Crazy idea on J_m and the homo solutions PhL 1.15.14

This crazy idea finally worked and is installed as the J_m Theorem in Appendix B. 6 (b)

Conjectured Theorem: If you add $\mathbf{J_m}$ to your $\mathbf{J_c}$, the extra $\mathbf{J_m}$ integral gives exactly the right homo solutions to make the A_z boundary conditions match.

Attempted Proof: Start with Appendix C.4's starting point

$$A_z(\mathbf{x}) = \frac{1}{4\pi} \int dV' \, [\mu_2 J_{cz}(\mathbf{x}') + \mu_1 J_{mz}(\mathbf{x}')] \frac{1}{R} \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad \omega = 0 \qquad (1.5.9)$$

$$= A_z^{(c)} + A_z^{(m)}$$

Why do I put μ_1 with J_m ? I guess I think of it as a "current in the dielectric". A pretty weak argument. I don't have any examples of this theorem currently sitting anywhere in lines doc, but I did do it for round conductor somewhere which made me think it might be a general theorem.

1. I know that the **"conduction current term"** has this property at the boundary between μ_1 and μ_2:

$$[A_z^{(c)}]^1 = [A_z^{(c)}]^2 \qquad 1 = \text{dielectric} = \text{outside wire}$$

$$[\partial_n A_z^{(c)}]^1 = [\partial_n A_z^{(c)}]^2$$

Therefore I know that

$$\frac{1}{\mu_1} [\partial_n A_z^{(c)}]^1 - \frac{1}{\mu_2} [\partial_n A_z^{(c)}]^2 = [\frac{1}{\mu_1} - \frac{1}{\mu_2}][\partial_n A_z^{(c)}]$$

This is the quantity that we want the J_m term to cancel out.

2. The **"magnetization current term"** is this

$$A_z^{(m)} = \mu_1 \int dV' \, J_{mz}(\mathbf{x}')] \frac{1}{4\pi R}$$

where $J_{mz}(\mathbf{x}')$ is confined to the conductor surface and this has the usual $(-\infty,\infty)$ dz' integral. The mag current density is in the form of a 3D density of amps/m^2.

However, with the usual set of assumptions I can replace this 3D integral and propagator with as 2D integral and propagator as follows:

$$A_z^{(m)} = \mu_1 \int dA' \, J_{mz}(\mathbf{x}')] \, [- \frac{1}{2\pi} \ln(R)]$$

Now since we know that $J_{mz}(\mathbf{x}')$ is confined to the 2D perimeter in this 2D picture, we then write

$$A_z^{(m)} = \mu_1 \int_C ds' K_{mz}(\mathbf{x}')] [- \frac{1}{2\pi} \ln(R)]$$

where we have now "taken out the delta function" so K is amps/m. I showed in Appendix B that

$$K_z = - (\mu_2/\mu_1 - 1) H_\theta \qquad \text{where } H_\theta > 0 \text{ and } K_z \leq 0.$$

so we then have

$$A_z^{(m)} = - (\mu_2-\mu_1) \int_C ds' H_\theta(\mathbf{x}')] [- \frac{1}{2\pi} \ln(R)]$$

$$= \int_C ds' [(\mu_1-\mu_2) H_\theta(\mathbf{x}')] [- \frac{1}{2\pi} \ln(R)]$$

$$= \int_C ds' a(\mathbf{x}')] E(x|x') . \qquad a(\mathbf{x}') = (\mu_1-\mu_2) H_\theta(\mathbf{x}')$$

3. Now recall from Stakgold these facts

$$u(\mathbf{x}) = \int_\sigma dS_\xi \, a(\boldsymbol{\xi}) E(x|\xi)$$

$$u(\mathbf{s}) = \int_\sigma dS_\xi \, a(\boldsymbol{\xi}) E(s|\xi)$$

$$\partial_\nu u(\mathbf{x}) = \int_\sigma dS_\xi \, a(\boldsymbol{\xi}) \partial_\nu E(x|\xi) \qquad \partial_\nu = \text{normal to surface } \sigma = \text{I write as } \partial_n$$

$$\partial_\nu u(\mathbf{s}) = [\partial_\nu u(\mathbf{x})]^{x=s+} = \int_\sigma dS_\xi \, a(\boldsymbol{\xi}) \partial_\nu E(x|\xi) - a(\mathbf{s})/2 \qquad \text{// the extra term!}$$

