This is the Title PhL 3.26.05

The plan of this doc has been installed.

C.3 The DC inductance of a round wire

We assume here that μ_i is the permeability of the wire, and μ_e of the region outside the wire.

The magnetic field for an isolated round wire is easily computed from Ampere's Law

$$\oint \mathbf{H} \bullet d\mathbf{s} = \int_S \mathbf{J} \bullet d\mathbf{S}$$

$$2\pi r\, H_i = I(\pi r^2/\pi a^2) \quad \Rightarrow \quad H_i = \frac{I}{2\pi}(r/a^2) \quad \Rightarrow \quad B_i = \frac{\mu_i I}{2\pi}(r/a^2)$$

$$2\pi r\, H_e = I \quad \Rightarrow \quad H_e = \frac{I}{2\pi}(1/r) \quad \Rightarrow \quad B_e = \frac{\mu_e I}{2\pi}(1/r)$$

The internal and external inductances of the wire (per unit length) are defined as

$$L_i dz = [\text{ total internal magnetic flux }] / I = \int_i \mathbf{B} \bullet d\mathbf{S} / I = \int_i (B_i/I)\, dS$$

$$L_e dz = [\text{ total external magnetic flux }] / I = \int_e \mathbf{B} \bullet d\mathbf{S} / I = \int_e (B_e/I)\, dS$$

The inductance of either the internal or external retion

We assume here that μ_i is the permeability of the wire, and μ_e of the region outside the wire.

The energy density (joules/m^3) stored in an electromagnetic field within a medium of negligible loss is given by $u_{em} = (\mathbf{E}\bullet\mathbf{D} + \mathbf{B}\bullet\mathbf{H})/2$ [Jackson p 259 Eq. (6.106)] . We are interested only in the portion of this energy density stored in the magnetic field, so $u = (1/2)\ \mathbf{B}\bullet\mathbf{H}$. Since μ and ε are assumed to be non-tensor in nature, we find that

$$u = (1/2)\ \mu\, H^2.$$

For any inductor of inductance L carrying current I and having potential difference V, we know that

$$V = L\, dI/dt$$

$$P = IV = I\,(L\, dI/dt) = d/dt\ [\ (1/2) L\, I^2\ ].$$

But since the power fed into an ideal inductor goes into the magnetic field energy, P = dU/dt and so

$$U = (1/2)L\, I^2$$

For our straight wire, we now redefine symbol L to mean inductance per unit length, so then

$$U = (1/2)[Ldz]\, I^2$$

For either the internal or external region we have $U = \int d\mathcal{V}\ u = dz \int dS\ u$ where $d\mathcal{V}$ is a volume element and dS is a cross sectional area element. Thus,

$$U = dz \int dS\ (1/2)\,\mu\, H^2 = (1/2)[Ldz]\, I^2$$

and then

$$L = \mu \int dS\ (H/I)^2$$

In particular

$$L_i = \mu_i \int dS\ (H_i/I)^2$$

$$L_e = \mu_e \int dS\ (H_e/I)^2$$

For the round wire it is a simple matter to compute H_i and H_e using Ampere's Law (1.1.37),

$$\oint \mathbf{H} \bullet d\mathbf{s} = \int_S \mathbf{J} \bullet d\mathbf{S}$$

$$2\pi r\, H_i = I(\pi r^2/\pi a^2) \Rightarrow \quad H_i/I = \frac{1}{2\pi}(r/a^2)\ .$$

$$2\pi r\, H_e = I \quad \Rightarrow \quad H_e/I = \frac{1}{2\pi}(1/r)$$

We then compute the two inductances as follows:

$$L_i = \mu_i \int dS\ (H_i/I)^2 = \mu_i \int_0^a r dr \int_{-\pi}^{\pi} d\theta\ [\frac{1}{2\pi}(r/a^2)]$$

The total energy U stored in the full field of the inductor must then be $U = (1/2)L I^2$ in order to obtain $P = dU/dt$.