Repair of Appendix B PhL 1.26.14

Here I realized the embarrassing situation that **E** • **B** ≠ 0 for my own rectangular conductor field plot! But I just got done proving that **E** • **B** = 0 !! This is now all resolved.

1. Statement of the Problem

I had this little "reader exercise" at the end of Chapter 3 which I was just going to discard since I didn't understand it,

" **Reader Exercise:** Reconcile the appearance of Appendix C Fig C.2 with the above proofs of Fact 7. "

I now understand the above reader exercise. I claim in Section 3.7 Fact 5 that B lines outside a conductor are parallel to the surface. One argument is that they have to be that way in order to be perp to the E field at the surface, in the transverse picture. But in Appendix C Fig C.2, the H lines do NOT seem to be parallel to the surface! Why is that?? The Maple file is rectangle v3.mws. I increased grid to 60x60, same problem. For me, this is a big problem!

(1) is my claim wrong that H should be parallel? [no]

(2) is my plot math wrong so graph is wrong? [no, graph is right]

I guess I now have to face this issue and stop proofing.

Here I think the problem has something to do with homo adder solutions which was a red herring.

I assume uniform current flow. Is that correct for a square conductor at DC? [I think yes]
I have a "particular solution" for H, how do I know there aren't adder homo solutions??? Reading at (B.2.6). Who says the particular solution meets BC's? And what are the BC's on H ?

$$H_{t2} = H_{t1} \quad \text{or} \quad (1/\mu_2)B_{t2} = (1/\mu_1)B_{t1} \qquad (1.1.44)$$

$$B_{n1} = B_{n2} \quad \text{or} \quad \mu_1 H_{n1} = \mu_2 H_{n2} \qquad (1.1.49)$$

I think I would make the same argument I did with A_a which is this: the particular solution,

$$\mathbf{H}(x,y) = -\frac{1}{4\pi}\int d^2\mathbf{x}'\, \ln(R^2)\ \text{curl}'\,\mathbf{J}(\mathbf{x}') \quad R = |\mathbf{x}-\mathbf{x}'| \qquad (B.2.6)$$

will be "naturally continuous" at the boundary and cannot possibly satisfy $\mu_1 H_{n1} = \mu_2 H_{n2}$!! So you must have to add a homo solution to this thing! But then my whole advertised method of computing J_m then falls apart! *However*, this (B.2.6) will "naturally satisfy" $H_{t2} = H_{t1}$ so perhaps it gives the correct tangential value! So maybe think of it as two equations

$$H_n(x,y) = -\frac{1}{4\pi}\int d^2\mathbf{x}'\, \ln(R^2)\ [\text{curl}'\,\mathbf{J}(\mathbf{x}')]_n \qquad R = |\mathbf{x}-\mathbf{x}'|$$

$$H_t(x,y) = -\frac{1}{4\pi}\int d^2\mathbf{x}'\, \ln(R^2)\ [\text{curl}'\,\mathbf{J}(\mathbf{x}')]_t \qquad R = |\mathbf{x}-\mathbf{x}'|$$

Maybe we can use this last one alone to compute the J_m stuff. In my round wire example, the normal H field is 0 because $[\text{curl}' J(\mathbf{x}')]_n = 0$ and therefore in that example, $\mu_1 H_{n1} = \mu_2 H_{n2}$ says $0 = 0$ which is then OK. That explains why the example happened to work.

The first can be written

$$H_n(x,y) = -\frac{1}{4\pi}\int d^2\mathbf{x}' \ln(R^2)\ [\text{curl}' J(\mathbf{x}')] \bullet \hat{\mathbf{n}} \qquad R = |\mathbf{x}-\mathbf{x}'|$$

2. How to repair the problem?

I need to find homo solutions to this equation

$$\nabla^2_{2D} H_n(x,y) = 0$$

such that $\mu_1 H_{n1} = \mu_2 H_{n2}$. This is a 2D scalar Laplace equation so I have a chance

$H_n(x,y)$ = Cartesian harmonics.

