How to plot the transverse electric field in a round wire PhL 12.12.13

This never got much off the ground. I proposed the reader do it at the end of Appendix D.

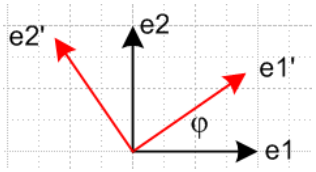
I have not done this, but I think this is how you could do it. First, pick z = 0. Then I think we have

$$\mathbf{E}_{\mathtt{transverse}}(r,\varphi) = \cos(-\omega t + m\varphi + \arg[E_{\varphi}(r)]\,)\,|E_{\varphi}|\;\hat{\boldsymbol{\varphi}} + \cos(-\omega t + m\varphi + \arg[E_{r}(r)]\,)\,|E_{r}|\;\hat{\mathbf{r}}$$

Then use

$$\mathbf{e}'_1 = \cos\varphi\,\mathbf{e}_1 + \sin\varphi\,\mathbf{e}_2$$
$$\mathbf{e}'_2 = -\sin\varphi\,\mathbf{e}_1 + \cos\varphi\,\mathbf{e}_2$$

Proof #1:



to replace

$$\hat{\boldsymbol{\varphi}} = -\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}}$$
$$\hat{\mathbf{r}} = \cos\varphi\,\hat{\mathbf{x}} + \sin\varphi\,\hat{\mathbf{y}}$$

and then you have

$$\mathbf{E}_{\mathtt{transverse}}(r,\varphi) = \cos(-\omega t + m\varphi + \arg[E_{\varphi}(r)]\,)\,|E_{\varphi}|\;\mathbf{[}-\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}}\mathbf{]}$$
$$+ \cos(-\omega t + m\varphi + \arg[E_{r}(r)]\,)\,|E_{r}|\;\mathbf{[}\cos\varphi\,\hat{\mathbf{x}} + \sin\varphi\,\hat{\mathbf{y}}\,\mathbf{]}$$

and then given the complex expressions for $E_{\mathtt{r}}(r,m)$ and $E_{\varphi}(r,m)$, you could plot the actual physical transverse electric field vector $\mathbf{E}_{\mathtt{transverse}}(r,\varphi)$ at this instant in time. This is for z = 0. The vector's direction and magnitude will change with t in a complicated way.

Ah, but for fixed r and φ this has the form

$$\mathbf{E}_{\mathtt{transverse}}(r,\varphi) = \cos(\omega t - a)\,\hat{\mathbf{x}} + \cos(\omega t - b)\,\hat{\mathbf{y}}$$

and I now know this describes an ellipse. So at any point, the $E_{\mathtt{t}}$ vector goes in an ellipse. This suggests to me that if you had a Maple arrow graph animation, the arrows would move in a very complicated way. The "current" in the wire then looks different geometrically at each instant in time! So you would have to print out a sequence of 2D arrow graphs to get the nature of things.

Idea. Go back to the complex form

$$\mathbf{E_t} = e^{jm\varphi}e^{-j\omega t}(E_{\varphi}\,\hat{\boldsymbol{\varphi}} + E_{r}\,\hat{\mathbf{r}})$$

$$= e^{jm\varphi}\, e^{-j\omega t}(E_{\varphi}[-\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}}] + E_{r}\,[\cos\varphi\,\hat{\mathbf{x}} + \sin\varphi\,\hat{\mathbf{y}}])$$

$$= e^{jm\varphi}\, e^{-j\omega t}(\,(E_{r}\cos\varphi - E_{\varphi}\sin\varphi)\,\hat{\mathbf{x}} + (E_{r}\sin\varphi + E_{\varphi}\cos\varphi)\,\hat{\mathbf{y}}\,)$$

Well never mind, there is no $E_r \pm jE_\varphi$ sitting here.