E fields outside the round wire? PhL 10.5.13

This has now been installed in appendix D, no need to keep long term.

How is Appendix D different if you are thinking outside the wire? Just page through the appendix.

First of all, the β appearing in the Helmholtz is then β_d since we are "in the dielectric". So we have

$$(\nabla^2 + \beta_d^2) \mathbf{E}(r,\varphi, z, t) = 0 . \qquad (D.1.1)$$

Assume the same form

$$\mathbf{E}(r,\varphi,z,t) = e^{j(\omega t - \beta_d z)} \mathbf{E}(r,\varphi) . \qquad (D.1.2)$$

As I read through the Helmholtz section, what comes out at the far end is this:

Summary of Examples:

$[\nabla^2\mathbf{E}]_z + \beta^2 E_z = 0$:

$$[r^2\partial_r^2 + r\,\partial_r - m^2]\, E_z(r,m) = 0 \qquad (D.1.16)$$

$[\nabla^2\mathbf{E}]_r + \beta^2 E_r = 0$:

$$[r^2\partial_r^2 + r\partial_r - (m^2+1)]\, E_r(r,m) - 2jm\, E_\varphi(r,m) = 0 \qquad (D.1.17)$$

$[\nabla^2\mathbf{E}]_\varphi + \beta^2 E_\varphi = 0$:

$$[r^2\partial_r^2 + r\partial_r - (m^2+1)]\, E_\varphi(r,m) + 2jmE_r(r,m) = 0 \qquad (D.1.18)$$

div $\mathbf{E} = 0$:

$$\partial_r [r\, E_r(r,m)] + jmE_\varphi(r,m) - j\beta_d\, r\, E_z(r,m) = 0 \qquad (D.1.19)$$

So basically you set $\beta' = 0$ it seems to me.

Now down to the E_z solution.

$$[r^2\partial_r^2 + r\,\partial_r - m^2]\, E_z(r,m) = 0$$

$$[\partial_r^2 + (1/r)\,\partial_r - m^2/r^2]\, E_z(r,m) = 0$$

$$[r\partial_r^2 + \partial_r - m^2/r]\, E_z(r,m) = 0$$

This is not Bessel's equation! Ouch, what is it?

$$r\partial_r(r\partial_r f(r)) = m^2 f(r)$$

How do I "look up" an ODE. Polyanin is so massive. Here is an offering he has

$[r^2\partial_r^2 + r\,\partial_r - m^2]\,E_z(r,m) = 0$ a = 1 and b = $-m^2$

118. $x^2y''_{xx} + axy'_x + by = 0$.

The Euler equation.

Solution:

$$y = \begin{cases} |x|^{\frac{1-a}{2}}\left(C_1|x|^{\mu} + C_2|x|^{-\mu}\right) & \text{if } (1-a)^2 > 4b, \\ |x|^{\frac{1-a}{2}}\left(C_1 + C_2\ln|x|\right) & \text{if } (1-a)^2 = 4b, \\ |x|^{\frac{1-a}{2}}\left[C_1\sin(\mu\ln|x|) + C_2\cos(\mu\ln|x|)\right] & \text{if } (1-a)^2 < 4b, \end{cases}$$

where $\mu = \frac{1}{2}|(1-a)^2 - 4b|^{1/2}$.

Then $\mu = 1/2*|\,0 - 4(-m^2)|^{1/2} = (1/2)\,|\,4m^2| = (1/2)2m = m$. The if condition is $0 > 4b = -4m^2$ so

$E_z(r,m) = C_1|r|^m + C_2|r|^{-m} = Ar^m + B\,r^{-m}$

Yes, this is the radial equation for the 2D Laplace equation $\nabla^2 u = 0$ as on Stak II page 90, equation (7.8) page 92. I never knew it by the name Euler Equation. For m = 0 it is the second case and the solutions are A + Bln r as I now well remember.