Study of my simple loss model PhL 6.22.14

This is a second doc on this topic (loss 1 is first). I again quote my old ω_c loss model that was installed into D.11. Here I get closer to the correct loss model which appears in the H&M text. I also question the fact that energy just flows along at v_d. I am now closer to installing a better loss model into lines doc, and completely getting rid of the ω_c idea which was based on a constant $v = v_d$. (10.8.14)

Here is the model I concocted for Appendix D.11 (quoted between lines):

(b) Derivation of the simple loss model

Start with this expression for power P in a wave,

$$P = IV = U v_d = [(1/2)L I^2 + (1/2)CV^2] v_d \quad . \qquad \dim(Uv_d) = J/m*m/sec = watts \qquad (D.11.7)$$

Then

$$dP/dz = [L I\, dI/dz + C V\, dV/dz] v_d$$

$$= [L I (-\mathbf{y} V) + C V (-\mathbf{z} I)] v_d \qquad \text{// using transmission line equations (4.11.14b)}$$

$$= - IV [L\mathbf{y} + C\mathbf{z}] v_d = - P[L(G + j\omega C) + C(R + j\omega L)] v_d$$

$$= -2\alpha P \qquad (D.11.8)$$

where

$$\alpha = [L(G + j\omega C) + C(R + j\omega L)] (v_d/2) \quad . \qquad (D.11.9)$$

So then

$$P(z) = e^{-2\alpha z} P(0)$$
$$|\mathbf{E}(z)| = e^{-\alpha z} |\mathbf{E}(0)| \quad . \quad \text{// electric field and all other non-power quantities} \qquad (D.11.10)$$

Ignoring G and assuming low ω, we obtain the result quoted above,

$$\alpha = [L(G + j\omega C) + C(R + j\omega L)] (v_d/2) \approx (LG+RC) (v_d/2) \approx (R_{dc}C) (v_d/2)$$

$$\omega_c = v_d\alpha = (R_{dc}C) (v_d^2/2) \qquad f_c = (R_{dc}C) (v_d^2/4\pi) \qquad (D.11.11)$$

The main point is that in this simple model α and ω_c are constants, independent of ω.

Does this make any sense at all?

Question 1. What is the *meaning* of P = IV or perhaps p(z) = i(z)V(z) at some point z in the transmission line?

It has the units of watts = joules/sec. In a resistor, this would be the power dissipation. I think it might be the rate at which energy is either burned or stored. For a pure capacitor, V(z) = q(z)/C and i(z) = ∂_tq(z) and we could then say p(z) = i(z)V(z) = ∂_tq(z) q(z)/C = ∂_t [1/2 C^{-1} $q(z)^2$] = ∂_t [1/2 C $V(z)^2$]= ∂_t u_C so yes, it is the rate of storage of energy in the capacitor. So perhaps we need to include ohmic loss in our expression

So in distance dz we can say this

u_C = 1/2 C $V(z)^2$dz = cap energy stored
u_L = 1/2 L $i(z)^2$dz = ind energy stored (as in (C.3.5))

p_C = Cdz V(z) ∂_tV(z) = rate of increase of the C stored energy
p_L = Ldz i(z) ∂_t i(z) = rate of increase of the L stored energy

p_R = $i(z)^2$Rdz = energy burned in R
p_G = $V(z)^2$Gdz = energy burned in G // V(z) $i_G(z)$ = V(z) [V(z) Gdz]

p(z) = energy/sec entering our little transmission line segment
p(z+dz) = energy/sec leaving our little transmission line segment

I think the claim is this:

p(z) = i(z) V(z) = interpretation of the product IV

If you had some i(z) and V(z) at z = z, and if you were to simply chop off the right part of the transmission line and replace it with a load (this would be Z_0) , you would claim that i(z)V(z) was the total power flowing into that load, since it has no where else to go. So OK, i(z)V(z) is the power flowing along the transmission line in the z direction.

In the full situation note that i(z) ≠ ∂_tq(z) but rather the transmission line equation!

