Old Sections B.2 thru B.5 for archive PhL retired 10.15.14

B.2 Calculation of H from the current J in a conductor

(a) An expression for H in terms of J

Start with Maxwell's equation (1.1.1), and we are now working inside a conductor so **E** = 0 and then

$$\text{curl } \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} = \mathbf{J} \tag{B.2.1}$$

where **J** is the conduction current. Apply curl to both sides and use curl curl = grad div $-\nabla^2$ to get

$$\text{grad div } \mathbf{H} - \nabla^2\mathbf{H} = \text{curl } \mathbf{J} \, . \tag{B.2.2}$$

But in a uniform medium div **H** = 0 since div **B** = 0 so

$$\nabla^2\mathbf{H} = -\text{ curl } \mathbf{J} \, . \tag{B.2.3}$$

Now let's assume that we have $\mathbf{J} = J_z(x,y)\hat{\mathbf{z}}$ and assume that the solution **H** does not depend on z. In that case we have

$$\nabla^2_{2D}\,\mathbf{H}(x,y) = -\text{ curl } \mathbf{J} \qquad \mathbf{J} = J_z(x,y)\hat{\mathbf{z}} \quad . \tag{B.2.4}$$

The particular solution to this PDE is shown in (I.1.8) to be

$$\mathbf{H}(x,y) = \int d^2\mathbf{x}' \, [\, \frac{1}{2\pi} \ln(1/R) \,] \, \text{curl}' \, \mathbf{J}(\mathbf{x}') \qquad R = |\mathbf{x}-\mathbf{x}'| \tag{B.2.5}$$

or

$$\mathbf{H}(x,y) = -\frac{1}{4\pi} \int d^2\mathbf{x}' \ln(R^2) \, \text{curl}' \, \mathbf{J}(\mathbf{x}') \qquad R = |\mathbf{x}-\mathbf{x}'| \tag{B.2.6}$$

where $\frac{1}{2\pi} \ln(1/R)$ is the Poisson 2D free-space propagator. This gives **H** in terms of **J**.

(b) An alternative derivation using the vector potential A_z

An *alternate* derivation of (B.2.5) makes use of the vector potential A_z. Start with (1.5.4),

$$(\nabla^2 + \beta^2)\mathbf{A}(\mathbf{x}) = -\Sigma_i \, \mu_i \mathbf{J}_i(\mathbf{x}) \, . \qquad \text{all of region R} \tag{1.5.4}$$

Apply this to a single conductor and assume $\mathbf{A}(x) = A_z(x,y)\hat{\mathbf{z}}$, so the above equation becomes

$$-\nabla^2_{2D} \, A_z(\mathbf{x}) = \mu J_z(\mathbf{x}) \, . \tag{B.2.7}$$

The particular integral from (I.1.8) is then

$$A_z(\mathbf{x}) = \int d^2x' \, [\ln(1/R)/2\pi] \, [\mu J_z(\mathbf{x}') = - (\mu/2\pi) \int d^2x' \, \ln(R) \, J_z(\mathbf{x}') \quad . \tag{B.2.8}$$

Then use

$$\mathbf{B} = \text{curl } \mathbf{A} = \hat{\mathbf{x}} \, (\partial_y A_z - \partial_z A_y) + \hat{\mathbf{y}} \, (\partial_z A_x - \partial_x A_z) + \hat{\mathbf{z}} \, (\partial_x A_y - \partial_y A_x)$$

$$= \hat{\mathbf{x}} \, (\partial_y A_z) + \hat{\mathbf{y}} \, (- \partial_x A_z) \tag{B.2.9}$$

so that

$$\mu \mathbf{H} = \hat{\mathbf{x}} \, (\partial_y A_z) + \hat{\mathbf{y}} \, (- \partial_x A_z) \, . \tag{B.2.10}$$