where σ is a surface of interest. We take now this surface to be our perimeter and then E is the 2D E. Thus we expect to find that

$$[\partial_n A_z^{(m)}(\mathbf{x})]^1 = \int_C ds' [(\mu_1-\mu_2) H_\theta(\mathbf{x}')] \partial_n E(\mathbf{x}|\mathbf{x}') - [(\mu_1-\mu_2) H_\theta(\mathbf{x})]/2 \quad (**)$$

$$[\partial_n A_z^{(m)}(\mathbf{x})]^2 = \int_C ds' [(\mu_1-\mu_2) H_\theta(\mathbf{x}')] \partial_n E(\mathbf{x}|\mathbf{x}') + [(\mu_1-\mu_2) H_\theta(\mathbf{x})]/2$$

Now compute **our "correction term"** for the difference boundary condition,

$$\frac{1}{\mu_1} [\partial_n A_z^{(m)}(\mathbf{x})]^1 - \frac{1}{\mu_2} [\partial_n A_z^{(m)}(\mathbf{x})]^2$$

$$= \{ \int_C ds' \, [(\mu_1-\mu_2)\, H_\theta(\mathbf{x}')] \, \partial_n E(\mathbf{x}|\mathbf{x}') \, [\frac{1}{\mu_1} - \frac{1}{\mu_2}] \;\; - [(\mu_1-\mu_2)\, H_\theta(\mathbf{x})] \, [\frac{1}{\mu_1} + \frac{1}{\mu_2}]/2$$

$$= (\mu_1-\mu_2) \, [\frac{1}{\mu_1} - \frac{1}{\mu_2}] \, \int_C ds' \, H_\theta(\mathbf{x}')] \, \partial_n E(\mathbf{x}|\mathbf{x}') \;\; - (1/2) \, (\mu_1-\mu_2) \, [\frac{1}{\mu_1} + \frac{1}{\mu_2}] \, H_\theta(\mathbf{x})$$

This then is our candidate correction term which we want to cancel the original term. I am not sure how that is going to happen! The original was this:

$$\frac{1}{\mu_1} [\partial_n A_z^{(c)}]^1 \; - \; \frac{1}{\mu_2} [\partial_n A_z^{(c)}]^2 \;\; = \; [\frac{1}{\mu_1} - \frac{1}{\mu_2}][\partial_n A_z^{(c)}]$$

Thus this is what I WANT to be true (add above two equations and hope for 0)

$$(\mu_1-\mu_2) \, [\frac{1}{\mu_1} - \frac{1}{\mu_2}] \, \int_C ds' \, H_\theta(\mathbf{x}')] \, \partial_n E(\mathbf{x}|\mathbf{x}') \;\; - (1/2) \, (\mu_1-\mu_2) \, [\frac{1}{\mu_1} + \frac{1}{\mu_2}] \, H_\theta(\mathbf{x}) + [\frac{1}{\mu_1} - \frac{1}{\mu_2}][\partial_n A_z^{(c)}] = 0$$

The μ factors don't look right, but let's plough ahead . Write

$$[\frac{1}{\mu_1} - \frac{1}{\mu_2}] \;\; = (\mu_2 - \mu_1)/(\mu_1\mu_2) \;\; = - \, (\mu_1-\mu_2) \, /(\mu_1\mu_2)$$

on the far right only, then cancel one factor $(\mu_1-\mu_2)$ to get

$$[\frac{1}{\mu_1} - \frac{1}{\mu_2}] \, \int_C ds' \, H_\theta(\mathbf{x}')] \, \partial_n E(\mathbf{x}|\mathbf{x}') \;\; - (1/2) \, [\frac{1}{\mu_1} + \frac{1}{\mu_2}] \, H_\theta(\mathbf{x}) - \frac{1}{\mu_1\mu_2} [\partial_n A_z^{(c)}] = 0$$

Further simplify to

$$(\mu_2 - \mu_1) \, \int_C ds' \, H_\theta(\mathbf{x}')] \, \partial_n E(\mathbf{x}|\mathbf{x}') \;\; - (1/2) \, (\mu_2+\mu_1) \, H_\theta(\mathbf{x}) - [\partial_n A_z^{(c)}] = 0 \qquad (*) \;\; ??$$