Now how about this claim:

p(z)-p(z+dz) = *loss* in power flow in the line (assume > 0 and there is a loss going over dz)

= rate of increase in energy stored + rate of energy burned
(some loss went into increased storage, some went into heat)

= (p_C + p_L) + (p_R+ p_G)

Then we write

$$p(z)-p(z+dz) = Cdz\, V(z)\, \partial_t V(z) + Ldz\, i(z)\, \partial_t i(z) + i(z)^2 Rdz + V(z)^2 Gdz$$

Divide by dz to get

$$-\partial_z p(z) = C\, V(z)\, \partial_t V(z) + Li(z)\, \partial_t i(z) + i(z)^2 R + V(z)^2 G$$

or

$$-\partial_z [\, i(z)V(z)] = \partial_t [1/2\, CV(z)^2 + 1/2 Li(z)^2] + i(z)^2 R + V(z)^2 G$$

I see something like this in HM on Sec 14.2 p 760. His result must be for a lossless line, then his result and my result agree. Yes, he says that he is holding off on losses until his Chapter 14.7

An incremental length, Δz, of a two-conductor system and its cross-section are pictured in Fig. 14.2.3. A one-dimensional version of the energy conservation law introduced in Sec. 11.1 can be derived from the transmission line equations using manipulations analogous to those used to derive Poynting's theorem in Sec. 11.2. We multiply (14.1.4) by V and (14.1.5) by I and add. The result is a one-dimensional statement of energy conservation.

$$\boxed{-\frac{\partial}{\partial z}(VI) = \frac{\partial}{\partial t}\left(\frac{1}{2}CV^2 + \frac{1}{2}LI^2\right)} \qquad (19)$$

This equation has intuitive "appeal." The power flowing in the z direction is VI, and the energy per unit length stored in the electric and magnetic fields is $\frac{1}{2}CV^2$ and $\frac{1}{2}LI^2$, respectively. Multiplied by Δz, (19) states that the amount by which the power flow at z exceeds that at $z + \Delta z$ is equal to the rate at which energy is stored in the length Δz of the line.

Now that I think I know what IV means, let's go back to my opening claim:

$$P = IV = U v_d = [(1/2)L I^2 + (1/2)CV^2]\, v_d \,. \qquad \dim(Uv_d) = J/m*m/sec = watts \qquad (D.11.7)$$

Question 2: Is this justifiable? I agree that U is the stored energy per length of line and I agree that Udz is the energy stored in the fields of a slice of the line of length dz. I think the field pattern really is moving to the right at some speed v, but with losses I am not sure exactly what v will be. It probably won't be the dielectric light velocity v_d. But apart from that problem, I think the power flow really is as Uv . The ohmic losses don't enter into that equation. It is the fields that carry the power.

So, with this careful warning about the meaning of v, I think my loss model looks OK and I get

$$p(z) = e^{-2\alpha z}\, p(0) \qquad \alpha = [\, L(G + j\omega C) + C(R + j\omega L)]\, (v/2)$$

Question 3. Assuming we are OK above, what does this say about β_d of Appendix D?

I suppose you could invoke Poynting at this point and say (all is complex)

$$p(z) = | E(z) \times B(z) | \sim e^{-2j\beta_d z} p(0)$$

and then we really can make the connection

$$-2\alpha z = -2j\beta_d z$$

or

$$\alpha = j\beta_d$$

or

$$\beta_d = -j\alpha = -j [L(G + j\omega C) + C(R + j\omega L)] (v/2) = -j [\mathbf{L}y + C\mathbf{z}] (v/2)$$

If I have done things right above, this should be a correct form for β_d including losses, except I don't have an expression for v !

Question 4. Can we compute v from the "true model" ?

The true model supposedly is this

$$\beta_d = k = -j\sqrt{\mathbf{zy}} = -j\sqrt{(R+j\omega L)(G+j\omega C)} \qquad \text{// from (5.3.5)}$$

Now I think the claim is that

$$\mathrm{Re}(\beta_d) = \omega/v$$

where v is the phase velocity. Is this the same as the field movement velocity? Let's just assume that this is the case (ignore group velox) and see where this leads. Then the claim is

$$v = \omega/\mathrm{Re}(\beta_d) = \omega / [\mathrm{Re}(-j\sqrt{\mathbf{zy}})] = \omega / \mathrm{Im}\sqrt{\mathbf{zy}} \qquad \text{// not simply } v_d$$

and then *my* model says

$$\beta_d = -j [\mathbf{L}y + C\mathbf{z}] (v/2) = -j [\mathbf{L}y + C\mathbf{z}] (1/2) \omega / \mathrm{Im}\sqrt{\mathbf{zy}}$$

