

C.5 The DC internal inductance of a thin strip conductor

The strip geometry is as follows, where we assume J_z is uniform and total current is I :

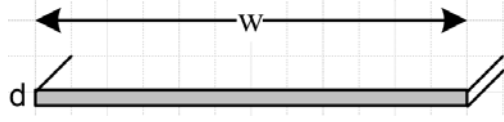


Fig C.5

We also assume $w \gg d$ so we can ignore end effects. The figure below is an end view of the flat conductor rotated to a vertical position, but only a central segment of the conductor is shown.

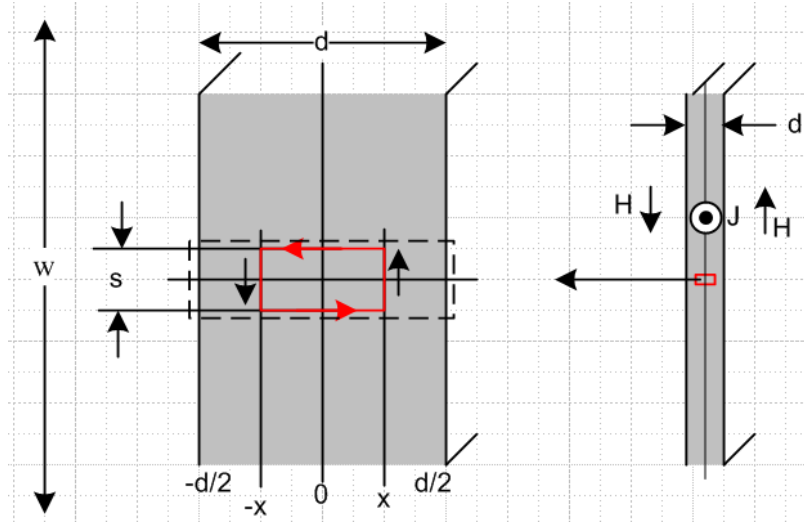


Fig C.6

The uniform current density J_z is out of the plane of paper as shown on the right. Based on the current direction we expect the H field to have the directions shown on the right, both inside and outside the surface of the conductor.

The current density is given by I/area ,

$$J_z = I/(wd) . \quad (\text{C.5.1})$$

Applying Ampere's Law to the red loop one finds

$$\int_{\mathbf{s}} \mathbf{J} \cdot d\mathbf{S} = \oint_{\mathbf{c}} \mathbf{H} \cdot d\mathbf{s}$$

so

$$J_z 2x s = 2H(x) s \Rightarrow H(x) = J_z x = Ix/(wd) \quad H/I = x/(wd) . \quad (\text{C.5.2})$$

To get the internal inductance per unit length L_i we compute its contribution from the dashed rectangle, then multiply by (w/s) to get L_i for the entire strip. Therefore,

$$L_i = (w/s) \mu \int_{\text{dashed}} dS (H/I)^2 = (w/s) \mu \int_{\text{dashed}} (s dx) [x/(wd)]^2$$

$$\begin{aligned}
&= (w/s) \mu s (wd)^{-2} \int_{-d/2}^{d/2} dx x^2 = (w/s) \mu s (wd)^{-2} 2 \int_0^{d/2} dx x^2 \\
&= (w/s) \mu s (wd)^{-2} 2 (1/3) (d/2)^3 = w^{-1} \mu d (2/3) (1/8) \\
&= (1/12) \mu (d/w) = (\mu_i/8\pi) [2\pi/3 (d/w)] \quad // \text{ strip } w \gg d \quad (C.5.3)
\end{aligned}$$

For some reason, this is 1/2 the value which appears in Holloway and Kuester equation (25).