Return of the Mu Paradox PhL 1.7.14

The resolution of this came when I realized that Chapter 4 cannot ever proceed unless you assume that the two μ's are the same, conductor and dielectric. I made that global change in Chap 4, all is well.

I thought I had this problem under control, but it has returned today with a vengeance. Here is the paradox of today. It arises at the end of my rewritten Chapter 4 where I am trying to show how K_L and K are related. I find that $\frac{K_L}{K} = \frac{\mu}{\mu_c}$ and yet I know that this ratio is 1 for symmetric geometries and can have nothing to do with the μ ratio shown! So here is a trace-back on this problem:

1. I claimed that the King Helmholtz equation reads

$$\mathbf{A}(\mathbf{x},\omega) = \frac{1}{4\pi} \Sigma_i \int \mu_i \mathbf{J}_i(\mathbf{x}',\omega) \frac{e^{-j\beta R}}{R} dV'$$

where μ_i is for the material of conductor i. I really busted my ass to get this equation clarified in a clean way. [it is correct!] If all conductors have the same μ_i then this says

$$\mathbf{A}(\mathbf{x},\omega) = \frac{\mu_c}{4\pi} \Sigma_i \int \mathbf{J}_i(\mathbf{x}',\omega) \frac{e^{-j\beta R}}{R} dV'$$

where μ_c is for the conductor, not for the dielectric. [correct]

2. In my rewritten Chapter 4.10 I show that

$$L_e = \frac{W(z)}{i(z)} = \frac{\mu_c}{4\pi} K_L$$

$$K_L \equiv \int_{C_1} dx_1' \, dy_1' \, b_1(x_1',y_1') \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2' \, dy_2' \, b_2(x_2',y_2') \ln(s_{22}^2/s_{12}^2) \qquad (4.10.9)$$

Before continuing to the paradox, I must say that this equation for L_e seems a little odd. Suppose the conductors are made of some super magnetic material. You would expect that to increase B *inside* the conductors, but not *outside* the conductors. You expect the internal wire flux to be related to μ_c and the external flux to be related to μ for the dielectric. Thus, you would not expect the *external* inductance L_e to double if μ_c doubles.

Now the relation $L_e = \frac{W(z)}{i(z)} = \frac{\mu_c}{4\pi} K_L$ is developed in (4.10.8) where I have my little picture showing a red math loop. I guess I assume that the loop runs just outside the conductor surfaces and that A_z is being sensed just outside the surface.

Back up some more. Look again at

$$\mathbf{A}(\mathbf{x},\omega) = \frac{\mu_c}{4\pi} \sum_i \int \mathbf{J}_i(\mathbf{x}',\omega) \frac{e^{-j\beta R}}{R} dV'$$

which is supposed to tell us **A** at point x in the dielectric. If have some currents flowing **J** and we suddenly double μ_c , that would double **A**. But since **B** = curl **A**, that would double **B** as well. But I just argued above that **B** outside a wire would stay the same if you doubled μ_c . Recall from Appendix C this result

$$L_e = \frac{\mu}{8\pi} \ln\frac{R}{a}$$

where μ is for the medium surrounding the wire, not for the wire. This reinforces my feeling that μ_c should not be sitting in my **A** equation above.

This means that my equation (1.5.9) which I spent so much time developing must be wrong!
[but it is not wrong]

$$\mathbf{A}(\mathbf{x},\omega) = \frac{1}{4\pi} \sum_i \int \mu_i \mathbf{J}_i(\mathbf{x}',\omega) \frac{e^{-j\beta R}}{R} dV' \qquad R = |\mathbf{x} - \mathbf{x}'| \quad . \qquad (1.5.9)$$

This may be traced back to (1.5.4) which says

$$(\nabla^2 + \beta^2)\mathbf{A} = -\sum_{i=2}^{N} \mu_i \mathbf{J}_i \qquad \text{all of region R} \qquad (1.5.4)$$

Tracing backwards some more, we get to

$$(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2 - \mu_1\sigma_1)\,\mathbf{A} = -\sum_{i=2}^{N+1} \mu_i \mathbf{J}_i \quad . \qquad \text{all of region R} \qquad (1.3.23)$$

and backwards more

$$\begin{aligned}
&(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2 - \mu_1\sigma_1\partial_t)\,\mathbf{A} = 0 && \text{region 1} \\
&(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2 - \mu_1\sigma_1\partial_t)\,\mathbf{A} = -\mu_2\mathbf{J}_2 && \text{region 2} \\
&(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2 - \mu_1\sigma_1\partial_t)\,\mathbf{A} = -\mu_3\mathbf{J}_3 && \text{region 3} \qquad (1.3.21)
\end{aligned}$$

This then leads back all the way to my KING GAUGE setup section with Fig 1.5 and following text. Recall that my plan there was to apply the region 1 King gauge to all three regions! [that is just fine]

Plan A. I think the resolution of all this business is related to homo solutions and can be perhaps best be understood if I do the simple example of a DC round wire. Recall that the solution for **A** has to work somehow both inside *and* outside the conductor where the μ's are different. This is the whole matter I swept under the rug, but now I need to face it directly. I think it is also related to the magnetization current at the boundary. This is going to take very many days to get cleaned up, but it has to be done,

otherwise my entire lines doc is completely worthless. Right now I don't know where my notes are on this subject. They are probably strewn around in 6 docs and discarded appendices etc.