In order for both models to be correct, we would have to have this be true (both β_d's the same)

$$-j [\mathbf{L}y + C\mathbf{z}] (1/2) \omega / \mathrm{Im}\sqrt{\mathbf{zy}} = -j\sqrt{\mathbf{zy}} \qquad ?$$

or

$$[\mathbf{L}y + C\mathbf{z}] (1/2) [\omega / \mathrm{Im}\sqrt{\mathbf{zy}}] = \sqrt{\mathbf{zy}} \qquad ?$$

or

$$\sqrt{\mathbf{zy}}\ \mathrm{Im}(\sqrt{\mathbf{zy}}) = \omega\ [\mathbf{L}y + C\mathbf{z}]/2 \qquad ? \qquad (**)$$

Is it even possible this could be valid? I doubt it. I will now find a counterexample.

Try R = G = 0 [this is the same as large ω]

$\mathbf{z} = j\omega L$ $\mathbf{y} = j\omega C$ $\mathbf{zy} = -\omega^2 LC$ $\sqrt{\mathbf{zy}} = j\omega\sqrt{LC}$

Then the last ? above says

$$(j\omega\sqrt{LC}) (\omega\sqrt{LC}) = \omega [Lj\omega C + Cj\omega L]/2$$

or

$$j\omega^2 LC = j\omega^2 LC$$

and so OK, both models agree at large ω. Try a different counterexample:

Try L = C = 0 [this is the same as small ω]

$\mathbf{z} = R$ $\mathbf{y} = G$ $\mathbf{zy} = RG$ $\sqrt{\mathbf{zy}} = \sqrt{RG}$

Then the last ? above says

$$\sqrt{RG}\ \ 0 = \omega\ 0/2$$

and I guess things agree there as well. Hmm. Let's try this more generally. In "math problem doc" I shows that

$$s \equiv \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{\mathbf{zy}}$$

$$\mathrm{Re}(\sqrt{\mathbf{zy}}) = (R^2+\omega^2L^2)^{1/4} (G^2+\omega^2C^2)^{1/4} \frac{1}{\sqrt{2}} \sqrt{1+\frac{(RG-\omega^2LC)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}}$$

$$\mathrm{Im}(\sqrt{\mathbf{zy}}) = (R^2+\omega^2L^2)^{1/4} (G^2+\omega^2C^2)^{1/4} \frac{1}{\sqrt{2}} \sqrt{1-\frac{(RG-\omega^2LC)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}}$$

Now introduce these symbols each of which is real, but b may have either sign:

$$a \equiv (R^2+\omega^2L^2)^{1/4} (G^2+\omega^2C^2)^{1/4} \qquad a^2 \to \omega^2 LC \ \text{ as } \omega \to \infty$$

$$b \equiv \frac{(RG-\omega^2LC)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} = \frac{(RG-\omega^2LC)}{a^2} \qquad b \to -1 \text{ as } \omega \to \infty;\ \ 1-b \to 2$$

so that

$$\mathrm{Re}(\sqrt{\mathbf{zy}}) = \frac{1}{\sqrt{2}}\, a \sqrt{1+b}$$

$$\mathrm{Im}(\sqrt{\mathbf{zy}}) = \frac{1}{\sqrt{2}}\, a \sqrt{1-b}$$

$$\sqrt{\mathbf{zy}} = \frac{1}{\sqrt{2}}\, a\, [\sqrt{1+b} + j\sqrt{1-b}\,]$$

Then the LHS of (**) is

$$\sqrt{\mathbf{zy}}\ \mathrm{Im}(\sqrt{\mathbf{zy}}) = \frac{1}{\sqrt{2}}\, a\, [\sqrt{1+b} + j\sqrt{1-b}\,] \frac{1}{\sqrt{2}}\, a\sqrt{1-b} = (a^2/2)\,[\sqrt{1-b^2} + j(1-b)]$$