Now compute,

$$\partial_y A_z(x) = \partial_y \, [- (\mu/2\pi) \int d^2x' \, \ln(R) \, J_z(\mathbf{x}') \,] = - (\mu/2\pi) \int d^2x' \, J_z(\mathbf{x}') \, \partial_y \ln(R) \, .$$

But $\partial_y \ln(R) = - \partial_{y'} \ln(R)$ since $R = |\mathbf{x}-\mathbf{x}'|$. But then do parts integration to move $\partial_{y'}$ to $J_z(\mathbf{x}')$ picking up an offsetting minus sign, and the parts vanish on a great circle surrounding the conductor. Thus,

$$\partial_y A_z(x) = - (\mu/2\pi) \int d^2x' \, \ln(R) \, \partial_{y'} J_z(\mathbf{x}')$$

$$\partial_x A_z(x) = - (\mu/2\pi) \int d^2x' \, \ln(R) \, \partial_{x'} J_z(\mathbf{x}') \quad . \tag{B.2.11}$$

Then from (B.2.10) one gets,

$$\mathbf{H} = - (1/2\pi) \int d^2x' \, \ln(R) \, [\hat{\mathbf{x}} \, \partial_{y'} J_z(\mathbf{x}') - \hat{\mathbf{y}} \, \partial_{x'} J_z(\mathbf{x}')] \, .$$

But the coordinates $\mathbf{x} = (x,y)$ and $\mathbf{x}' = (x',y')$ have the same unit vectors $\hat{\mathbf{x}} = \hat{\mathbf{x}}'$ and $\hat{\mathbf{y}} = \hat{\mathbf{y}}'$. Since $\mathbf{J} = J_z \hat{\mathbf{z}}$ we then end up with

$$\mathbf{H}(\mathbf{x}) = - (1/2\pi) \int d^2x' \, \ln(R) \, \text{curl}' \, J_z(\mathbf{x}') = + (1/2\pi) \int d^2x' \, \ln(1/R) \, \text{curl}' \, \mathbf{J}(\mathbf{x}')$$

which agrees with (B.2.5).

(c) Boundary conditions

Recall from (B.1.1) that the tangential component of H is continuous through the boundary between conductor and dielectric, even if $\mu_1 \neq \mu_2$. Consider then the transverse component of (B.2.5) at some point on the conductor surface such as the point shown in Fig 3.3. We have (t = transverse)

$$H_t(\mathbf{x}) = \int d^2x' \left[\frac{1}{2\pi} \ln(1/R) \right] [\text{curl}' \mathbf{J}(\mathbf{x}')]_t \qquad R = |\mathbf{x}-\mathbf{x}'| \quad , \qquad \text{(B.2.12)}$$

This particular integral is naturally continuous at the boundary between the media, and this agrees with the fact that $H_t(\mathbf{x})$ must have this property. Therefore, no homogeneous solutions of (B.2.4) $\nabla^2_{2D} H_t = 0$ need be added in, so (B.2.12) is the complete solution for $H_t(\mathbf{x})$. This solution can then be used in (B.1.10) to find the magnetization surface current.

(d) The Biot-Savart Law in 3D and 2D

Recall the above 3D vector Helmholtz equation,

$$\nabla^2\mathbf{H} = -\ \text{curl}\ \mathbf{J}\ . \qquad \textit{(B.2.3)}$$

In Cartesian coordinates it is three scalar Helmholtz equations which can be solved as in (H.1.8) to give

$$\mathbf{H}(\mathbf{x}) = \int d^3x'\ [\ \frac{1}{4\pi R}\ ]\ \text{curl}'\ \mathbf{J}(\mathbf{x}') \qquad\qquad R \equiv |\mathbf{x}-\mathbf{x}'| \qquad \text{(B.2.13)}$$

where $1/4\pi R$ is the 3D free-space Poisson propagator. Write this in components and define $\mathbf{R}$ as shown,

$$H_i(\mathbf{x}) = \int d^3x'\ [\ \frac{1}{4\pi R}\ ]\varepsilon_{ijk}\ \partial'_j J_k(\mathbf{x}')\ , \qquad \mathbf{R} \equiv \mathbf{x} - \mathbf{x}'\ = \text{points to observation point } \mathbf{x}\ . \qquad \text{(B.2.14)}$$

