I removed this section from Appendix C after I realized that my strip L_i derivation was wrong. The basic facts presented here I think are correct in terms of the H plots, but the whole subject of transverse H fields ruins things. In the rolled state, all the B field is "along the strip", but in the unrolled state half the L_i energy is in the newly appearing " perp to the strip" L_i B field. So the comparison is very weak and not worth writing about.

(c) Unrolling the thin shell?

Suppose we were to unroll the thin shell of the previous section to arrive at a wire whose cross section was a strip of with w = 2πa and thickness d,

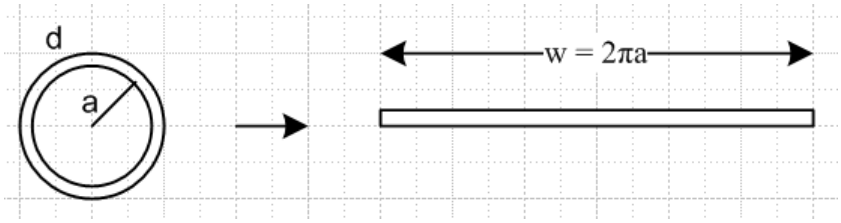
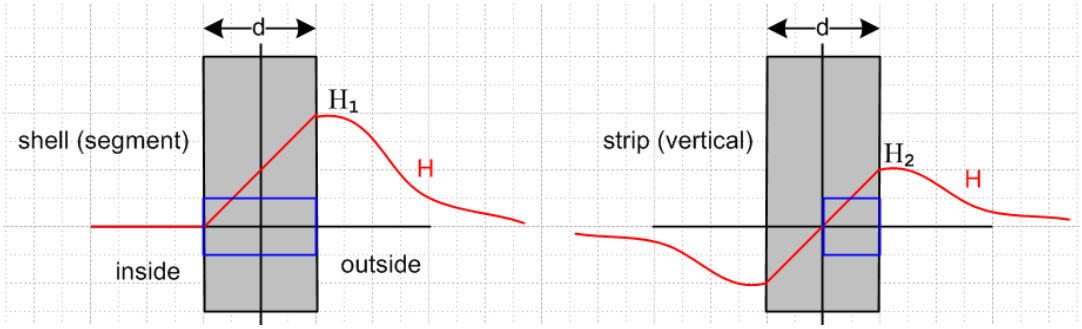


Fig C.8

If this unrolling could be done without affecting L_i, we would then have an expression for L_i for the strip, which would be

$$L_i = \frac{\mu_i}{8\pi}\,[\,(4/3)(d/a)\,] \;=\; = \frac{\mu_i}{8\pi}\,[\,(4/3)(2\pi d/w)\,] \;=\; \frac{\mu_i}{8\pi}\,[\,(8\pi/3)(d/w)\,] \qquad \text{// "if ..."}$$

Although the current density J_z is the same for rolled and unrolled, the H field pattern is in fact not the same in the two situations:



$J_z = I/(wd)$ $\quad w = 2\pi a$ $\qquad\qquad$ $J_z = I/(wd)$ Fig C.9

In the above figures we plot the H field in red, giving it linear form inside the conductor. This linear form is justified for the thin shell from (C.3.6),

$$H_\theta(r) = I_{enc}(r)/\,(2\pi r) \;= \frac{1}{2\pi}\,\frac{1}{r}\,\frac{r^2 - a^2}{b^2 - a^2}\;I \;\approx\; \frac{1}{2\pi}\,\frac{1}{a}\,\frac{(r+a)(r-a)}{(b+a)(b-a)}\,I \approx \frac{1}{2\pi}\,\frac{1}{a}\,\frac{r-a}{b-a}\;I \;=\; \frac{1}{2\pi}\,\frac{1}{a}\,\frac{r-a}{d}\;I \qquad (C.6.9)$$

For the wide strip, we found the linear form H(x) = Ix/(wd) in (C.5.2). Ignoring these specific expressions and just using the fact that H is linear inside the conductor in both cases, and using the J_x expressions above, we will now show that the thin shell has four times more L_i than the equivalent strip.

For each of the blue loops in Fig * apply Ampere's Law:

$$\text{shell:} \quad H_1 s = J_z\, sd \quad \Rightarrow \quad H_1 = J_z d = I/2\pi a = I/w \quad \Rightarrow \quad H_{shell}(x) = H_1 \left(\frac{x}{d} + \frac{1}{2}\right)$$

$$\text{strip:} \quad H_2 s = J_z\, s(d/2) \Rightarrow \quad H_2 = J_z d/2 = I/(2w) \quad \Rightarrow \quad H_{strip}(x) = H_2 \left(2\frac{x}{d}\right) \qquad \text{(C.6.10)}$$

Then integrating the squared H field,

$$f_1 = H_1^2 \int_{-d/2}^{d/2} dx \left(\frac{x}{d} + \frac{1}{2}\right)^2 = H_1^2 \,(d/3) = (I/w)^2 (d/3) = (1/3)\, I\, (d/w^2) \qquad \text{// shell}$$

$$f_2 = H_2^2 \int_{-d/2}^{d/2} dx \left(2\frac{x}{d}\right)^2 = H_2^2 \,(d/3) = [I/(2w)]^2 (d/3) = (1/12)\, I\, (d/w^2) \qquad \text{// strip} \qquad \text{(C.6.11)}$$

Thus $f_1 = 4f_2$. Since $L_i = \mu_i\, w\, (f_i/I^2)$ we conclude that $L_1 = 4L_2$. This is consistent with our results obtained above,

$$L_i = \mu_i \frac{1}{6\pi} (d/a) = \frac{\mu_i}{8\pi} [\,(4/3)(d/a)\,] \qquad \text{// thin shell, valid for } d << a \qquad \textit{(C.6.8)}$$

$$L_i = \frac{\mu_i}{8\pi} [\,2\pi/3\,(d/w)\,] = \frac{\mu_i}{8\pi} [\,(1/3)\,(d/a)\,] \qquad \text{// strip } w = 2\pi a >> d \qquad \textit{(C.5.3)}$$

As noted earlier, this strip result is half the value quoted in (C.4.11).