(R + j\omega L)]\,(v_d/2) \approx -j[\,L(j\omega C) + C(j\omega L)]\,(v_d/2) = -j[\,2L(j\omega C)]\,(v_d/2)$$

$= \omega LC\, v_d = \omega/v_d$ if we assume β_d is just the β of the dielectric, then we have

$\Rightarrow \quad LC = 1/v_d^2$ // another equation I need to work into things!

Meanwhile

$$\beta_{d2} = -j\sqrt{(R+j\omega L)(G+j\omega C)} = -j\sqrt{j\omega L j\omega C} = -j\sqrt{-\omega^2 LC} = -j\,j\,\omega\sqrt{LC} = \omega\sqrt{LC}$$

and then again we set this to ω/v_d to get $v_d = 1/\sqrt{LC}$. So v_d is the correct phase velocity at high ω regardless of the values of the 4 parameters.

The models are completely different however at ω = 0 We have

$$\beta_{d1} = -j[\,L(G + j\omega C) + C(R + j\omega L)]\,(v_d/2) = -j[\,L(G) + C(R)]\,(v_d/2)$$

$$= -j[\,L(G) + C(R)]\,(1/2)\,1/\sqrt{LC} = (-j/2)\,[G\sqrt{L/C} + R\sqrt{C/L}\,]$$

$$\beta_{d2} = -j\sqrt{(R+j\omega L)(G+j\omega C)} = -j\sqrt{RG}$$

Question: <u>What is wrong with my model bracketed above?</u> Let's look at the opening statement

$$P = IV = U\,v_d = [(1/2)L\,I^2 + (1/2)CV^2\,]\,v_d$$

Here $(1/2)L\,I^2$ is the total mag energy stored in the inductance of the line per unit length.
Here $(1/2)CV^2$ is the total elec energy stored in the capacitance of the line per unit length.

So I take a little dz of the line, and U is how much energy is stored in that dz. Energy is being burned in that dz as well, and maybe that is what I have omitted? Well, think of this as

$$p(z) = i(z)V(z) = U(z)\, v_d = [(1/2)L\, i(z)^2 + (1/2)CV(z)^2]\, v_d$$

and at some z, this IS the total stored energy regardless of loss rate in that dz. Ah, but is this correct for complex i(z) and V(z)? OK, I will go through that song and dance. We have

$$i(z) = e^{j(\omega t - \beta_d z)}\, i(0) \qquad i_{phy}(z) = \mathrm{Re}[e^{j(\omega t - \beta_d z)}\, i(0)]$$
$$V(z) = e^{j(\omega t - \beta_d z)} V(0) \qquad V_{phy}(z) = \mathrm{Re}[e^{j(\omega t - \beta_d z)}\, V(0)]$$

The actual power burned is

$$p_{ph}(z) = i_{ph}(z)V_{ph}(z) = \mathrm{Re}[i(z)]\, \mathrm{Re}[V(z)]$$

$$= \mathrm{Re}[e^{j(\omega t - \beta_d z)}\, i(0)]\, \mathrm{Re}[e^{j(\omega t - \beta_d z)}\, V(0))]$$

$$= \{ \cos(\omega t - \beta_d)\mathrm{Re}[i(0)] - \sin(\omega t - \beta_d)\mathrm{Im}[i(0)] \} \{ \cos(\omega t - \beta_d)\mathrm{Re}[V(0)] - \sin(\omega t - \beta_d)\mathrm{Im}[V(0)] \}$$

$$= \cos^2(\omega t - \beta_d)\, \mathrm{Re}[i(0)]\, \mathrm{Re}[V(0)] + \sin^2(\omega t - \beta_d)\, \mathrm{Im}[i(0)]\, \mathrm{Im}[V(0)]$$

$$- \sin(\omega t - \beta_d) \cos(\omega t - \beta_d) \{ \mathrm{Re}[i(0)]\, \mathrm{Im}[V(0)] + \mathrm{Im}[i(0)]\, \mathrm{Re}[V(0)]$$

The trick I think is now to <u>average this result over time</u>, so the squared trig terms

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Int((sin(x))^2,x=0..2*Pi)/(2*Pi);
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$$\frac{1}{2}\,\frac{\int_0^{2\pi} \sin(x)^2\, dx}{\pi}$$

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value(%);
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$$\frac{1}{2}$$

So we then get

$$< p_{ph}(z) > = 1/2\, \mathrm{Re}[i(0)]\, \mathrm{Re}[V(0)] + 1/2\, \mathrm{Im}[i(0)]\, \mathrm{Im}[V(0)]$$

$$= 1/2 \{ \mathrm{Re}[i(0)]\, \mathrm{Re}[V(0)] + \mathrm{Im}[i(0)]\, \mathrm{Im}[V(0)] \}$$

This has the form

$$f = \mathrm{Re}(a)\mathrm{Re}(b) + \mathrm{Im}(a)\mathrm{Im}(b)$$

But notice that

$$Re(ab^*) = Re\,[\,Re(a)+j\,Im(a)][Re(b)-jIm(b)] = Re(a)Re(b) + Im(a)Im(b) = Re(a^*b)$$

Therefore we have shown that

$$< p_{ph}(z) > = (1/2)\,Re\{\, i(0)V^*(0)\,\}$$

But putting in the phases, they then cancel and this becomes

$$< p_{ph}(z) > = (1/2)\,Re\{\, i(z)V^*(z)\,\} \qquad \text{watts}$$

What