More on the μ fix PhL 2.5.14

This is an earlier version of what is now called the J_m theorem. The essential proof succeeds right here. I then rewrote it a bit, and it is all installed.

In Lines Appendix B, I somewhat lose the thread of how the surface current J_m is supposed to "repair" things for a transmission line, so maybe I can bail it out here. The main problem is that I did not treat more than one conductor so the result is hazy.

The starting point should really be an equation like

$$A_z(\mathbf{x}) = \frac{\mu_1}{4\pi}\int_{C_1} J_{z1}(\mathbf{x}_1')\frac{e^{-j\beta R1}}{R}\,d^3x_1' \;+\; \frac{\mu_2}{4\pi}\int_{C_2} J_{z2}(\mathbf{x}_2')\frac{e^{-j\beta R2}}{R}\,d^3x_2'$$

$$R_1 = |\mathbf{x} - \mathbf{x}_1'| \qquad R_2 = |\mathbf{x} - \mathbf{x}_2'|$$

where this is the total potential due to all conductors integrations. Then we consider the boundary condition at the surface first of conductor C_1 . How do we do the repair? If I trace through B.7 I would start going just 1/R factors. Then B.6.1 would say

$$-A_z(\mathbf{x}) = \int_{C_2} dA_2'\,[\mu_2\, J_{cz2}(\mathbf{x}_2') + \mu_0\; J_{mz2}(x_2')]\,\frac{\ln(R_2^2)}{4\pi}$$

$$+ \int_{C_3} dA_3'\,[\mu_3\, J_{cz3}(\mathbf{x}_3') + \mu_0\; J_{mz3}(x_3')]\,\frac{\ln(R_3^2)}{4\pi}$$

where the dielectric has μ_1 to be consistent with Appendix B, so the two conductors are now 2 and 3. I then have to restate the theorem more carefully. This form should fix the boundary conditions at both conductor surfaces.

I would then have (1) Preliminaries with no change.

Then I guess for section (2) I would have to have a break up into four pieces instead of 2:

$$A_z^{(c2)}(\mathbf{x}) = \frac{1}{4\pi}\int_{C_2}[\mu_2\, J_{cz2}(\mathbf{x}_2')]\,\frac{e^{-j\beta R2}}{R_2}\,dV_2' \;. \qquad R = |\mathbf{x} - \mathbf{x}_2'|$$

$$A_z^{(c3)}(\mathbf{x}) = \frac{1}{4\pi}\int_{C_3}[\mu_3\, J_{cz3}(\mathbf{x}_3')]\,\frac{e^{-j\beta R3}}{R_3}\,dV_3' \;. \qquad R = |\mathbf{x} - \mathbf{x}_3'|$$

$$A_z^{(m2)}(\mathbf{x}) = \frac{1}{4\pi}\int_{C_2}[\mu_0\, J_{mz2}(\mathbf{x}_2')]\,\frac{e^{-j\beta R2}}{R_2}\,dV_2' \;. \qquad R = |\mathbf{x} - \mathbf{x}_2'|$$

$$A_z^{(m3)}(\mathbf{x}) = \frac{1}{4\pi}\int_{C_3}[\mu_0\, J_{mz3}(\mathbf{x}_3')]\,\frac{e^{-j\beta R3}}{R_3}\,dV_3' \quad . \qquad R = |\mathbf{x} - \mathbf{x}_3'|$$

$$A_z(\mathbf{x}) = A_z^{(c2)}(\mathbf{x}) + A_z^{(m2)}(\mathbf{x}) + A_z^{(c3)}(\mathbf{x}) + A_z^{(m3)}(\mathbf{x})$$

Now I would *specialize* so that $\mathbf{x} \to \mathbf{s}$ which is a point on C_2. The claim is that each $A_z^{(ci)}(\mathbf{x})$ is smooth at such a point and we have

$$\begin{aligned} &A_z^{(ci)}(\mathbf{s}+) = A_z^{(ci)}(\mathbf{s}-) \\ &\partial_n A_z^{(ci)}(\mathbf{s}+) = \partial_n A_z^{(ci)}(\mathbf{s}-)\,. \qquad\qquad i = 2,\,3 \end{aligned} \tag{B.6.7}$$

Then for the sum of the two (c) terms I would write

$$\frac{1}{\mu_1}\,\partial_n[A_z^{(c2)}(\mathbf{s}+) + A_z^{(c3)}(\mathbf{s}+)] \;-\; \frac{1}{\mu_2}\,\partial_n[A_z^{(c2)}(\mathbf{s}-) + A_z^{(c3)}(\mathbf{s}-)]$$

$$= \;+\,[\,\frac{1}{\mu_1} - \frac{1}{\mu_2}\,]\,\partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})]. \tag{B.6.8}$$

where I could have used either s+ or s- on the right, since each is continuous at s on C_1. So this is the fancier version of (B.6.8). The goal is then to show that

$$\frac{1}{\mu_1}\,\partial_n[A_z^{(m2)}(\mathbf{s}+) + A_z^{(m3)}(\mathbf{s}+)] \;-\; \frac{1}{\mu_2}\,\partial_n[A_z^{(m2)}(\mathbf{s}-) + A_z^{(m3)}(\mathbf{s}-)]$$

$$= \;-\;[\,\frac{1}{\mu_1} - \frac{1}{\mu_2}\,]\,\partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})]\,. \tag{B.6.9}$$

So it then remains to demonstrate this new and fancier (B.6.9).