Then move ∂_j' from J_k to $(1/R)$ by parts integration (pick up minus sign) and throw out the parts for the usual reasons (see end of Appendix A.1),

$$H_i(\mathbf{x}) = -\ \int d^3x'\ \frac{1}{4\pi}\ \partial'_j\ (\frac{1}{R})\ \varepsilon_{ijk}\ J_k(\mathbf{x}')\ . \qquad \text{(B.2.15)}$$

Then note that $\partial'_j R^{-1}\ = -R^{-2}\ \partial'_j R$ and

$$\partial'_j R\ = \partial'_j\ \sqrt{\Sigma_k (x'_k - x_k)^2}\ = (1/2)(1/R)\ 2(x'_j - x_j) = \ R^{-1}\ (x'_j - x_j)\ \ = -\ R^{-1}\ R_j \qquad \text{(B.2.16)}$$

so that $\partial'_j R^{-1} = +R^{-3} R_j$. Only the parts minus sign remains, so

$$H_i(\mathbf{x}) = -\ \int d^3x'\ [\ \frac{1}{4\pi R^3}\ ]\varepsilon_{ijk}\ R_j\ J_k(\mathbf{x}') \qquad \text{(B.2.17)}$$

or reversing the cross product order,

$$\mathbf{H}(\mathbf{x}) = \int d^3x'\ \ \frac{1}{4\pi R^3}\ \ \mathbf{J}(\mathbf{x}')\ \text{x}\ \mathbf{R}\ \ . \qquad\qquad \mathbf{R} \equiv \mathbf{x} - \mathbf{x}' \qquad \text{(B.2.18)}$$

This equation is basically the **3D Biot-Savart Law**, see for example Panofsky and Philips p 125 (7.31). For a short piece d$\mathbf{s}'$ of thin wire carrying current I, one writes $\mathbf{J}(\mathbf{x}')\ d^3\mathbf{x}' = I\ d\mathbf{s}'$ so the above becomes,

$$\mathbf{H}(\mathbf{x}) = \oint \frac{1}{4\pi R^3}\ I\ d\mathbf{s}'\ \text{x}\ \mathbf{R} \qquad \text{or} \qquad d\mathbf{H}(\mathbf{x})\ \ = \frac{1}{4\pi R^3}\ I\ d\mathbf{s}'\ \text{x}\ \mathbf{R}\ . \qquad \text{(B.2.19)}$$

We can apply the same process to obtain a 2D Biot-Savart Law as follows. Start with

$$\nabla^2_{2D}\,\mathbf{H}(x,y) = -\,\mathrm{curl}\,\mathbf{J} \qquad \mathbf{J} = J_z(x,y)\hat{\mathbf{z}} \tag{B.2.4}$$

and its solution (B.2.5)

$$\mathbf{H}(x,y) = \int d^2x' \left[\frac{1}{2\pi}\ln(1/R)\right] \mathrm{curl}'\,\mathbf{J}(\mathbf{x}') \qquad R = |\mathbf{x}-\mathbf{x}'| \tag{B.2.5}$$

where $\frac{1}{2\pi}\ln(1/R)$ is the Poisson 2D free-space propagator. Then, inverting 1/R,

$$H_i(x,y) = -\int d^2x' \left[\frac{1}{2\pi}\ln(R)\right] \varepsilon_{ijk}\,\partial'_j J_k(\mathbf{x}') \,. \tag{B.2.20}$$

