More on Surface Current PhL 11.16.13

This is early work on the mag surface current issue. This eventually led to the new BC on $\partial_n Az$ to be included in Chapter 1, and this BC later became important for many reasons! I also first discovered the little formula for K_z surface current right here that became so important later on for the general theory.

1. I think I may have a bug. First, in $(1.1.9g)_m$ I make this claim, derived before your eyes:

curl $\mathbf{B} = \mu_0(\text{curl}\mathbf{H} + \text{curl}\mathbf{M}) = \mu_0(\partial_t\mathbf{D} + \mathbf{J}_c) + \mu_0\mathbf{J}_m$

$= \mu_0(\partial_t\mathbf{D} + \mathbf{J}_c + \mathbf{J}_m)$ line 1 " **B** sees all currents" $(1.1.9g)_m$

[now (1.1.24)]

But then in (1.1.24) [this is (1.1.1)] I state basically that

curl $\mathbf{H} = \partial_t\mathbf{D} + \mathbf{J}_c$ => curl $(\mathbf{B}/\mu) = \partial_t\mathbf{D} + \mathbf{J}_c$ =>

curl $\mathbf{B} = \mu\,\partial_t\mathbf{D} + \mu\mathbf{J}_c$ line 2 [this seems correct] (1.1.24)

Suppose we are doing statics so $\partial_t\mathbf{D} = 0$. Then I have from the above two lines,

curl $\mathbf{B} = \mu_0\,\mathbf{J}_c + \mu_0\mathbf{J}_m$ // line 1

curl $\mathbf{B} = \mu\mathbf{J}_c$ // line 2 [both OK]

How can these both be valid? If they are valid, we are claiming that

$\mu\mathbf{J}_c = \mu_0\mathbf{J}_c + \mu_0\mathbf{J}_m$

$\mathbf{J}_c\,(\mu-\mu_0) = \mu_0\mathbf{J}_m$

$\mathbf{J}_m = (\mu/\mu_0 - 1)\mathbf{J}_c$ [this still seems right on 2/9/14, inside a medium of μ]

This says mag current is determined in this way by conduction current? Is this really true? Well, let's test it

$\mathbf{J}_m = (\mu/\mu_0 - 1)\mathbf{J}_c$

curl $\mathbf{M} = (\mu/\mu_0 - 1)$ curl $\mathbf{H}$

$\mathbf{M} = (\mu/\mu_0 - 1)\mathbf{H}$

$\chi_m\,\mathbf{H} = (\mu/\mu_0 - 1)\mathbf{H}$

$\chi_m = (\mu/\mu_0 - 1)$

$1 + \chi_m = \mu/\mu_0$ and this is correct !!!

So OK, there is no bug after all. [I think it is all correct, away from boundaries]

2. I want to redo the H BC if there is a surface current?

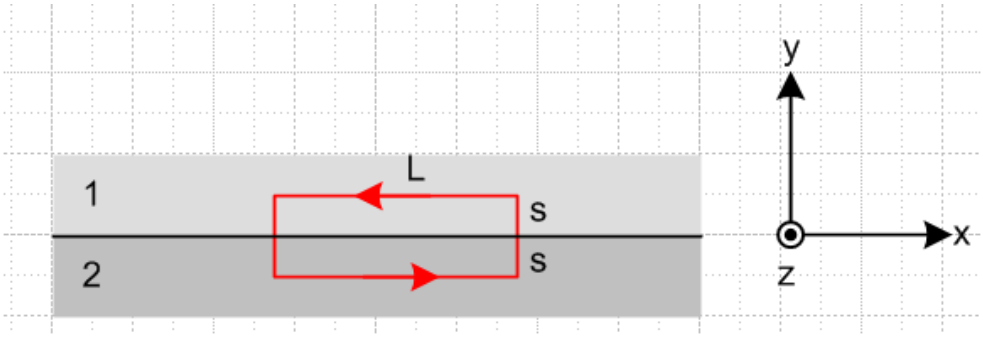
I can work with either of the forms above:

curl $\mathbf{B} = \mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$

curl $\mathbf{B} = \mu\, \partial_t \mathbf{D} + \mu \mathbf{J_c}$

The second form is not useful since I won't be able to "capture" the surface current $\mathbf{J_m}$ with it, so use the first equation (even though both are valid)

Now consider this surface geometry



Suppose there is a surface current flowing toward the viewer,

$\mathbf{J_m}(x) = \delta(y)\mathbf{K}(x) = J_m(x)\, \hat{\mathbf{z}}$

Start with the result above

curl $\mathbf{B} = \mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$

curl $\mathbf{B} = \mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m}) \quad \Leftrightarrow \quad \oint \mathbf{B} \bullet d\mathbf{s} = \int_S [\mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m})] \bullet d\mathbf{A}$

Then

LHS = $(B_{2x} - B_{1x})L$

And if the $\mathbf{D}$ and $\mathbf{J_c}$ are non-singular at the boundary, we just have

$$\text{RHS} = \mu_0 \int_S \mathbf{J_m} \bullet d\mathbf{A} = \mu_0 \int_S \delta(y)\mathbf{K}(\mathbf{x}) \bullet d\mathbf{A} = \mu_0 L\, K_z$$

This seems to say that

$$\mu_0 K_z = B_{2x} - B_1$$

How does this compare with my Appendix B claim?

$$\mu_0 K_z = (B_{x2} - B_{x1})$$ [this is the current Appendix B claim]

Question: Why did I get μ_1 on the left in Appendix B ? [an earlier version of App B]

Answer: In App B I quoted (1.1.24) correctly, but in (1.1.24) the **J** there does not include $\mathbf{J_m}$. since the rule is originating from curl **H** = $\partial_t \mathbf{D} + \mathbf{J}$ which does not see $\mathbf{J_m}$. But then in App B I wrongly say that this **J** does include $\mathbf{J_m}$.

