These are notes for my Appendix B exercise in B.7. Keep !!

$$\nabla^2 \mathbf{A}(\mathbf{x}) = -\mu_1 \mathbf{J_m}(\mathbf{x}) - \mu_2 \mathbf{J_c}(\mathbf{x})$$ // compare to (B.6.1) which is 2D version

$$\mathbf{A}(\mathbf{x}) = (1/4\pi) \int dV' \, [\mu_1 \mathbf{J_m}(\mathbf{x}') + \mu_2 \mathbf{J_c}(\mathbf{x}')] \, (1/R) \qquad R = |\mathbf{x}-\mathbf{x}'|$$

$$A_z(\mathbf{x}) = (1/4\pi)\,(1/2\pi a)\, I \; \{ (\mu_1-\mu_2) \int dS' \, (1/R) + (2/a)\, \mu_2 \int dV' \, (1/R) \; \}$$ // compare (B.6.4)

$$\int_{surf} dS' \, (1/R) = \int (a d\theta') \int_{-\infty}^{\infty} dz' \, (1/R) = a \int_{-\pi}^{\pi} d\theta' \int_{-\infty}^{\infty} dz' \, (1/\sqrt{s^2 + z'^2})$$

$$\int dV' \, (1/R) = \int (r' d\theta') \int dr' \int_{-\infty}^{\infty} dz' \, (1/R) = \int_0^a r' dr' \int_{-\pi}^{\pi} d\theta' \int_{-\infty}^{\infty} dz' \, (1/\sqrt{s^2 + z'^2})$$

$$\int_{-\infty}^{\infty} dz' \, (1/\sqrt{s^2 + z'^2}) = -2\ln(s/\Lambda)$$

$$\int dS' \, (1/R) = -2a \int_{-\pi}^{\pi} d\theta' \ln(s/\Lambda)$$

$$\int dV' \, (1/R) = -2 \int_0^a r' dr' \int_{-\pi}^{\pi} d\theta' \ln(s/\Lambda)$$

$$s^2 = (x-x')^2 + (y-y')^2 = R^2$$

$$\int dS' \, (1/R) = -a \int_{-\pi}^{\pi} d\theta' \ln(R^2) + a \int_{-\pi}^{\pi} d\theta' \ln(\Lambda^2) = -a \int_{-\pi}^{\pi} d\theta' \ln(R^2) + 2\pi a \ln(\Lambda^2)$$

$$\int dV' \, (1/R) = -\int_0^a r' dr' \int_{-\pi}^{\pi} d\theta' \ln(R^2) + \int_0^a r' dr' \int_{-\pi}^{\pi} d\theta' \ln(\Lambda^2)$$

$$= -\int_0^a r' dr' \int_{-\pi}^{\pi} d\theta' \ln(R^2) + \ln(\Lambda^2)\, 2\pi\, (1/2) a^2$$

Now

$$\int_{-\pi}^{\pi} d\theta' \ln(R^2) = 2 \int_0^{\pi} dx \, \ln [r'^2 + r^2 - 2rr' \cos x] = 2\, Q(a,r)$$

$$\int_0^a r' dr' \int_{-\pi}^{\pi} d\theta' \ln(R^2) = \int_0^a r' \, dr' \; 2\, Q(r',r)$$

So we then have

$$A_z(\mathbf{x}) = (1/4\pi)\,(1/2\pi a)\,I\ \{\ (\mu_1-\mu_2)\ \int dS'\ (1/R) + (2/a)\,\mu_2 \int dV'\ (1/R)\ \}$$

$$= (1/4\pi)\,(1/2\pi a)\,I\ \{\ (\mu_1-\mu_2)\ [\ -\ a\ \int_{-\pi}^{\pi}\ d\theta'\ \ln(R^2) + 2\pi a\ \ln(\Lambda^2)]$$

$$+ (2/a)\,\mu_2\ [-\ \int_0^a\ r'dr'\ \int_{-\pi}^{\pi}\ d\theta'\ \ln(R^2)\ + \ln(\Lambda^2)\ 2\pi\ (1/2)a^2]\ \ \}$$

$$= (1/4\pi)\,(1/2\pi a)\,I\ \{\ (\mu_1-\mu_2)\ [\ -\ a\ \ 2\ Q(a,r) + 2\pi a\ \ln(\Lambda^2)]$$

$$+ (2/a)\,\mu_2\ [-\ \int_0^a\ r'\ dr'\ \ 2\ Q(r',r)\ + \ln(\Lambda^2)\ 2\pi\ (1/2)a^2]\ \ \}$$

$$= -\ (I/4\pi^2 a)\ \ \{\ (\mu_1-\mu_2)\ [\ \ a\ Q(a,r) - \pi a\ \ln(\Lambda^2)]$$

$$+ (1/a)\,\mu_2\ [\ \int_0^a\ r'\ dr'\ \ 2\ Q(r',r)\ \ -\ \ln(\Lambda^2)\ \pi\ a^2]\ \ \}$$

$$= -\ (I/4\pi^2 a^2)\ \ \{\ (\mu_1-\mu_2)\ [\ \ a^2\ Q(a,r) - \pi a^2\ \ln(\Lambda^2)]$$

$$+\ \ 2\ \mu_2\ [\ \int_0^a\ r'\ dr'\ \ \ Q(r',r)\ \ -\ \ln(\Lambda^2)\ \pi\ a^2]\ \ \}$$

Using the 2D method I got instead

$$A_z = -\ [I/(4\pi^2 a^2)]\ \{\ (\mu_1-\mu_2)\ (a/2)\ \int dS'\ \ln(R^2)\ + \mu_2 \int dV'\ \ln(R^2)\ \}$$

$$= -\ [I/(4\pi^2 a^2)]\ \{\ (\mu_1-\mu_2)\ (a/2)\ \ 2a\ Q(a,r)\ + \mu_2 2\ \ \int_0^a\ r'\ dr'\ \ Q(r',r)\ \}$$

$$= -\ [I/(4\pi^2 a^2)]\ \{\ (\mu_1-\mu_2)\ (a/2)\ \ 2a\ Q(a,r)\ + \mu_2 2\ \ \int_0^a\ r'\ dr'\ \ Q(r',r)\ \}$$

$$= -\ [I/(4\pi^2 a^2)]\ \{\ (\mu_1-\mu_2)\ a^2\ Q(a,r)\ + \mu_2\ \ 2\ \ \int_0^a\ r'\ dr'\ \ Q(r',r)\ \}$$

The non-Λ terms agree!!!! The Λ terms are these

$$= -\ (I/4\pi^2 a^2)\ \{\ (\mu_1-\mu_2)\ [-\ \pi a^2\ \ln(\Lambda^2)] + 2\mu_2\ [-\ \ln(\Lambda^2)\ \pi\ a^2]\ \ \}$$

$$= +\ (I/4\pi^2 a^2)\ \pi a^2\ \ln(\Lambda^2)\ \{\ (\mu_1-\mu_2) + 2\mu_2\ \ \}$$

$$= + (I/4\pi)\ \ln(\Lambda^2)\ \{\mu_1 + \mu_2\}$$