Doing the same parts integration gives

$$H_i(x,y) = +\int d^2x' \left[\frac{1}{2\pi}\,\partial'_j \ln(R)\right] \varepsilon_{ijk}\,J_k(\mathbf{x}') \tag{B.2.21}$$

and now using result (B.2.16) from above,

$$\partial'_j \ln(R) = R^{-1}\partial'_j R = R^{-1}\,[-R^{-1}\,R_j] = -R^{-2}R_j \tag{B.2.22}$$

we get

$$H_i(x,y) = -\int d^2x' \left[\frac{1}{2\pi}\,R^{-2}R_j\right] \varepsilon_{ijk}\,J_k(\mathbf{x}') = -\int d^2x' \,\frac{1}{2\pi R^2}\,\varepsilon_{ijk}\,R_j\,J_k(\mathbf{x}') \tag{B.2.23}$$

or

$$\mathbf{H}(x,y) = \int d^2x' \,\frac{1}{2\pi R^2}\,\mathbf{J}(\mathbf{x}') \times \mathbf{R} \qquad \mathbf{R} \equiv \mathbf{x} - \mathbf{x}' \tag{B.2.24}$$

which is the **2D Biot-Savart Law**. It provides an alternate way to obtain **H** from **J** in a 2D problem.

B.3 General Method for computing the surface current J_m on a wire

Here are the steps for a wire of arbitrary cross sectional shape:

1. Compute the **H** field at all points in the wire cross-sectional plane section using (B.2.6) or (B.2.24)

$$\mathbf{H}(x,y) = -\frac{1}{4\pi}\int d^2x' \ln(R^2)\;\mathrm{curl}'\,\mathbf{J}(\mathbf{x}') \qquad R = |\mathbf{x}-\mathbf{x}'| \,. \tag{B.2.6}$$

$$\mathbf{H}(x,y) = \int d^2x' \,\frac{1}{2\pi R^2}\,\mathbf{J}(\mathbf{x}') \times \mathbf{R} \qquad \mathbf{R} \equiv \mathbf{x} - \mathbf{x}' \tag{B.2.24}$$

or use the third method of first computing A_z,

$$A_z(\mathbf{x}) = -(\mu/2\pi) \int d^2x' \ln(R)\, J_z(\mathbf{x}') \qquad (B.2.8)$$

$$\mathbf{B} = \text{curl } \mathbf{A} = \hat{\mathbf{x}}\,(\partial_y A_z) + \hat{\mathbf{y}}\,(-\partial_x A_z) \qquad \mathbf{H} = \mathbf{B}/\mu \qquad (B.2.9)$$

2. Evaluate this **H** field at $\mathbf{x}_b = (x_b, y_b)$ for all points $\mathbf{x}_b$ on the cross section boundary.

3. Compute the component of **H** which is tangential to the boundary in the cross sectional plane. Call this component H_θ.

4. The surface current density is then given by (B.1.10),

$$K_z = -\left(\frac{\mu_2}{\mu_0} - \frac{\mu_1}{\mu_0}\right) H_\theta \qquad \text{amps/m} \qquad (B.1.10)$$

B.4 Surface current on a round wire with uniform J

For a round wire of radius a with uniform J_z (as would be the DC case $\omega = 0$), geometric symmetry makes the calculation of **H** very easy. One need only apply Ampere's Law separately for a point r outside the wire, and for another point r inside the wire. For the outside case one finds

$$2\pi r\, H_\theta(r) = I \quad \Rightarrow \quad H_\theta(r) = I/(2\pi r) \qquad r \geq a \quad . \qquad (B.4.1)$$

And then for the inside case the "current enclosed" is determined by a simple area fraction.

$$2\pi r\, H_\theta(r) = I\,(\pi r^2/\pi a^2) \quad \Rightarrow \quad H_\theta(r) = I\, r/(2\pi a^2) \qquad r \leq a \quad . \qquad (B.4.2)$$