(3) Verification of (B.6.9). Now suppose I just regard this whole section as a applying to C_2 only, so imagine little "2" subscripts as needed. We just march down through this section thinking this way. In (B.6.18) we would then have

$$\partial_n A_z^{(m2)}(\mathbf{s}\pm) = \oint_{C2} ds_2'\,[(\mu_1-\mu_2)\,H_{\theta 2}(\mathbf{x}')]\,\partial_n E_2(\mathbf{s}|\mathbf{x}_2') \mp [(\mu_1-\mu_2)\,H_{\theta 2}(\mathbf{s})]/2 \quad .$$

$$\partial_n A_z^{(m3)}(\mathbf{s}\pm) = \oint_{C3} ds_3'\,[(\mu_1-\mu_3)\,H_{\theta 3}(\mathbf{x}')]\,\partial_n E_2(\mathbf{s}|\mathbf{x}_3') \tag{B.6.18}$$

The second line does not have the Stakgold "extra term" because as $\mathbf{x} \to \mathbf{s}$, since $\mathbf{s}$ lies on C_2 and not on C_3, we are not approaching the monopole later $H_{\theta 3}$ on C_3 so this singularity effect does not occur. Notice that each line is an integral over its own conductor with its own H_θ value.

The big equation after (B.6.18) is now even bigger:

$$\frac{1}{\mu_1}\partial_n[A_z^{(m2)}(\mathbf{s}+) + A_z^{(m3)}(\mathbf{s}+)] \;-\; \frac{1}{\mu_2}\partial_n[A_z^{(m2)}(\mathbf{s}-) + A_z^{(m3)}(\mathbf{s}-)]$$

$$= \frac{1}{\mu_1}\{ \oint_{C2} ds_2' [(\mu_1-\mu_2) H_{\theta 2}(\mathbf{x}_2')]\, \partial_n E_2(\mathbf{s}|\mathbf{x}_2') - [(\mu_1-\mu_2) H_{\theta 2}(\mathbf{s})]/2 \;\}$$
$$- \frac{1}{\mu_2}\{ \oint_{C2} ds_2' [(\mu_1-\mu_2) H_{\theta 2}(\mathbf{x}_2')]\, \partial_n E_2(\mathbf{s}|\mathbf{x}_2') + [(\mu_1-\mu_2) H_{\theta 2}(\mathbf{s})]/2 \;\}$$

$$+ \frac{1}{\mu_1}\{ \oint_{C3} ds_3' [(\mu_1-\mu_3) H_{\theta 3}(\mathbf{x}_3')]\, \partial_n E_2(\mathbf{s}|\mathbf{x}_3') \;\}$$
$$- \frac{1}{\mu_2}\{ \oint_{C3} ds_3' [(\mu_1-\mu_3) H_{\theta 3}(\mathbf{x}_3')]\, \partial_n E_2(\mathbf{s}|\mathbf{x}_3') \;\}$$

$$= (\mu_1-\mu_2)\; [\frac{1}{\mu_1} - \frac{1}{\mu_2}] \oint_{C2} ds_2' H_{\theta 2}(\mathbf{x}_2')\, \partial_n E_2(\mathbf{s}|\mathbf{x}_2') \;-\; [\frac{1}{\mu_1} + \frac{1}{\mu_2}] (\mu_1-\mu_2) H_{\theta 2}(\mathbf{s})/2$$

$$+ [\frac{1}{\mu_1} - \frac{1}{\mu_2}] (\mu_1-\mu_3) \oint_{C3} ds_3' H_{\theta 3}(\mathbf{x}_3')]\, \partial_n E_2(\mathbf{s}|\mathbf{x}_3')$$

where now we have an extra term on the last line which was not present before.