Once again, I am trying to show that (*) is true, I have not shown it is true! I do know from (**) that

$$\int_C ds' \, [(\mu_1-\mu_2) \, H_\theta(\mathbf{x}')] \, \partial_n E(\mathbf{x}|\mathbf{x}') \;\; = [\partial_n A_z^{(m)}(\mathbf{x})]^1 \;\; + \;\; [(\mu_1-\mu_2) \, H_\theta(\mathbf{x})]/2$$

so then here is what I want to show is true:

$$-[\partial_n A_z^{(m)}(\mathbf{x})]^1 \;\; - \;\; [(\mu_1-\mu_2) \, H_\theta(\mathbf{x})]/2 - (1/2) \, (\mu_2+\mu_1) \, H_\theta(\mathbf{x}) - [\partial_n A_z^{(c)}] = 0 \qquad ??$$

or

$$[\partial_n A_z^{(m)}(\mathbf{x})]^1 \;\; + \;\; [(\mu_1-\mu_2) \, H_\theta(\mathbf{x})]/2 + (1/2) \, (\mu_2+\mu_1) \, H_\theta(\mathbf{x}) + [\partial_n A_z^{(c)}] = 0 \qquad ??$$

or

$$[\partial_n A_z^{(m)}(\mathbf{x})]^1 \;\; + \;\; \mu_1 \;\; H_\theta(\mathbf{x}) + \; [\partial_n A_z^{(c)}] = 0 \qquad ??$$

Now how is H_θ related to A_z ? If we define a little local cylindrical coordinate system, then we know that

$$\mathbf{B} = \text{curl } \mathbf{A} = \hat{\mathbf{r}} \left[r^{-1}\partial_\theta A_z - \partial_z A_\theta \right] + \hat{\boldsymbol{\theta}} \left[\partial_z A_r - \partial_r A_z\right] + \hat{\mathbf{z}} \left[r^{-1}\partial_r(rA_\theta) - r^{-1}\partial_\theta A_r \right.$$

$$= \hat{\mathbf{r}} \left[r^{-1}\partial_\theta A_z\right] + \hat{\boldsymbol{\theta}} \left[- \partial_r A_z\right]$$

For a circular boundary this would be just $\mathbf{B} = \hat{\boldsymbol{\theta}} \left[- \partial_r A_z\right]$ since A_z is constant on the surface. Let us now **conjecture** that for a general perimeter we have

$$\mathbf{B} = \hat{\boldsymbol{\theta}} \left[- \partial_n A_z \right]$$

where $\hat{\boldsymbol{\theta}}$ is a local tangent vector as I often use. Then we could claim that

$$B_\theta = - \partial_n A_z \ .$$

What A_z is this? It should be the "true one" where BC's are met, so this says

$$B_\theta = - \partial_n A_z^{(c)} - \partial_n A_z^{(m)}$$

But is this inside or outside? I guess we have to say

$$B_\theta^1 = - [\partial_n A_z]^1$$
$$B_\theta^2 = - [\partial_n A_z]^2$$

Then we can write

$$H(\theta) = B_\theta^1/\mu_1 = B_\theta^2/\mu_2 \qquad \text{take your choice}$$

So I choose 1 and outside to get

$$H(\theta) = B_\theta^1/\mu_1 = -(1/\mu_1) \left[\partial_n A_z^{(c)} + [\partial_n A_z^{(m)}]^1\right]$$

or

$$\mu_1 H(\theta) = B_\theta^1 = - \left[\partial_n A_z^{(c)} + [\partial_n A_z^{(m)}]^1\right]$$

Now try this in our ?? equation:

$$[\partial_n A_z^{(m)}(\mathbf{x})]^1 + \mu_1 H_\theta(\mathbf{x}) + [\partial_n A_z^{(c)}] = 0 \qquad ??$$

$$[\partial_n A_z^{(m)}(\mathbf{x})]^1 - \left[\partial_n A_z^{(c)} + [\partial_n A_z^{(m)}]^1\right] + [\partial_n A_z^{(c)}] = 0 \qquad ??$$

$$0 = 0 \qquad !!!!!$$

Did I just go in a circle, or have I really succeeded here? Time is 8:13 AM 1.16.14.