Ouch, that would be

$A,\ B \ln(x^2+y^2),\ (x^2+y^2)^{n/2}\quad n = \pm 1,2,3....$

I can rule out negative n integers to keep finite at the origin. So I then have

$$H^{homo}_n(x,y) = A + B \ln(x^2+y^2) + \Sigma_{n=1}^{\infty} C_n (x^2+y^2)^{n/2}$$

with lots of TBD coefficients! I then have to solve a particular problem and then add this mess and find all the coefficients!

For example, in the rectangular wire cross section I know that the normal field on the right edge is given by

$$H^{part}_x(a,y) = \frac{1}{16\pi ab} F(a,y,a,b)$$

$$H^{homo}_n(a,y) = A + B \ln(a^2+y^2) + \Sigma_{n=1}^{\infty} C_n (a^2+y^2)^{n/2}$$

Then I have to find coefficients such that

$$\mu_1 H_{n1} = \mu_2 H_{n2}$$

3. Back up and start again

Suppose I assume that $\mu_1 = \mu_2$. *Then* my particular solution should be the correct solution since it will then be "automatically continuous" at the boundary in both normal and transverse sense. In Appendix B I never say anything about the μ values. I might add a comment near (C.4.1) that it is valid for $\mu_1 = \mu_2$. But then why don't the results look right??? I think probably the H field is continuous at all my arrows, and then the real problem is E and H being perpendicular! So we still have a big mystery here! My Section 3.7 Fact 4 that $\mathbf{E} \bullet \mathbf{B} = 0$ everywhere still seems good, so what is wrong at my surface?

(a) Let's examine one piece at a time. Here is a chunk from Appendix B:

Start with Maxwell's equation (1.1.1), and we are now working inside a conductor so $\mathbf{E} = 0$ and then

$$\text{curl } \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} = \mathbf{J} \qquad \text{(B.2.1)}$$

where **J** is the conduction current. Apply curl to both sides and use curl curl = grad div $-\nabla^2$ to get

$$\text{grad div } \mathbf{H} - \nabla^2\mathbf{H} = \text{curl } \mathbf{J} \, . \qquad \text{(B.2.2)}$$

But in a uniform medium div **H** = 0 since div **B** = 0 so

$$\nabla^2\mathbf{H} = -\text{ curl } \mathbf{J} \, . \qquad \text{(B.2.3)}$$

Since I am at DC, ω = 0 so that is another reason the E term goes away. But this is value both inside and outside the conductor if ω = 0. I am just trying to get the DC H field, cannot be that hard! The curl curl theorem is correct, I checked.

(b) My next chunk is this:

Now let's assume that we have $\mathbf{J} = J_z(x,y)\hat{\mathbf{z}}$ and assume that the solution **H** does not depend on z. In that case we have

$$\nabla^2_{2D}\,\mathbf{H}(x,y) = -\text{ curl } \mathbf{J} \qquad \mathbf{J} = J_z(x,y)\hat{\mathbf{z}} \quad . \qquad \text{(B.2.4)}$$

The particular solution to this PDE is shown in (I.1.8) to be

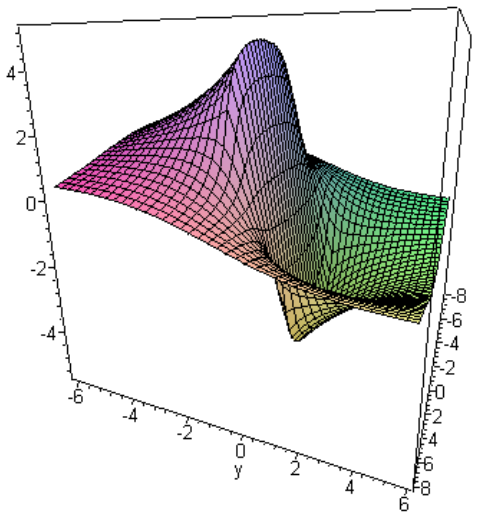
$$\mathbf{H}(x,y) = \int d^2\mathbf{x}' \left[\frac{1}{2\pi} \ln(1/R) \right] \text{curl}' \, \mathbf{J}(\mathbf{x}') \qquad R = |\mathbf{x}-\mathbf{x}'| \qquad \text{(B.2.5)}$$

or

$$\mathbf{H}(x,y) = -\frac{1}{4\pi} \int d^2\mathbf{x}' \ln(R^2) \text{ curl}' \, \mathbf{J}(\mathbf{x}') \qquad R = |\mathbf{x}-\mathbf{x}'| \qquad \text{(B.2.6)}$$

Two items here. First, H does in fact depend on z, so there is a β'^2 term missing in (B.2.4) where this is whatever β' describes the traveling wave. Second, we have a vector Helmholtz, not a scalar, so not clear that this is the particular solution! But I guess OK in Cartesians. Near DC we have very LONG λ and thus very small β' so I guess both items are OK.

