First Paradox of the Day for 9_19_14 PhL 9.19.14

This relates to the relation between I and N_0 ending up with $I = (\omega/k)\, CV$ which is a relatively new equation in lines doc. I added it just now as (D.31.2c) so it will be locatable. I use the result in Chapter 7 when taking $\omega \rightarrow 0$ where I want to see k and ω clearly indicated. I also use it in (D.9.25) where I turn on G in Appendix D.

This started as a little paradox, but that soon went away.

This relates directly to Appendix D, so it gets stored there. This review done 9/23/14.

1. In Appendix D I claim that

$$I = 2\pi\omega\,(a/k)\,N_0 = 2\pi a\,(\omega/k)\,N_0$$

But I also claim that

$$N_0 = \langle n(\theta)\rangle = q/(2\pi a) = CV/(2\pi a)$$

This I can write

$$I = 2\pi a\,(\omega/k)\,N_0 = 2\pi a\,(\omega/k)\,CV/(2\pi a)$$

$$= (\omega/k)\,CV.$$

2. On the other hand, I have always claimed that

$$I = V/Z_0$$

3. Comparing these two expressions for the same quantity I requires that

$$(\omega/k)\,CV = V/Z_0 \qquad \text{or} \qquad (\omega/k)\,C = 1/Z_0$$

Cancelling V, this says

$$\omega C = k/Z_0$$

Is this true? We know that

$$k = -j\sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{(R+j\omega L)/(G+j\omega C)}$$

Therefore

$$k/Z_0 = -j\sqrt{(R+j\omega L)(G+j\omega C)} * \sqrt{(G+j\omega C)/(R+j\omega L)} = -j\,(G+j\omega C)$$

But in App D for the CP BC I assume that G = 0, so then

$$k/Z_0 = -j\,(0+j\omega C) = -j^2\omega C = \omega C$$

and so there is no paradox!

4. Let's redo this where we allow G > 0 using the App D generalization to this condition.
The rule as I show in Section D.9 (?) is to replace $N_m \rightarrow (\xi_d/\varepsilon_d)\, N_m$. Thus, the above discussion would say

$$N_0(\text{new}) = CV/(2\pi a) * (\xi_d/\varepsilon_d) = C'\,V/(2\pi a)$$

$$I = 2\pi\omega\,(a/k)\,N_0(\text{new}) = 2\pi a\,(\omega/k)\,C'\,V/(2\pi a) = (\omega/k)\,C'\,V$$

and

$$I = V/Z_0$$

so we then want to show that

$$\omega C' = k/Z_0 \qquad \text{where } j\omega C' = (\xi_d/\varepsilon_d)j\omega C = G + j\omega C \qquad (4.11.24)$$

Then we find that

$$k/Z_0 = -j\,(G+j\omega C) = -j\,(j\omega C') = \omega C'$$

and result is reconfirmed.