App D Actions 3 PhL 12.1.13

In this doc I ponder that "second boundary condition" that $E_\varphi(r=a,m) = 0$. I was very unhappy with it for a while, ready to throw it out, but then I bailed it out. The explanation is now given in App D just below equation (D.1.2). I regard this current doc as fully reviewed.

I have now accurate (I think) expressions for all six E and B field components inside a round wire. Each partial wave has two coefficients a_m and K_m. I am searching for proper boundary conditions which will set these coefficients. Here are the field expressions:

$$E_z(r,m) = -j\,(\beta'/\beta_d)\,\frac{K_m}{2}\,J_m(x) \qquad x = \beta' r$$

$$E_r(r,m) = a_m\,J_m(x)/x + \frac{K_m}{2}\,J_{m+1}(x)$$

$$jE_\varphi(r,m) = -\,a_m\,J_m(x)/x + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x)$$

$$(\omega/\beta')B_z(r,m) = \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_m(x)$$

$$(\omega/j\beta_d)B_r(r,m) = \left[\,(\beta'^2/\beta_d^2)\,m\,\frac{K_m}{2} - a_m\,\right] J_m(x)/x + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x)$$

$$(\omega/\beta_d)B_\theta(r,m) = -\left[\,(\beta'^2/\beta_d^2)\,m\,\frac{K_m}{2} - a_m\,\right] J_m(x)/x + \frac{K_m}{2}\,(1 + \beta'^2/\beta_d^2)\,J_{m+1}(x)$$

In a transmission line, a capacitative analysis determines the surface charge distribution. This happens because the capacitor problem determines the potential φ which then determines E, all in 2D, and E_r at the surface determines surface charge n there, and that then gives one condition on each partial wave. This is already incorporated into Appendix D where I have

$$a_m\,x_a^{-1}\,J_m(x_a) + \frac{K_m}{2}\,J_{m+1}(x_a) = (j\omega/\sigma)\,N_m \qquad (1)$$

My problem is finding a *second* condition, and that is the topic of this doc. Now it is true that the 2D capacitor problem *assumes* a constant potential around the wire!!! That is how you are able to solve that capacitance problem. So we have this logic

1) assume constant potential on the two conductor surfaces in a given $z = z_z$ plane.
2) solve the 2D potential problem for φ, and from that get **E**
3) evaluate $\mathbf{E}\bullet\hat{\mathbf{n}}$ at the surface of your round wire conductor. That determines surface charge n
and in this way you can compute n(θ) around the perimeter, and from that you can get the moment N_m which appears in the BC above.

So this whole plan only makes sense with the assumption of step 1. Notice that we only need constant potential on the surface because that is all we are using in the capacitor potential problem.

Now, this capacitor problem is not done in partial waves, it is done "as a whole". So let's consider:

$$\mathbf{E}(r,\varphi) = \sum_{m=-\infty}^{\infty} \mathbf{E}(r,m)\, e^{jm\varphi} \qquad \Leftrightarrow \qquad \mathbf{E}(r,m) = (1/2\pi) \int_{-\pi}^{\pi} d\varphi\, \mathbf{E}(r,\varphi)\, e^{-jm\varphi} \tag{D.1.3}$$

and then the specific component of interest

$$E_\varphi(r,\varphi) = \sum_{m=-\infty}^{\infty} E_\varphi(r,m)\, e^{jm\varphi} \qquad \Leftrightarrow \qquad E_\varphi(r,m) = (1/2\pi) \int_{-\pi}^{\pi} d\varphi\, E_\varphi(r,\varphi)\, e^{-jm\varphi} \tag{D.1.3}$$

If I assume that $E_\varphi(a,\varphi) = 0$ to get that constant potential, then we have this condition

$$E_\varphi(a,m) = (1/2\pi) \int_{-\pi}^{\pi} d\varphi\, E_\varphi(a,\varphi)\, e^{-jm\varphi} = (1/2\pi) \int_{-\pi}^{\pi} d\varphi\, 0\, e^{-jm\varphi} = 0$$

Thus, the condition must be $E_\varphi(a,m) = 0$ in each partial wave! This happens to be exactly the condition I have already used in App D.

So how do we justify this general idea? First of all, we know that E_φ is always small relative to E_z if we are in the "transmission line limit" which is that $(a\beta_d) << 1$. Thus, we can consider this idea

Argument A. Although E_φ is not exactly 0 in a round wire, we shall start off by assuming that it is 0 at the round wire surface. This really is an ansatz. Having done this, we then have a 2D capacitor problem which we can solve for n and then N_m and then we get one condition on the a_m and K_m coefficients. In order to get this condition, we had to assume that the potential was constant, and that in turn means we had to assume that $E_\varphi(a,m) = 0$ which is a second condition on a_m and K_m. These two boundary conditions, although perhaps not exactly right, are at least consistent with one another.

OK, I have elaborated on this argument in a new App D Section just written, and it really is an ansatz type argument indeed, heavy on approximation. I think it is OK, but not great.