(c) I claim H_n and H_t are naturally continuous at the boundaries for H from the above particular solution. Do plots bear this out? For example, H_x along the right edge. A 3D plot of Hx shows that H_x is continuous, though its slope is not continuous. Here is that plot



where the top and bottom edges are at y = ±b = ±1

Let's go with a square just to get less complexity. My field plot still shows the problem that H is not tangent to the square edge.

What does the web say about the mag field of a square conductor? Maybe i should at least learn what the right answer is, then I will know where to look for my error.

Galvan Paper: They start with this equation

$$A = \frac{I\mu_0}{8\pi ab} \int_{-a}^{a} \int_{-b}^{b} \log r \, dx' \, dy' \qquad (1)$$

which I think is my 2D vector potential formula for rectangular conductor with a uniform current. This is a 2D version of an equation I usually show in 3D. I do have this in Appendix I

$-\nabla_{2D}^2 f(\mathbf{x}) = s(\mathbf{x})$ => $f(\mathbf{x}) = \int d^2x' \, [\ln(1/R)/2\pi] \, s(\mathbf{x}')$ + homogeneous solutions

The Poisson Equation (I.1.8)

I do have in (1.5.4),

$(\nabla^2 + \beta^2)\mathbf{A} = -\Sigma_i \, \mu_i \mathbf{J}_i$ all of region R (1.5.4)

and then at ω = 0 this becomes

$$(\nabla^2)\mathbf{A} = -\Sigma_i \mu_i \mathbf{J}_i$$

and then for one wire and with the 2D assumption it becomes

$$-(\nabla_{2D}^2)\mathbf{A} = \mu\mathbf{J}$$

and then the solution is

$$\mathbf{A} = + \int d^2x' [\ln(1/R)/2\pi] \mu\mathbf{J}(\mathbf{x}') + \text{homogeneous solutions}$$

and then if J = I/4ab this says

$$A_z = \mu I/(8\pi ab) \int d^2x' [\ln(1/R)]$$

or

$$A_z = -\mu I/(8\pi ab) \int d^2x' [\ln(R)]$$

or

$$A_z = -\mu I/(16\pi ab) \int d^2x' [\ln(R^2)]$$

So this agrees with their equation apart from sign. I will ignore this sign issue, does not matter much for what I am doing. Then they compute this A_z for rectangle and get,

$$A=\frac{I\mu_0}{16\pi ab}\int_{-a}^{a}\int_{-b}^{b}\log[(x'-x)^2+(y'-y)^2]dx'dy' \quad (2)$$

Using Strutt simplification, (2) becomes,

$$\begin{aligned}A=\frac{I\mu_0}{16\pi ab}\{&(a-x)(b-y)\log[(a-x)^2+(b-y)^2]+\\&+(a+x)(b-y)\log[(a+x)^2+(b-y)^2]+\\&+(a-x)(b+y)\log[(a-x)^2+(b+y)^2]+\\&+(a+x)(b+y)\log[(a+x)^2+(b+y)^2]+\\&+(a-x)^2\left[\tan^{-1}\left(\frac{b-y}{a-x}\right)+\tan^{-1}\left(\frac{b+y}{a-x}\right)\right]+\\&+(a+x)^2\left[\tan^{-1}\left(\frac{b-y}{a+x}\right)+\tan^{-1}\left(\frac{b+y}{a+x}\right)\right]+\\&+(b-x)^2\left[\tan^{-1}\left(\frac{a-x}{b+y}\right)+\tan^{-1}\left(\frac{a+x}{b-y}\right)\right]+\\&+(b-y)^2\left[\tan^{-1}\left(\frac{a-x}{b-y}\right)+\tan^{-1}\left(\frac{a+x}{b+y}\right)\right]\}\end{aligned}$$