Now compute

$$1-b^2 = 1 - \frac{(RG-\omega^2LC)^2}{a^4} = [\, a^4 - (RG-\omega^2LC)^2]/a^4$$

$$= [\, (R^2+\omega^2L^2)\,(G^2+\omega^2C^2) - (RG-\omega^2LC)^2\,]/a^4$$

$$= [\, R^2G^2 + \omega^2L^2G^2 + \omega^2R^2C^2 + \omega^4L^2C^2 - R^2G^2 - \omega^4L^2C^2 + 2\omega^2RGLC\,]/a^4$$

$$= [\omega^2L^2G^2 + \omega^2R^2C^2 + 2\omega^2RGLC\,]/a^4$$

$$= \omega^2\,[L^2G^2 + R^2C^2 + 2RGLC\,]/a^4$$

$$= \omega^2(LG+RC)^2/a^4$$

$$\Rightarrow \quad \sqrt{1-b^2} = \omega\,(LG+RC)/\,a^2 \qquad \text{//feeling lucky}$$

Meanwhile.

$$1-b = 1 - \frac{(RG-\omega^2LC)}{a^2} = [\, a^2 - (RG-\omega^2LC)\,]/a^2$$

$$= [\sqrt{(R^2+\omega^2L^2)\,(G^2+\omega^2C^2)} - (RG-\omega^2LC)\,]\,/\,a^2 \qquad \text{// } \rightarrow 2 \text{ at high } \omega$$

Then I get:

$$\sqrt{\mathbf{zy}}\ \mathrm{Im}(\sqrt{\mathbf{zy}}) = (a^2/2)\,[\sqrt{1-b^2} + j(1-b)]$$

$$= (a^2/2)\,\{\,\omega\,(LG+RC)/\,a^2 + j\,[\,a^2 - (RG-\omega^2LC)\,]/a^2\,\}$$

$$= (1/2)\,\{\,\omega\,(LG+RC) + j\,[\,a^2 - (RG-\omega^2LC)\,]\,\}$$

$$= (1/2)\,\{\,\omega\,(LG+RC) + j\,[\sqrt{(R^2+\omega^2L^2)\,(G^2+\omega^2C^2)} - (RG-\omega^2LC)\,]\,\}$$

$$= (1/2)\,\{\,\omega\,(LG+RC) + j\,[\omega^2LC + \sqrt{(R^2+\omega^2L^2)\,(G^2+\omega^2C^2)} - RG\,]\,\}$$

Meanwhile, the right side of (**) is

$$\omega\ [L\mathbf{y} + C\mathbf{z}]/2 \ = \omega\ [L(G+j\omega C) + C(R+j\omega L)]\ /2$$

$$= (1/2)\ \omega(LG+RC)/2\ + j\ \omega^2\ LC$$

Interestingly, the REAL parts of the two sides exactly match. Here are the two imaginary parts

LHS: $(1/2)\ [\ \omega^2 LC\ +\ \sqrt{(R^2+\omega^2L^2)\ (G^2+\omega^2C^2)}\ -\ RG]$

RHS: $\omega^2\ LC$

So the imaginary parts agree when

$$\omega^2 LC\ +\ \sqrt{(R^2+\omega^2L^2)\ (G^2+\omega^2C^2)}\ -\ RG\ =\ 2\omega^2\ LC$$

At very low ω they agree but seems like a factor of 2 error somehow before we reach the limit.
They agree *at* ω = 0.

At very high ω they also agree exactly.

It is true that my model uses the transmission line equations which are not valid at ω = 0, whereas the true model is perhaps more accurate near ω = 0.

Conclusion: my model and the true model are pretty close!

<u>Maybe the group velocity is what you need to use for the energy motion.</u> In that case

$v = v_g = \partial\omega/\partial k$ says wiki, so

$1/v_g = \partial\beta_d/\partial\omega$

Then:

$$\beta_d\ = k\ = -j\ \sqrt{\mathbf{zy}}\ \ \ \ = -j\sqrt{(R+j\omega L)(G+j\omega C)}$$

Then I have to do this derivative which will run me off the end for today.

$$\partial_\omega\ \sqrt{(R+j\omega L)(G+j\omega C)}\ \ = (1/2)\ 1/\sqrt{(R+j\omega L)(G+j\omega C)}\ \ *\ \partial_\omega\ [(RG-\omega^2 LC) + j\omega(LG+RC)]$$

$$=\ (1/2)\ 1/\sqrt{(R+j\omega L)(G+j\omega C)}\ [\ -2\omega LC + j(Lg+RC)\ ]$$

$$= 1/v_g \qquad\qquad ??$$

What does it mean for group velocity to be complex?