At the boundary the two expressions agree and we have

$$H_\theta = I/(2\pi a) \quad . \qquad (B.4.3)$$

If this wire has magnetic permeability μ_2 and is embedded in an infinite medium of μ_1, then the surface magnetization current induced on the wire is

$$K_z = -\left(\frac{\mu_2}{\mu_0} - \frac{\mu_1}{\mu_0}\right) H_\theta = -\left(\frac{\mu_2}{\mu_0} - \frac{\mu_1}{\mu_0}\right) I/(2\pi a) \qquad \text{amp/m} \qquad (B.4.4)$$

$$\mathbf{K}_z = K_z \hat{\mathbf{z}} \qquad (B.4.5)$$

and this surface current is in the direction opposite **J** if $\mu_2 > \mu_1$. If $\mu_1 = \mu_2$, the surface current vanishes. This surface current could be expressed in volume density form as

$$\mathbf{J}_m = K_z \delta(r-a) \hat{\mathbf{z}} \qquad \text{amp/m}^2 \quad . \qquad (B.4.6)$$

B.5 Computing H_θ for a round wire using the General Method of B.3

For a wire of some general cross section, symmetry is not available to allow the simple solution for H_θ outlined in the previous section. We then have to use the more general method outlined in Section B.3 above. As a check on the viability of this general method, we shall carry out "step 1" of the method and show how H_θ may be computed from **J** using the formula (B.2.6).

The conduction current density in a round wire with uniform J_z is given by

$$J_z(r) = J_0\theta(a-r) \tag{B.5.1}$$

where θ is the Heaviside step function. Our first step is to compute curl **J**, and we do this in cylindrical coordinates by just staring at the cylindrical-coordinates curl formula,

$$\text{curl } \mathbf{J} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta J_z - \partial_z J_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z J_r - \partial_r J_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rJ_\theta) - r^{-1}\partial_\theta J_r\,] \tag{B.5.2}$$

and finding the only non-zero piece which is this (uniform J_z),

$$\text{curl } \mathbf{J} \;=\; [-\partial_r J_z(r)]\,\hat{\boldsymbol{\theta}}\;. \tag{B.5.3}$$

Inserting $J_z(r)$ from above we find

$$\partial_r\, J_z(r) = J_0\;\partial_r\theta(a-r) \;= -\,J_0\;\delta(r-a) \tag{B.5.4}$$

$$\Rightarrow\;\text{curl } \mathbf{J}(r) = \hat{\boldsymbol{\theta}}\;J_0\;\delta(r-a)\;. \tag{B.5.5}$$

so we have a "ring source of curl **J**". For use in our integral for **H** we then have

$$\text{curl}'\,\mathbf{J}(r') = \hat{\boldsymbol{\theta}}'\;J_0\;\delta(r'-a)\;. \tag{B.5.6}$$

For a current distribution which tapers off smoothly to 0 at the wire edge one would not have this singular contribution, but for a wire with prescribed uniform current, it is present, and curl **J** vanishes everywhere but on the boundary. The relevant picture is this:

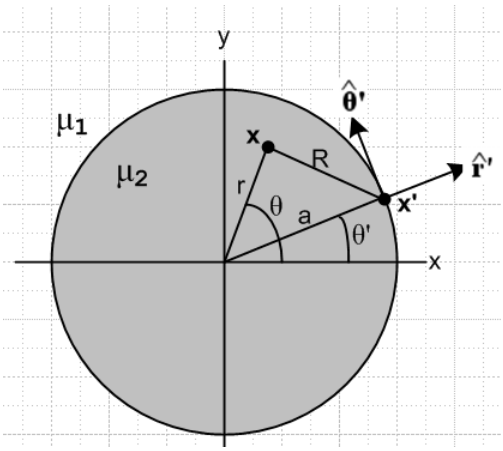


Fig B.4

From (B.2.5) the **H** field at any point **x** = (x,y) is then given by

$$\mathbf{H}(x,y) = -\frac{1}{4\pi}\int d^2\mathbf{x}'\, \ln(R^2)\, \mathrm{curl}'\, \mathbf{J}(\mathbf{x}') = -\frac{1}{4\pi}\int d^2\mathbf{x}'\, \ln(R^2)\, \hat{\boldsymbol{\theta}}'\, J_0\, \delta(r'-a)$$