…(3)

This certainly has functions similar to my expression say for H_x. The phrase " "Strutt simplification" has only their paper as a web hit! They then end up with a form of the result in terms of angles

$$H_x=\frac{1}{8\pi ab}\left[w(\theta_1-\theta_2)+u(\theta_4-\theta_3)+v\log\frac{r_2}{r_3}+t\log\frac{r_1}{r_4}\right]$$

…(6)

$$H_y=\frac{1}{8\pi ab}\left[v(\theta_2-\theta_3)+t(\theta_1-\theta_4)+w\log\frac{r_2}{r_1}+u\log\frac{r_3}{r_4}\right]$$

…(7)

The magnitude of the magnetic field intensity is given by:

$$H_{TOTAL}=\sqrt{H_x^2+H_y^2} \quad (8)$$

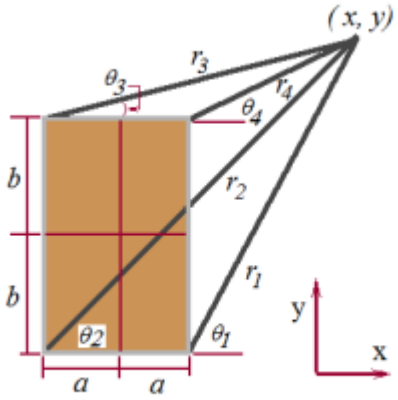


Figure 2: Geometry of the rectangular conductor for the calculation of H_x and H_y

What is the meaning of A_z contours as they plot? In electrostatics the E field is perp to the φ contours, but I don't know what that means for the B case. I see this paper is mainly about using a software package, so enough.

Web site: http://www.ntmdt.com/spm-basics/view/magnetic-field-rectangular-wire

They treat the rectangular conductor as a set of thin wires and use superposition. He has x,z intead of x,y and he uses h = 2b and 2 = 2a for transverse and he finds that

$$\tilde{H}_x(p_0,z_0)=\frac{\gamma}{2}\left[p_0\ln\left(1+\frac{h^2+2z_0h}{p_0^2+z_0^2}\right)+2(z_0+h)\arctan\left(\frac{p_0}{z_0+h}\right)-2z_0\arctan\left(\frac{p_0}{z_0}\right)\right]$$

$$\tilde{H}_z(p_0,z_0)=\frac{\gamma}{2}\left[z_0\ln\left(1+\frac{p_0^2}{z_0^2}\right)-(z_0+h)\ln\left(1+\frac{p_0^2}{(z_0+h)^2}\right)-2p_0\left(\arctan\left(\frac{z_0+h}{p_0}\right)-\arctan\left(\frac{z_0}{p_0}\right)\right)\right] \quad (5)$$

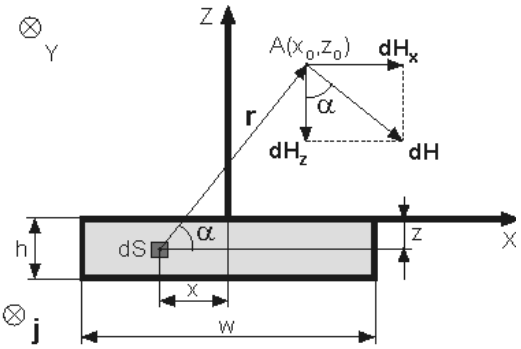
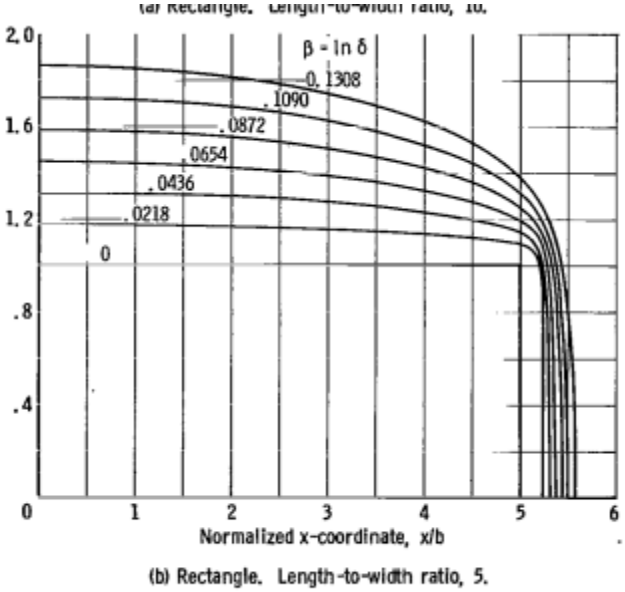


