Try to show φ = constant for low ω PhL 5.12.14

Review 5.14.14: This is a very recent doc (2 days old) written after Section 3.7 was updated. In that section now I allow as you might say that φ ≈ constant in the low ω regime based on the little picture where A_z is "ballpark constant" at DC. But I obtain here no silver bullet to prove what ought to be easy to prove: that φ = constant applies at ω = 0 up at least to some low ω_1 .

Part I of 2

I tried to show this for about an hour today and I got nowhere, so for now I give up.

It seems odd that I can show that φ = constant for ω = 0 and ω = strong and extreme skin effect, but I can't show it for low frequencies. That just seems wrong to me.

Basically we have to show (ignore abs values in this doc) that on the surface at least,

$$\omega A_t << \nabla_t \varphi$$

In terms of my little coordinate system of Section 3.7, this says I have to show

$$\omega A_x << \partial_x \varphi \qquad \text{and same for y}$$

You might think the gauge condition would play a role in the proof.

I do claim in Step 2 that

$$|\partial_x \varphi| \approx \frac{\Delta\varphi}{\Delta x} \approx \frac{V}{D} \approx (1/D)\,|\varphi| \qquad \text{ballpark}$$

which at least gets rid of the ∂_x problem. But then I have to show that

$$\omega A_x << \varphi/D$$

How do I get an estimate for A_x ? I do think I know that $A_x << A_z$. Suppose I could show that

$$\omega A_z << \varphi/D$$

Again, I need some estimate for A_z. In Appendix G I compute A_z for a DC round wire and find

$$A_z(a) = -\,[I\mu_1/2\pi]\,\ln(a) \qquad \text{at the surface.}$$

So then I would have to show that

$$\omega\, I\mu_1/2\pi]\,\ln(a) << V/D \qquad \text{say}$$

This requires a relation between I and V, but I don't have that yet! Also, by using $A_x << A_z$ I am giving up a lot of space relative to <<, so that is not a good pathway.

Can Appendix M help? It says that

$$A_x(x) = -\frac{1}{4\pi} \Sigma_i \mu_i \int J_x(x',y') \ln(s^2)\, dx'\, dy'$$

I just don't see at this point how to relate any current like J_x to V.

Maybe one could do an "after the fact" ansatz thing here. Let's go back to showing

$$\omega\, I\mu_1/2\pi]\, \ln(a) << V/D$$

This then becomes

$$\omega\, \mu\, \ln(a) << 2\pi\, Z_0/a$$

But as ω increases, this gets into trouble, and my estimate for A_z is DC only.

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I can use the equation

$$E_z = j\beta_d\varphi - j\omega A_z = j(\omega/v_d)\varphi - j\omega A_z = j\omega\, [\, \varphi/v_d - A_z\,]\,. \qquad \beta_d = (\omega/v_d) \qquad (3.7.6)$$

or

$$\varphi/v_d - A_z = E_z/(j\omega)$$

Maybe I need some better words at this point. I know in a round wire that

$$E_z = J_z/\sigma \approx (I/\text{area}*\sigma) \qquad \text{low } \omega$$

but on the other hand skin effect boosts J_z so that

$$J_z = I / [\delta\, 2\pi a\,]$$

so then in that limit

$$E_z = I / [\delta\, 2\pi a\, \sigma\,] \qquad \delta \equiv \sqrt{2/\omega\mu\sigma} \qquad \delta^2 = 2/(\omega\mu\sigma)$$

$$E_z = I / [\sqrt{2/\omega\mu\sigma}\; 2\pi a\, \sigma\,] = \frac{I\sqrt{\mu\sigma\omega}}{\sqrt{2}\, 2\pi\alpha\sigma} = \frac{I\sqrt{\mu\omega}}{2\sqrt{2\sigma}\, \pi a}$$

$E_z/\omega = \text{const} / \omega^{1/2}$

But this is all for strong and extreme skin. As noted above, for **low ω** I get

$E_z = J_z/\sigma \approx (I/\pi a^2\sigma)$

$E_z/\omega = (I/\pi a^2\sigma)(1/\omega)$

Then I would have to show that the RHS is small here:

$\varphi/v_d - A_z = E_z/(j\omega) \approx (I/\pi a^2)(1/\sigma\omega)$

But that is only true as $(1/\omega\sigma) \rightarrow \infty$. But

$(1/\omega\sigma) = \mu\delta^2/2$

so I am back to needing $\delta \rightarrow 0$ to make this work.