Argument B. Suppose you say this: the surface is the only place free charge is found. If the surface were NOT an equipotential surface, then that free charge would quickly adjust in such a way to make the surface be an equipotential surface. That is to say, the free charge would see $E_\varphi \neq 0$ and would move to neutralize it. This does not happen inside the wire because there is no free charge there and that is why you could have swirling current $E_\varphi \neq 0$ inside the wire. In Section 3.1 it is shown that the surface charge adjustment to make $E_\varphi = 0$ happens in a time associated with 10^{19} Hz and so in any practical transmission line the local situation at some fixed point on the wire surface is almost static relative to this 10^{19} Hz adjustment rate, so adjustment happens and the surface is an equipotential.

But then since there is also E_z at the surface, why doesn't the charge move longitudinally in order to cancel the E_z field in this same way? The answer is that the charge DOES move along the surface, and it does so at the dielectric speed of light, but as old charge moves down the line, new charge moves into its

place. The charge pattern moves as a wave down the surface in the z direction. This charge is constantly moving in response to the local E_z field, it is not neutralizing the E_z field as it does in the φ direction.

I think this is a better argument than my convoluted Argument A.

If this is correct, does that mean my "azimuthal wave" idea is not viable? Yes, this wave is a standalone solution of the wave equations and Maxwell, but it travels at the same rate as any other wave. It might exist because there is no surface charge? But there is always "charge available" on a metal surface, so I think Argument B says any E_φ that existed would be killed off instantly. You could try to drive such a wave down the wire, but a charge pattern would arise at any z to kill off E_φ very quickly and then the wave is gone. In other words, you could try, but it would damp out very quickly. So then I don't have to mention this wave thing at all!

OK, I saved off my convoluted previous argument and installed my new argument right at the very start of Appendix B where I list off my assumptions. I sure hope this is stable and won't change again! [it was not stable!]

I think now that Appendix D is OK through the big box (D.4.6) which outlines the E fields inside a round wire. I am not sure right now how this appendix should proceed from that point.

- want to compute the B fields and comment on them
- want to look at the m = 0 case and compare to Chapter 2
- want to say something about the E field *outside* the wire

Question. In the E field final box in App D, can I just take the m = 0 limits and get the correct m=0 result?

(1) $$E_z(r,0) = (1/4)\,\eta_0\, I\, R_{dc}\left[\frac{x_a J_0(x)}{J_1(x_a)} - \frac{x_a J_0(x)}{J_{-1}(x_a)}\right]$$

$$= (1/2)\,\eta_0\, I\, R_{dc}\frac{x_a J_0(x)}{J_1(x_a)} \qquad \text{// yes this one works}$$

(2) $$E_r(r,0) = (j/4)\,(a\beta_d)\,\eta_0\, I\, R_{dc}\left[+\frac{J_1(x)}{J_1(x_a)} - \frac{J_1(x)}{J_{-1}(x_a)}\right]$$

$$= (j/2)\,(a\beta_d)\,\eta_0\, I\, R_{dc}\frac{J_1(x)}{J_1(x_a)} \qquad \text{// works}$$

(3) $$E_\varphi(r,0) = (1/4)\,(a\beta_d)\,\eta_0\, I\, R_{dc}\left[-+\frac{J_1(x)}{J_1(x_a)} + \frac{J_1(x)}{J_{-1}(x_a)}\right] = 0 \quad \text{// works}$$

So the answer is YES.

Here is the old box I had.

E Fields Inside a Round Wire

$x = \beta' r \qquad \beta'^2 = \beta^2 - \beta_d^2 \approx \beta^2 \qquad \beta = \omega\sqrt{\mu\xi} \qquad \xi = [\varepsilon + \sigma/(j\omega)] \approx \sigma/(j\omega)$ conductor

$x_a = \beta' a \qquad \beta_d = \omega\sqrt{\mu_d\xi_d} \qquad \xi_d = [\varepsilon_d + \sigma_d/(j\omega)] \approx \varepsilon_d$ dielectric

$\mathbf{E}(r,\varphi,z,t) = e^{j(\omega t - \beta_d z)}\,\mathbf{E}(r,\varphi)$ *(D.1.2)* $\qquad \mathbf{E} = E_r\hat{\mathbf{r}} + E_\varphi\hat{\boldsymbol{\varphi}} + E_z\hat{\mathbf{z}}$ (for any arguments)

$$\mathbf{E}(r,\varphi) = \mathbf{E}(r,0) + 2\sum_{m=1}^{\infty}[\mathrm{Re}\{\mathbf{E}(r,m)\}\cos(m\varphi) - \mathrm{Im}\{\mathbf{E}(r,m)\}\sin(m\varphi)] = \text{real} \qquad (D.1.4)$$

where:

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\left[\frac{x_a J_m(x)}{J_{m+1}(x_a)} - \frac{x_a J_m(x)}{J_{m-1}(x_a)}\right] \qquad a = \text{radius} \qquad \eta_m \equiv \frac{N_m}{N_0}$$

$$E_z(r,0) = (1/2)\, I\, R_{dc}\left[\frac{x_a\, J_0(x)}{J_1(x_a)}\right] \qquad \text{// = (2.2.22) for } \beta_d = 0 \qquad \eta_0 = 1$$

$$E_r(r,m) = (j/4)\,(a\beta_d)\,\eta_m\, I\, R_{dc}\left[\frac{2mx^{-1}J_m(x)}{J_{m-1}(x_a)} + \frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m+1}(x)}{J_{m-1}(x_a)}\right]$$

$$E_r(r,0) = (j/2)\,(a\beta_d)\, I\, R_{dc}\left[\frac{J_1(x)}{J_1(x_a)}\right] \qquad R_{dc} = \frac{1}{\sigma\pi a^2}$$

$$E_\varphi(r,m) = (1/4)\,(a\beta_d)\,\eta_m\, I\, R_{dc}\left[-\frac{2mx^{-1}J_m(x)}{J_{m-1}(x_a)} + \frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m+1}(x)}{J_{m-1}(x_a)}\right]$$

$$E_\varphi(r,0) = 0$$

(D.4.6)

And here is a section I think I will remove

<u>What about $E_\varphi(r,m=0)$?</u>

We have already noted that $a_0 = 0$. For m=0, (D.2.15) is invalid because we divided (D.2.13) by m. In fact, we really know nothing at all about the field $E_\varphi(r,m=0)$ from our solution method because E_φ is not even present in our starting equation (D.2.2) when m = 0,

$$-jmE_\varphi(r,m) = [1 + r\partial_r]\,E_r(r,m) - r\,(j\beta_d)E_z(r,m)\,. \qquad (D.2.2)$$

However, we can use the E_φ Helmholtz equation (D.1.18) to learn more about $E_\varphi(r,m)$ with m = 0,

$$[r^2\partial_r^2 + r\partial_r - (m^2+1) + r^2\beta'^2]\,E_\varphi(r,m) + 2jmE_r(r,m) = 0 \qquad (D.1.18)$$

With m = 0 we get

$$[r^2\partial_r^2 + r\partial_r - 1 + r^2\beta'^2]\, E_\varphi(r,0) = 0$$

or

$$[x^2\partial_x^2 + x\partial_x - 1 + x^2]\, E_\varphi(r,0) = 0 \qquad x = \beta' r \qquad \text{(D.2.16)}$$

Since [..] is the $\nu = 1$ Bessel operator and since E_φ must be finite at $r = 0$, we find that

$$E_\varphi(r,0) = C_{\varphi 0}\, J_1(\beta' r) \qquad \text{(D.2.17)}$$

where $C_{\varphi 0}$ is some constant.

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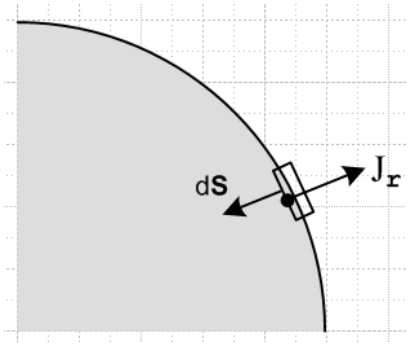
(c) The Charge Pumping Boundary Condition

The reason we are interested in surface charge $n(\varphi)$ of (D.1.5) is that it acts as a driving source of the radial electric field in the wire. Recall the equation of continuity (1.1.8) converted to the ω domain

$$\text{div}J = -j\omega\rho \quad \Leftrightarrow \quad \int_S dS \bullet J = -j\omega \int_V dV\, \rho \ . \quad \text{// divergence theorem} \qquad \text{(D.1.20)}$$

When applied to a thin box of radial area dS straddling the wire surface,



one finds that $\int_S dS \bullet J = -J_r(r=a,\varphi)dS$ and $\int_V dV\, \rho = n(\varphi)\, dS$ so that

$$J_r(r=a,\varphi) = j\omega\, n(\varphi) \ . \qquad \text{(D.1.21)}$$

We assume that there is no free current outside the wire to get this result. Since $\mathbf{J} = \sigma\mathbf{E}$, this is really a boundary condition on the radial electric field,

$$E_r(r=a,\varphi) = (j\omega/\sigma)\, n(\varphi) \ . \qquad \text{(D.1.22)}$$

We convert this to m-space using the conversion rules (D.1.15) to trivially obtain

$$E_{\mathbf{r}}(r=a,m) = (j\omega/\sigma)\, N_{\mathbf{m}} \ . \qquad \text{(D.1.23)}$$

Thus, the radial electric field must have a certain value at the r=a boundary in each partial wave. And the value it must have is determined by the moment of the charge distribution.

By way of interpretation, the surface charge of a transmission line is "pumped" by the radial current in the wire. Since div**J** = 0 inside the wire, this radial current is accompanied by the usual longitudinal current one expects to find inside the conductors of a transmission line.
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