Fig. 1. Cross-section of rectangular conductor.

His results look similar to mine in the nature of the terms, but he makes no plot of H. I cannot seem to find a plot anywhere.

NASA Article

It has this interesting plot supporting my parallel claim: but this assumes extreme skin effect!



(b) Rectangle. Length-to-width ratio, 5.

Figure 3. - Lines of constant magnetic potential around corner.

How come I don't have Biot-Savart anywhere? ******* [I later added this I think in App B] This gives the B field directly from the current in theory.

I am amazed that I cannot find any plots of the type I made, so I am blocked from knowing the correct answer. Let's look at texts:

Bleaney. Nada.

Purcell. He has a picture on page 209 that is like my plot and does NOT have B parallel to the surfaces of his two busbars. So that is pretty convincing that my claim of B being purely tangent is wrong! Maybe that is only true at high ω [correct] . My perp proof is just above (3.7.1), I wish I had given all my Facts equation numbers *********. Here is my proof:

Fact 4: In a cross sectional sketch of a transmission line, the **E** and **B** field lines are perpendicular at every point in the dielectric.

Proof: Recall these two equations from earlier sections

$$\text{curl } \mathbf{B} = \mu\ (j\omega\varepsilon\ \mathbf{E} + \mathbf{J}\ ) = \mu\ (j\omega\varepsilon\ \mathbf{E} + \sigma\mathbf{E}\ ) = \mu(j\omega\varepsilon + \sigma)\mathbf{E} \qquad (2.2.1)$$

$$\beta^2 = \omega^2\ \mu\ [\ \varepsilon + \sigma/j\omega] = \omega^2\ \mu(\ j\omega\varepsilon + \sigma)\ /j\omega = \omega\mu(\ j\omega\varepsilon + \sigma)\ /j \quad . \qquad (1.5.1)$$

If follows from these equations that,

$$\text{curl } \mathbf{B} = j(\beta^2/\omega)\ \mathbf{E} \equiv C\ \mathbf{E}\ . \qquad (3.7.1)$$

$$C = j(\beta^2/\omega) = j\ \mu(\ j\omega\varepsilon + \sigma)\ /j = \mu(\ j\omega\varepsilon + \sigma)$$

To show that the **E** and **B** fields are perpendicular, we will show that **E•B** = 0. We have from (3.7.1),

$$C\ \mathbf{E}\bullet\mathbf{B} = \text{curl}\ \mathbf{B} \bullet \mathbf{B} = (\partial_x B_y - \partial_y B_x)B_z + (\partial_y B_z - \partial_z B_y)B_x + (\partial_z B_x - \partial_x B_z)B_y\ .$$

Since $B_z \approx 0$ (see estimate in previous section), we are left with only two terms

$$C\ \mathbf{E}\bullet\mathbf{B} = B_y(\partial_z B_x) - B_x(\partial_z B_y) = B_y^2\ \partial_z\ (B_x/B_y)\ .$$

However, we argued in Fact 3 that the shape of fields does not vary with z. Thus, the ratio of two components like B_x/B_y cannot vary with z. Thus, **E•B** = 0 so the **E** and **B** lines are perpendicular everywhere in the dielectric. (We also assume $E_z \approx 0$ since it is small in the dielectric).

Now of course in the Purcell case, we are just looking at a DC pair of busbars, there is no electric field at all anywhere. We are not looking at a TEM wave on a transmission line I guess except as $\omega \to 0$.

Idea #1. Maybe at very low ω, we still have large J_z but E_r is very small.