Part II of 2

Again, I am trying to show that I can ignore the last term

$$\mathbf{E}_t = -\nabla_t\varphi - j\omega\mathbf{A}_t \qquad (3.7.2)$$

I do know that $|\mathbf{A}_t| < 10^{-4} |A_z|$ so can I find some upper bound for ωA_z ? I guess I can continue with the assumption of Step 2 that $|\nabla_t\varphi| \approx (1/D)|\varphi|$.

Question: Do I have some real world numbers for A_z anywhere? The way I think of $\varphi \sim V$, what can I say about the general size of A_z? How about this argument. I know that

$$\varphi(x,y,z) = \frac{1}{4\pi\xi}\, q(z)\, \varphi_t(x,y) \qquad \textit{(5.1.1)}$$

$$A_z(x,y,z) = \frac{\mu}{4\pi}\, i(z)\, A_{zt}(x,y) \ . \qquad \textit{(5.2.1)}$$

and I know that

$$[\nabla_t^2 + (\beta^2-k^2)]\, \varphi_t(x,y) = 0 \qquad \varphi_t(C_1) = K_1 \quad \varphi_t(C_2) = K_2 \quad K_1 - K_2 = K \qquad (5.3.10)$$

$$[\nabla_t^2 + (\beta^2-k^2)]\, A_{zt}(x,y) = 0 \qquad A_{zt}(C_1) = W_1 \quad A_{zt}(C_2) = W_2 \quad W_1 - W_2 = K \qquad (5.3.11)$$

So if would seem that both φ_t and A_{zt} have the same scale, which is K, and I do know a few things about K. So then I get something like this:

$$\varphi(x,y,z) \approx \frac{1}{4\pi\varepsilon}\ q(z)\ K \qquad (5.1.1)$$

$$A_z(x,y,z) = \frac{\mu}{4\pi}\ i(z)\ K \qquad (5.2.1)$$

It the first one realistic? It is in fact exact for a difference, so it says

$$V = \frac{1}{4\pi\varepsilon}\ q(z)\ K \qquad \text{or} \qquad C = V/q\ = \frac{1}{4\pi\varepsilon}\ K$$

and this is the result I always use, so yes it is realistic. Then I should be able to say

$$A_z(x,y,z) = \frac{\mu}{4\pi}\ i(z)\ K\ =\ \frac{\mu}{4\pi}\ \frac{i(z)}{q(z)}\ q(z)\ K\ \frac{1}{4\pi\varepsilon}\ \ 4\pi\varepsilon$$

$$= \frac{\mu}{4\pi}\ v_d\ \ \varphi(x,y,z)\ \ 4\pi\varepsilon$$

$$= v_d\ \mu\varepsilon\ \varphi(x,y,z)$$

$$= v_d\ (1/v_d)^2\ \varphi(x,y,z)$$

$$= \varphi/v_d$$

But this is the result (3.7.7) that I claimed was only valid for skin effect. But now I am claiming this is ballpark reasonable at ANY frequency all the way down to DC, for a lossless line. Of course then lossless means skin effect!

Question: Are $\varphi_t(x,y)$ and $A_{zt}(x,y)$ exactly the same? Well, maybe at extreme skin effect where then the boundary conditions on A_{zt} really are constant on both those conductors. But the answer is NO at lower frequency where we KNOW that A_z is not constant on the conductors.