Our task of showing that (B.6.9) is true then boils down to showing that the last expression above is equal to $-\; [\frac{1}{\mu_1} - \frac{1}{\mu_2}]\, \partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})]$. I smell trouble brewing. I will first write this big equation:

$$(\mu_1-\mu_2)\; [\frac{1}{\mu_1} - \frac{1}{\mu_2}] \oint_{C2} ds_2' H_{\theta 2}(\mathbf{x}_2')\, \partial_n E_2(\mathbf{s}|\mathbf{x}_2') \;-\; [\frac{1}{\mu_1} + \frac{1}{\mu_2}] (\mu_1-\mu_2) H_{\theta 2}(\mathbf{s})/2$$

$$+ [\frac{1}{\mu_1} - \frac{1}{\mu_2}] (\mu_1-\mu_3) \oint_{C3} ds_3' H_{\theta 3}(\mathbf{x}_3')]\, \partial_n E_2(\mathbf{s}|\mathbf{x}_3') = \; - \; [\frac{1}{\mu_1} - \frac{1}{\mu_2}]\, \partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})] \quad ?$$

Cancelling $(\mu_1-\mu_2)$ factors gives

$$[\frac{1}{\mu_1} - \frac{1}{\mu_2}] \oint_{C2} ds_2' H_{\theta 2}(\mathbf{x}_2')\, \partial_n E_2(\mathbf{s}|\mathbf{x}_2') \;-\; [\frac{1}{\mu_1} + \frac{1}{\mu_2}] H_{\theta 2}(\mathbf{s})/2$$

$$- \frac{1}{\mu_1\mu_2} (\mu_1-\mu_3) \oint_{C3} ds_3' H_{\theta 3}(\mathbf{x}_3')]\, \partial_n E_2(\mathbf{s}|\mathbf{x}_3') = \; + \; \frac{1}{\mu_1\mu_2}\, \partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})] \qquad ?$$

or

$$(\mu_2-\mu_1) \oint_{C2} ds_2' H_{\theta 2}(\mathbf{x}_2')\, \partial_n E_2(\mathbf{s}|\mathbf{x}_2') \;-\; (\mu_2+\mu_1) H_{\theta 2}(\mathbf{s})/2$$

$$- (\mu_1-\mu_3) \oint_{C3} ds_3' H_{\theta 3}(\mathbf{x}_3')] \partial_n E_2(\mathbf{s}|\mathbf{x}_3') = + \partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})] \quad ? \quad \text{(B.6.19)}$$

Now we want to replace both integrals using (B.6.18) where we make the s+ choice

$$\partial_n A_z^{(m2)}(\mathbf{s}+) = \oint_{C2} ds_2' [(\mu_1-\mu_2) H_{\theta 2}(\mathbf{x}_2')] \partial_n E_2(\mathbf{s}|\mathbf{x}_2') - (1/2)[(\mu_1-\mu_2) H_{\theta 2}(\mathbf{s})$$

$$\partial_n A_z^{(m3)}(\mathbf{s}\pm) = \oint_{C3} ds_3' [(\mu_1-\mu_3) H_{\theta 3}(\mathbf{x}_2')] \partial_n E_2(\mathbf{s}|\mathbf{x}_3') \quad \textit{(B.6.18)+}$$

Our equation of interest (B.6.19) then becomes

$$- \partial_n A_z^{(m2)}(\mathbf{s}+) - (1/2)[(\mu_1-\mu_2) H_{\theta 2}(\mathbf{s}) - (1/2)(\mu_2+\mu_1) H_{\theta 2}(\mathbf{s})$$

$$- \partial_n A_z^{(m3)}(\mathbf{s}\pm) = + \partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})] \quad ?$$

or

$$- \partial_n A_z^{(m2)}(\mathbf{s}+) - \mu_1 H_{\theta 2}(\mathbf{s})$$

$$- \partial_n A_z^{(m3)}(\mathbf{s}\pm) = + \partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})] \quad ?$$

or

$$\partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s}) + A_z^{(m2)}(\mathbf{s}+) + \partial_n A_z^{(m3)}(\mathbf{s}\pm)] = - \mu_1 H_{\theta 2}(\mathbf{s})$$

or

$$\partial_n A_z(s) = - \mu_1 H_{\theta 2}(\mathbf{s}) \quad \mathbf{?}$$

But this equation is (B.6.23) in the language $C \rightarrow C_2$ on which point s lies. Thus, we can erase all the question marks!!!!!!!!

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Is there a faster way? Let's go back to the point where I have the "fancier B.6.9

$$\frac{1}{\mu_1} \partial_n[A_z^{(m2)}(\mathbf{s}+) + A_z^{(m3)}(\mathbf{s}+)] - \frac{1}{\mu_2} \partial_n[A_z^{(m2)}(\mathbf{s}-) + A_z^{(m3)}(\mathbf{s}-)]$$

$$= - [\frac{1}{\mu_1} - \frac{1}{\mu_2}] \partial_n[A_z^{(c2)}(\mathbf{s}) + A_z^{(c3)}(\mathbf{s})] . \quad \text{(B.6.9)}$$

I don't think I can claim that the "2" part of this is true on its own. The reason is the last step above where you have to add *all four* elements to get $H_{\theta 2}$. The philosophy is that we "force" J in the conductors, and that forces $H_{\theta 2}$ for example. Then the total A_z has to satisfy $\partial_n A_z(s) = -\mu_1 H_{\theta 2}(s)$.

OK, I first redid equation numbers in parts (1), (2) and (3) . Now I add section (c) and I will do this in a separate doc so I can copy from here. .