$$= -\frac{J_0 a}{4\pi}\int_{-\pi}^{\pi} d\theta'\, \ln(R^2)|_{r'=a}\, \hat{\boldsymbol{\theta}}'$$

or

$$\mathbf{H}(r,\theta) = -\frac{J_0 a}{4\pi}\int_{-\pi}^{\pi} d\theta'\, \ln\left[\, r^2 + a^2 - 2ar\cos(\theta'-\theta)\,\right]\hat{\boldsymbol{\theta}}'\,. \qquad \text{(B.5.7)}$$

The figure shows that

$$\hat{\boldsymbol{\theta}}' = \cos\theta'\,\hat{\mathbf{y}} - \sin\theta'\,\hat{\mathbf{x}} \qquad \text{(B.5.8)}$$

so then

$$\mathbf{H}(r,\theta) = -\frac{J_0 a}{4\pi}\int_{-\pi}^{\pi} d\theta'\, \ln\left[\, r^2 + a^2 - 2ar\cos(\theta'-\theta)\,\right]\left[\cos\theta'\,\hat{\mathbf{y}} - \sin\theta'\,\hat{\mathbf{x}}\right]\,. \qquad \text{(B.5.9)}$$

Next, let $x \equiv \theta'-\theta$. Since the $\int d\theta'$ has full range 2π, one can replace $\int_{-\pi}^{\pi} d\theta' = \int_{-\pi}^{\pi} dx$. Then

$$\mathbf{H}(r,\theta) = -\frac{J_0 a}{4\pi}\int_{-\pi}^{\pi} dx\, \ln\left[\, r^2 + a^2 - 2ar\cos(x)\,\right]\left[\cos(x+\theta)\,\hat{\mathbf{y}} - \sin(x+\theta)\,\hat{\mathbf{x}}\right]\,. \qquad \text{(B.5.10)}$$

Now writing $\mathbf{H} = H_x\,\hat{\mathbf{x}} + H_y\,\hat{\mathbf{y}}$, decompose the above into two equations

$$H_x(r,\theta) = +\frac{J_0 a}{4\pi}\int_{-\pi}^{\pi} dx\, \ln\left[\, r^2 + a^2 - 2ar\cos(x)\,\right]\sin(x+\theta)$$

$$H_y(r,\theta) = -\frac{J_0 a}{4\pi}\int_{-\pi}^{\pi} dx\, \ln\left[\, r^2 + a^2 - 2ar\cos(x)\,\right]\cos(x+\theta) \qquad \text{(B.5.11)}$$

or

$$H_x(r,\theta) = + \frac{J_0 a}{4\pi} \int_{-\pi}^{\pi} dx \ln [\, r^2 + a^2 - 2ar\cos(x)\,] [\, \sin x\cos\theta + \cos x \sin\theta \,]$$

$$H_y(r,\theta) = - \frac{J_0 a}{4\pi} \int_{-\pi}^{\pi} dx \ln [\, r^2 + a^2 - 2ar\cos(x)\,] [\cos x\cos\theta - \sin x\sin\theta \,] \,. \qquad \text{(B.5.12)}$$

Since $\int_{-\pi}^{\pi} dx$ is over an even range, throw out odd integrand terms, and *then* fold the negative range into the positive adding a factor of 2 to get

$$H_x(r,\theta) = + \sin\theta \frac{J_0 a}{2\pi} \int_{0}^{\pi} dx \ln [\, r^2 + a^2 - 2ar\cos(x)\,] \cos x \quad \equiv \quad \sin\theta \frac{J_0 a}{2\pi} Q$$

$$H_y(r,\theta) = - \cos\theta \frac{J_0 a}{2\pi} \int_{0}^{\pi} dx \ln [\, r^2 + a^2 - 2ar\cos(x)\,] \cos x \quad = -\cos\theta \frac{J_0 a}{2\pi} Q \qquad \text{(B.5.13)}$$

where

$$Q \equiv \int_{0}^{\pi} dx \ln [\, r^2 + a^2 - 2ar\cos(x)\,] \cos x \,. \qquad \text{(B.5.14)}$$