Idea #2. Where have I stated the **E** and **B** fields for a transmission line? I have given expressions for $\varphi_{12}(x)$ at any point in the dielectric in (4.2.1). At one point I have this

$$\varphi_{12}(\mathbf{x}) = \frac{1}{4\pi\xi}\int_{-\infty}^{\infty} dz'\ q(z')\{\ \int_{C_1} dx_1'\ dy_1'\ \alpha_1(x_1',y_1')\frac{1}{R_1} - \int_{C_2} dx_2'\ dy_2'\ \alpha_2(x_2',y_2')\frac{1}{R_2}\ \} \qquad (4.3.7)$$

But then I never pursue this further. I go right to $V(z) \equiv \varphi_{12}(\mathbf{x}_1) - \varphi_{12}(\mathbf{x}_2)$ and I eventually get

$$V(z) = q(z)\frac{1}{4\pi\xi}\ \{\ \int_{C_1} dx_1'\ dy_1'\ \alpha_1(x_1',y_1')\ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2'\ dy_2'\ \alpha_2(x_2',y_2')\ln(s_{22}^2/s_{12}^2)\ \}$$

I go on to get the transmission line parameters, but I never compute the potentials or the fields IN the transmission line. That seems a major omission.

But maybe I do it in the transverse problem chapter. $\varphi(x,y,z) = \frac{1}{4\pi\xi}\ q(z)\ \varphi_t(x,y)$ and I then get a capacitor problem for φ_t and seemingly the same problem for A_{zt}. See (5.3.8) and (5.3.9).

Possible Issue Spotted: In my low less approximation I try to get rid of the Helmholtz parameter sitting in these equations

$$[\nabla_t^2 + (\beta^2-k^2)]\ \varphi_t(x,y) = 0 \qquad \varphi_t(C_1) = K_1 \quad \varphi_t(C_2) = K_2 \quad K_1 - K_2 = K \qquad (5.3.8)$$

$$[\nabla_t^2 + (\beta^2-k^2)]\ A_{zt}(x,y) = 0 \qquad A_{zt}(C_1) = W_1 \quad A_{zt}(C_2) = W_2 \quad W_1 - W_2 = K \qquad (5.3.9)$$

$$\text{where}\quad (\beta^2 - k^2) = \frac{j\omega\ Z_s\ 4\pi\xi}{K}\ .$$

I then make the "low loss assumption" which says

$|Z_s| << (1/4\pi)\,\omega\mu\,K$ $\qquad\qquad$ $\sec^{-1}$ henry/m $=$ ohm/m

BUT THEN I take the high ω limit of Z_s for the round wire to get $(\delta/a) << K/\sqrt{2}$.

But in the low ω limit, this is going to basically say

$$\left| \frac{1}{\sigma\pi a^2} + j\omega \frac{\mu}{8\pi} \right| << (1/4\pi)\,\omega\mu\,K$$

or really just

$$\frac{1}{\sigma\pi a^2} << (1/4\pi)\,\omega\mu\,K$$

or

$$\omega >> \frac{1}{\sigma\pi a^2}\ 4\pi\,(1/\mu)(1/K)$$

or

$\omega >> 4\pi\,R_{dc}/\mu\ \ (1/K)$.

so this does seem to put a lower limit on the validity of my solutions, something I have been ignoring.

Let's look at the Belden cable again on this issue. We have a = 394 μ ans then

$$R_{dc} = \frac{1}{\sigma\pi a^2}$$

$1/R_{dc} = \pi\sigma a^2 = \pi\ 5.81 \times 10^7 \text{mho/m} * (394)^2 * 10^{-12}$

$= \pi\ 5.81\ (394)^2 10^{-5}\ = 28.3$ m/ohm

So then

$R_{dc} = .0353$ ohms/m

```
> evalf(Pi*5.81*394*394*1e-5);
                                28.33468891
```

Then we find for μ_0 that we require

$\omega >> 4\pi * .0353 * 1/(4\pi x 10^{-7})* (1/K)$

or

$2\pi f >> .0353 \times 10^7\ (1/K)$

or

$f >> .0056 * 10^7* (1/K) = .056 * 10^6* (1/K) = 5.6 \times 10^3\ (1/K) = (1/K)* 5.6$ KHz

So if you operate near DC, then you do *not* have f >> (1/K)* 5.6 KHz and you are then *not* in the low loss limit of a transmission line. That means you can*not* treat the transverse equations for φ_t and A_{zt} as simple Laplace equations, and that means the "capacitor problem solution" is *not* valid.

