**Electric Field Outside a Round Wire at DC** PhL 12.11.13

My conclusion is shown in the picture below, I think this issue is resolved. I found a physical situation which gives fields which satisfy the Max equations. The Jackson paper reference is noted.

**Question 1.** Current I flows in a DC round wire of radius a.

(a) What is the $E_z$ field just outside the surface?

(b) What is $E_z$ outside the wire?

(a) I think the current profile is uniform and just drops to zero as a step function at r = a. If so, then we have $E_z = J/\sigma$ at $r = a-\varepsilon$ where $J = I/(\pi a^2)$. If $E_z$ is continuous across the boundary as I think it is, then the answer is that just outside the wire, $E_z = I(\pi a^2 \sigma)$. [ I still think this is correct ]

(b) There are perhaps several approaches here. We seem to know that dive $\mathbf{E} = \partial_z E_z(r) = 0$ if there are no other field components in existence, but that fact just says $E_z$ does not vary with z which we already know. We know that

$$(\nabla^2 - \mu\varepsilon\, \partial_t^2)\mathbf{E} = \mu\partial_t \mathbf{J} + (1/\varepsilon)\ \text{grad}\ \rho \qquad (1.2.1)$$

In our case, outside the wire, I assume $\mathbf{J} = 0$ since perfect insulator say, and $\rho = 0$ and $\mathbf{E}$ is static, so

$$\nabla^2 E_z(r) = 0$$

or

$$\nabla^2_{2D} E_z(r) = 0$$

This means that we must have

$$E_z(r) = A + B \ln (r/a) \qquad \text{ok}$$

Then the boundary condition says

$$E_z(a) = A = I(\pi a^2 \sigma) \qquad \text{ok}$$

So then we have

$$E_z(r) = I(\pi a^2 \sigma) + B \ln (r/a)$$

What sets the other constant? Maybe one of the Maxwell equations?

$$\text{curl}\ \mathbf{E} = 0$$

$$\text{curl}\ \mathbf{E} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta E_z - \partial_z E_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z E_r - \partial_r E_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r\,]$$

$= + \hat{\boldsymbol{\theta}} [- \partial_r E_z]$

This seems to say that $\partial_r E_z(r) = 0$ everywhere and that would set B = 0. ok

Well, we know there is an H field outside the wire given by

$2\pi r H_\theta = I \qquad H_\theta(r) = I/2\pi r.$

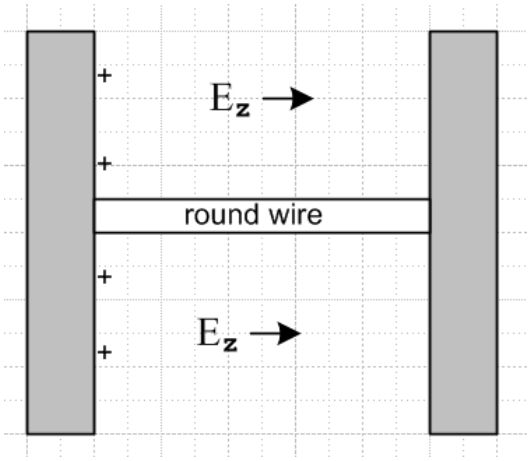
We should have curl H = 0 since $\partial_t = 0$ and J = 0 outside, so then

curl $\mathbf{H} = \hat{\mathbf{r}} [ r^{-1}\partial_\theta H_z - \partial_z H_\theta] + \hat{\boldsymbol{\theta}} [\partial_z H_r - \partial_r H_z] + \hat{\mathbf{z}} [ r^{-1}\partial_r(rH_\theta) - r^{-1}\partial_\theta H_r ] = 0$

$+ \hat{\mathbf{z}} [ r^{-1}\partial_r(rH_\theta) ] = 0$ // and this works properly since $H_\theta(r) = I/2\pi r$.

So my tentative conclusion is rather astounding. It says $E_z(r) = I(\pi a^2 \sigma)$ everywhere outside the wire, all the way out to $r = \infty$.

**Conclusion**: I think this is correct, that $E_z$ = constant in all 3D space for this problem. Here is my interpretation of this solution: This uniform $E_z$ field is provided by a huge parallel plate capacitor at the ends of the round wire (at $\pm\infty$). The capacitor makes $E_z$ uniform everywhere between the plates. It just happens that in the round wire it causes a current to flow. This is certainly A possible Maxwell equation solution, and I "found it" above but in retrospect it is quite obvious.




7. J. D. Jackson, *Am. J. Phys.* **64**, 855–870 (1996).

"**Surface charges on circuit wires and resistors play three roles**" by J. D. Jackson, in American Journal of Physics -- July 1996 -- Volume 64, Issue 7, pp. 855 .

Scribd has it but I am too tired to upload something.

There are lots of fancy **papers** on this subject addressing induced charge, relativistic effects and so on. But for me, I think the answer is this:

For an idealized isolated infinite round wire, $E_z$ = constant everywhere in space. This is consistent with Maxwell's Equations. Notice above how the curl E equation killed off the lnr term.

In the more realistic situation of two oppositely directed wires, if they were forced to have a uniform current density even, things are messier. Now we have surface charges and external fields etc.

The notion of fields outside a uniform wire is slippery I feel. agreed

**Question 2.** Can a round wire have an $E_\varphi \neq 0$ field at the surface?

I keep coming back to this painful question. If I argue that $E_\varphi = 0$ due to "quasi-static", then why doesn't that mean $E_z = 0$ as well at the surface. Hold on this question. [ Section D.8 at least faces this question ]

**Question 3.** In the round wire with wave going down it, there is $E_z$ at the surface I think. I know that $E_z$ creates a current $J_z$ inside the wire and pushes those conduction electrons along. Does this $E_z$ push on the surface charge density and make it do something? Does it make a surface current in the z direction? If this surface charge is really in a little channel at the surface, then yes, there should be a surface current. The free electrons then oscillate back and forth in the z direction along with z. [ Final Reader Exercise ]