How is G related to σ ? PhL 6.27.14

Another confusion rather late in the game, a land mine every place I look, it never stops.

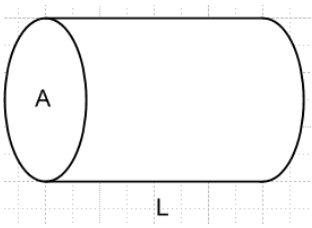
In Appendix C I show that

$$R = 1/(\sigma A)$$

for a medium of cross sectional area A. This is resistance per unit length.

$$\dim R = \text{ohm-m}/m^2 = \text{ohm/m}$$

I often apply this to a round wire with $A = \pi a^2$.



Question: In a transmission line with conductance G of the dielectric, is there a simple relation between G and σ_d similar to what I have written above?

The answer is not immediately obvious as I first thought it was.

Consider a resistor of area A and length L. We know that

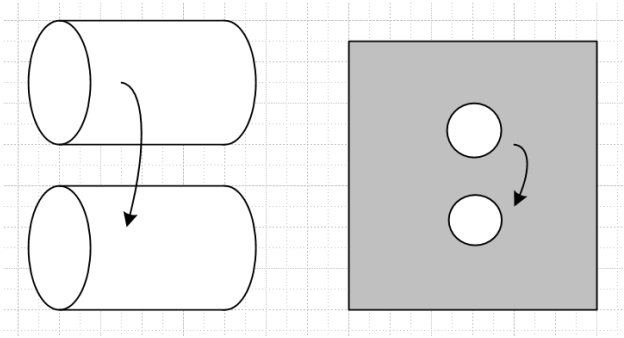
$$\mathcal{R} = 1/(\sigma A) * L \qquad \text{ohms}$$

We can certainly think of this "resistor" as being a "conductor" having

$$\mathcal{G} = 1/\mathcal{R} = (\sigma A/L) \qquad \text{mhos}$$

So this would be the conductance of the above round wire slice for current going left to right. But this has nothing whatsoever to do with the scenario of the transmission line.

In the transmission line scenario, we are interested in this geometry and its cross section



Here there is no A and L as in the previous example. Current flows through the entire infinite medium from one conductor to another. So it would make no sense to look for some formula like

$G = \sigma A/L^2$ // wrong

because as I just said, there is no A or L for this geometry.

The trick here is to remember that

$G/C = \sigma_d/\varepsilon_d$ (4.11.34)

Thus, if you want to know G for the above geometry, you have to know C, and then

$G = C\ (\sigma_d/\varepsilon_d)$

Dimension check:

dim G = far/m * mho/m * m/far = 1/m * mho = mho/m

We also know from writing G +jωC many times that we must have

$\dim G = \dim(\omega C) = \sec^{-1}$ far/m = $\sec^{-1}$ sec/ohm 1/m = mho/m

This is consistent with the fact that

dim (σ_d/ε_d) = mho/m * m/farad = mho/farad = mho * ohm/sec = $\sec^{-1}$ = dim(ω)