So would this invalidate the idea that **E** is perp to the surface transversely speaking? I think that would remain valid because there is no strong E_φ field to make this not be true.

So even at 1 Hz, I think there exists some φ and there is some n and **E** is perp to the conductor surfaces in the transverse picture. Now once again, let us examine the E perp B argument,.

Proof: Recall these two equations from earlier sections

$$\text{curl } \mathbf{B} = \mu\ (j\omega\varepsilon\ \mathbf{E} + \mathbf{J}\ ) = \mu\ (j\omega\varepsilon\ \mathbf{E} + \sigma\mathbf{E}\ ) = \mu(j\omega\varepsilon + \sigma)\mathbf{E} \qquad (2.2.1)$$

$$\beta^2 = \omega^2\ \mu\ [\ \varepsilon + \sigma/j\omega] = \omega^2\ \mu(\ j\omega\varepsilon + \sigma)\ /j\omega = \omega\mu(\ j\omega\varepsilon + \sigma)\ /j \quad . \qquad (1.5.1)$$

Now of ω = 1 Hz, then $\omega\varepsilon_0 \sim 10^{-11}$ and perhaps $\sigma = 10^{-15}$ for the dielectric, so we have

$$\text{curl } \mathbf{B} = 4\pi \text{ x } 10^{-7} * 10^{-11}\ \mathbf{E} = 10^{-17}\mathbf{E}$$

Of course in magnetostatics you take curl B = 0 and then $\mathbf{B} = -\mu_0\nabla\varphi$ as in Bleaney p 132 and this then works because curl grad = 0. In electrostatics you have curl **E** = 0 and then E = -∇φ, so electrostatics and magnetostatics are pretty symmetric in this regard.

Now back to the proof. I would then argue that

$$\mathbf{E} \bullet \mathbf{B} = 10^{17} \text{ curl } \mathbf{B} \bullet \mathbf{B}$$

$$= 10^{17}\ \{\ (\partial_x B_y - \partial_y B_x)B_z + (\ \partial_y B_z - \partial_z B_y)B_x + (\ \partial_z B_x - \partial_x B_z)B_y\ .\ \}$$

Notice the huge amplifying factor 10^{17} sitting there. The next step is to say $B_z = 0$ exactly. But in my Appendix D analysis of a round wire, I found that $B_z \neq 0$. But let's look again at (D.4.9).

$$\beta' = \beta^2 - \beta_d^2$$

At DC both β and β_d are 0 so I guess in that case App D does say $B_z = 0$, and it does say that for m = 0 which is where the DC limit lives. But even at 1 Hz, there will be some $B_z \neq 0$ and I guess that 10^{17} could them amplify it and make it enter into the calculation and then it would be wrong to ignore B_z. I am just guessing that is how this works. SO yes, this is a very subtle issue that I completely ignored when I stated this "fact".

[So finally I arrived here at the cause of the problem. It is all written up in lines doc now. It took me a long time to realize what was going on here, that 10^{17} was the first clue.]

Question: Looking at the m=0 results (D.6.1), what can you say about the relative size of E_r and B_φ ? (these are the two strong field components). Well, these are inside the wire, but I want to know E_r just outside the wire, so this is not going to help.

I should somewhere add a section on "low frequency characteristics of a transmission line" . I think I could show that Z_0 is very small, so $V = Z_0 I$ means V and this E_r is very small in the dielectric relative say to the B field, and then the $E \bullet B = 0$ argument changes its nature. But somehow V is very large on a power line, so what gives there?

I read up on this. Z_0 = 400 ohms typical, it is in the highω limit for Z_0 formula even at 60 Hz!

Now how might I *qualify* my $\mathbf{E} \bullet \mathbf{B} = 0$ theorem? Look once again at the derivation:

curl $\mathbf{B}$ = C$\mathbf{E}$ where $C = \mu(j\omega\varepsilon + \sigma)$ where μ and σ are for the dielectric.