Before evaluating this integral, we see that

$$\mathbf{H} = H_x \hat{\mathbf{x}} + H_y \hat{\mathbf{y}} = - \frac{J_0 a}{2\pi} Q [\, \cos\theta \hat{\mathbf{y}} - \sin\theta \hat{\mathbf{x}} \,] = - \frac{J_0 a}{2\pi} Q \hat{\boldsymbol{\theta}} = H_\theta \hat{\boldsymbol{\theta}} \,. \qquad \text{(B.5.15)}$$

Thus we find that the resulting **H** is entirely in the $\hat{\boldsymbol{\theta}}$ direction and

$$H_\theta = - \frac{J_0 a}{2\pi} Q \,. \qquad \text{(B.5.16)}$$

We seek now to evaluate this integral Q,

$$Q \equiv \int_{0}^{\pi} dx \ln [\, r^2 + a^2 - 2ar\cos(x)\,] \cos x$$

$$= \int_{0}^{\pi} dx \ln [\, \{a^2\}\{ (r/a)^2 + 1 - 2(r/a)\cos(x)\} \,] \cos x$$

$$= \int_{0}^{\pi} dx \{ \ln (a^2) + \ln[(r/a)^2 + 1 - 2(r/a)\cos(x)] \} \cos x$$

$$= \ln (a^2)[\int_{0}^{\pi} dx \cos x \,] + \int_{0}^{\pi} dx \ln[(r/a)^2 + 1 - 2(r/a)\cos(x)] \cos x$$

$$= \ln (a^2)[0] + \int_{0}^{\pi} dx \ln[\alpha^2 + 1 - 2\alpha\cos(x)] \cos x \qquad \text{where} \quad \alpha \equiv r/a$$

$= \int_0^{\pi} dx \ln[\alpha^2 + 1 - 2\alpha \cos(x)] \cos x \ . \qquad // = Q$ (B.5.17)

This integral is the n=1 special case of the following integral from GR7 page 589 4.379.6,

6. $$\int_0^{\pi} \ln\left(1 - 2a\cos x + a^2\right) \cos nx \, dx$$

$$= \frac{1}{2}\int_0^{2\pi} \ln\left(1 - 2a\cos x + a^2\right) \cos nx \, dx$$

$$= -\frac{\pi}{n} a^n \qquad \left[a^2 < 1\right] \quad \text{BI (330)(11), BI (332)(5)}$$

$$= -\frac{\pi}{n a^n} \qquad \left[a^2 > 1\right] \quad \text{GW (338)(13a)}$$

Therefore we find

$$Q = \begin{cases} -\pi(r/a) & r<a \\ -\pi(a/r) & r>a \end{cases} \quad \text{(B.5.18)}$$

so

$$H_\theta = -\frac{J_0 a}{2\pi} Q = \begin{cases} (aJ_0/2)\,(r/a) & r<a \\ (aJ_0/2)\,(a/r) & r>a \end{cases} \ . \quad \text{(B.5.19)}$$

Now the total current in the wire is $I = J_0 \pi a^2$ so $(aJ_0/2) = (I/2\pi a)$ and then

$$H_\theta = \begin{cases} (I/2\pi a)\,(r/a) & r<a \\ (I/2\pi a)\,(a/r) & r>a \end{cases} = \begin{cases} I\,(r/2\pi a^2) & r<a \\ I\,(1/2\pi r) & r>a \end{cases} \ . \quad \text{(B.5.20)}$$

Thus, we finally arrive at the same results for H_θ as obtained in (B.4.2) and (B.4.1).