Question: If the correct μ is μ_0 in the K formula, how does this alter later sections of App B?

Now consider:

$$\mathbf{B} = \text{curl } \mathbf{A} \Rightarrow \quad B_x = \partial_y A_z - \partial_z A_y$$

Then we have shown that

$$\mu_0 K_z = (B_{x2} - B_{x1}) = (\partial_y A_{z2} - \partial_z A_{y2}) - (\partial_y A_{z1} - \partial_z A_{y1})$$

$$= [\partial_y A_{z2} - \partial_y A_{z1}] - [\partial_z A_{y2} - \partial_z A_{y1}]$$

Since y is the normal direction, if we could somehow ignore the second term this says

$$\mu_0 K_z = \partial_n A_{z2} - \partial_n A_{z1}$$

and this at least resembles something I quoted long ago from the web

$$\frac{\partial \vec{A}_2}{\partial n} - \frac{\partial \vec{A}_1}{\partial n} = -\mu_o \vec{K}$$

// I finally made sense of this early finding

Now suppose we knew that $\mathbf{A} = A_z \hat{\mathbf{z}}$ in both regions. Then *my* result would say

$$\mu_0 K_z = \partial_n A_{z2} - \partial_n A_{z1}$$

$$\mu_0 K_z \hat{\mathbf{z}} = (\partial_n A_{z2})\, \hat{\mathbf{z}} - (\partial_n A_{z1})\, \hat{\mathbf{z}}$$

Now $\partial_n \hat{\mathbf{z}} = 0$, so this says

$$\mu_0 \mathbf{K} = \partial_n \mathbf{A_1} - \partial_n \mathbf{A_2}$$

and this agrees exactly with the quoted result.

Can we show that in general $[\partial_z A_{y2} - \partial_z A_{y1}] = 0$? That is to say

$$\partial_z (A_{y2} - A_{y1}) = 0 \text{ ?}$$

If we are in some situation where everything varies slowly in the z direction (transmission line limit?), then we could say that $\partial_z A_{y2} \approx 0$ and $\partial_z A_{y1} \approx 0$ separately, and then this is true. Another case would be where we argue that we can neglect A_y.

Is it possible that $A_{y2} - A_{y1} = 0$ all the time? In our King gauge for all of region R we have

$$\text{div } \mathbf{A} = - \mu_1 \varepsilon_1 \partial_t \varphi - \mu_1 \sigma_1 \varphi \qquad \text{// applied to all of R .} \qquad (1.3.18)$$

$$\text{div } \mathbf{A} = - \mu_1 \varepsilon_1 j\omega \varphi - \mu_1 \sigma_1 \varphi \qquad \text{// applied to all of R .} \qquad (1.3.18)$$

$$\text{div } \mathbf{A} = [- \mu_1 \varepsilon_1 j\omega - \mu_1 \sigma_1] \varphi = -j\omega \mu_1 \xi_1 \varphi \qquad \text{// applied to all of R .} \qquad (1.3.18)$$

If we apply the divergence theorem we get

$$\text{div } \mathbf{E} = \rho/\varepsilon \quad \Leftrightarrow \quad (1/\varepsilon) \int_v \rho \, dV = \int_s \mathbf{E} \bullet d\mathbf{S} \quad \text{if } \varepsilon \text{ is constant in space} \qquad (1.1.19)$$

$$\text{div } \mathbf{A} = -j\omega \mu_1 \xi_1 \varphi \quad \Leftrightarrow \quad \int_v -j\omega \mu_1 \xi_1 \varphi \, dV = \int_s \mathbf{A} \bullet d\mathbf{S}$$

$$\Leftrightarrow \quad -j\omega \mu_1 \xi_1 \left[\int_v \varphi \, dV \right] = \int_s \mathbf{A} \bullet d\mathbf{S}$$

Now if φ is non-singular and continuous at the boundary, then $\int_v \varphi \, dV = 0$. Then we really can conclude that

$$A_{y2} - A_{y1} = 0$$

and

$$A_{y1} = A_{y2}$$

Wow. This has to be added to the arsenal somewhere! It is King gauge specific, or Lorenz gauge, either one. Coulomb gauge as well!

What about continuity of A_z ? In that case we want curl **A** appearing somewhere, so try this

$$\text{curl } \mathbf{A} = \mathbf{B} \qquad \Leftrightarrow \qquad \oint \mathbf{A} \bullet d\mathbf{s} \; = [\int_S \mathbf{B} \bullet d\mathbf{A}]$$

The left side will then pick up $A_{1t} - A_{t2}$. Then if B is finite at the surface, you have it! Wow!