Repair of Appendix D for m = 0 error PhL 3.6.14

Just as it says, I found an error ONLY in the Appendix D results for m = 0. I guess this error did not affect the comparison with Chapter 2 or I would have found it earlier. I think is all OK.

I luckily found an error today and will now attempt to fix it.

Here first is a review of the math with Maple verification. I start off with

First summary of the E field solutions (D.2.21)

$$E_z(r,m) = -j\,(\beta'/\beta_d)\,\frac{K_m}{2}\,J_m(x) \qquad x = \beta' r \qquad \textit{(D.1.27)}$$

$$E_r(r,m) = a_m\,x^{-1}\,J_m(x) + \frac{K_m}{2}\,J_{m+1}(x) \qquad \beta'^2 = \beta^2 - \beta_d^2 \qquad \textit{(D.2.11)}$$

$$jE_\varphi(r,m) = -a_m\,x^{-1}\,J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x) \qquad \textit{(D.2.15)}$$

and I want to apply these two boundary conditions,

$$E_r(r=a,m) = (j\omega/\sigma)\,N_m \qquad \text{(D.2.26)}$$

$$E_\varphi(r=a,m) = 0 \qquad \text{(D.2.27)}$$

The boundary conditions then say, in terms of the last two equations in (D.2.21),

$$a_m\,x_a^{-1}\,J_m(x_a) + \frac{K_m}{2}\,J_{m+1}(x_a) = (j\omega/\sigma)\,N_m \qquad (1)$$

$$-a_m\,x_a^{-1}\,J_m(x_a) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x_a) = 0\ . \qquad (2)$$

OK

I now enter these equations into Maple and have it solve for a_m (= am_s) and K_m (= Km_s),

```
restart; alias(I=I,j=sqrt(-1),J = BesselJ);
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$$j, J$$

```
eq1 := am*(1/x)*J(m,x) + (1/2)*Km*J(m+1,x) = (j*omega/sigma)*Nm;
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$$eq1 := \frac{am\,J(m,x)}{x} + \frac{1}{2}Km\,J(m+1,x) = \frac{j\,\omega\,Nm}{\sigma}$$

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eq2 := -am*(1/x)*J(m,x) +(Km/2 + am/m)*J(m+1,x) = 0;
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$$eq2 := -\frac{am\,J(m,x)}{x} + \left(\frac{1}{2}Km + \frac{am}{m}\right)J(m+1,x) = 0$$

```
s := solve({eq1,eq2},{am,Km});
```

$$s := \{am = -\frac{j\,\omega\,Nm\,x\,m}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)},\ Km = 2\,\frac{j\,\omega\,Nm\,(-J(m,x)\,m + J(m+1,x)\,x)}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)\,J(m+1,x)}\}$$

```
s[1];
```

$$am = -\frac{j\,\omega\,Nm\,x\,m}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)}$$

```
am_s := rhs(s[1]);
```

$$am_s := -\frac{j\,\omega\,Nm\,x\,m}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)}$$

```
Km_s := rhs(s[2]);
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$$Km_s := 2\,\frac{j\,\omega\,Nm\,(-J(m,x)\,m + J(m+1,x)\,x)}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)\,J(m+1,x)}$$

Then I enter *my* solutions and see if they are the same,

```
am_p := m*(j*omega/(2*sigma))*2*Nm/J(m-1,x);
```

$$am_p := \frac{j\,m\,\omega\,Nm}{\sigma\,J(m-1,x)}$$

```
Km_p := (j*omega/(sigma))*Nm*(1/J(m+1,x) - 1/J(m-1,x));
```

$$Km_p := \frac{j\,\omega\,Nm\left(\frac{1}{J(m+1,x)} - \frac{1}{J(m-1,x)}\right)}{\sigma}$$

```
am_s-am_p; simplify(%);
```

$$-\frac{j\,\omega\,Nm\,x\,m}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)} - \frac{j\,m\,\omega\,Nm}{\sigma\,J(m-1,x)}$$

$$0$$

```
Km_s-Km_p; simplify(%);
```

$$2\,\frac{j\,\omega\,Nm\,(-J(m,x)\,m + J(m+1,x)\,x)}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)\,J(m+1,x)} - \frac{j\,\omega\,Nm\left(\frac{1}{J(m+1,x)} - \frac{1}{J(m-1,x)}\right)}{\sigma}$$

$$0$$

The only other thing to check is the sum thing:

```
sum_s := Km_s/2 + am_s/m;
```

$$sum_s := \frac{j\,\omega\,Nm\,(-J(m,x)\,m + J(m+1,x)\,x)}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)\,J(m+1,x)} - \frac{j\,\omega\,Nm\,x}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)}$$

```
sum_p := (j*omega/(2*sigma))*Nm*(1/J(m+1,x) + 1/J(m-1,x));
```

$$sum_p := \frac{1}{2}\,\frac{j\,\omega\,Nm\left(\frac{1}{J(m+1,x)} + \frac{1}{J(m-1,x)}\right)}{\sigma}$$

```
sum_s-sum_p;simplify(%);
```

$$\frac{j\,\omega\,Nm\,(-J(m,x)\,m + J(m+1,x)\,x)}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)\,J(m+1,x)} - \frac{j\,\omega\,Nm\,x}{\sigma\,(-2\,J(m,x)\,m + J(m+1,x)\,x)} - \frac{1}{2}\,\frac{j\,\omega\,Nm\left(\frac{1}{J(m+1,x)} + \frac{1}{J(m-1,x)}\right)}{\sigma}$$

$$0$$

So I am convinced that my general results are correct as stated in (D.2.28).

$$a_m = (j\omega/2\sigma)\ 2m\ N_m \frac{1}{J_{m-1}(x_a)}\ . \qquad \Rightarrow \qquad a_0 = 0 \qquad \text{(D.2.28)}$$

$$\frac{K_m}{2} = (j\omega/2\sigma)\ N_m \left[\frac{1}{J_{m+1}(x_a)} - \frac{1}{J_{m-1}(x_a)}\right] \qquad \Rightarrow \qquad \frac{K_0}{2} = (j\omega/\sigma)\ N_0 \frac{1}{J_1(x_a)} \qquad // J_{-1}(z) = -J_1(z)$$

$$\left(\frac{K_m}{2} + \frac{a_m}{m}\right) = (j\omega/2\sigma)\ N_m \left[\frac{1}{J_{m+1}(x_a)} + \frac{1}{J_{m-1}(x_a)}\right]$$

and I rewrite the first one as

$$\frac{a_m}{m} = (j\omega/2\sigma)\ 2\ N_m \frac{1}{J_{m-1}(x_a)}\ . \qquad (5)$$

The issue at hand is the above ratio when m = 0. We know that $a_0 = 0$ from (D.2.28) and of course m = 0 so we have a 0/0 situation. The correct ratio is given by (5) above,

$$\frac{a_0}{0} = (j\omega/2\sigma)\ 2\ N_0 \frac{1}{J_{-1}(x_a)} = -(j\omega/2\sigma)\ 2\ N_0 \frac{1}{J_1(x_a)}$$

but I accidentally set this ratio to to zero in several places! So I keep marching through Appendix D now.

I verified (visual tracking) all the algebra of inserting the coefficients(D.2.28) into (D.2.21) to get (D.2.33). That algebra luckily is written out above (D.2.33).

Second summary of the E field solutions : $\quad R_{dc} = \frac{1}{\sigma\pi a^2} \quad \beta'^2 = \beta^2 - \beta_d^2 \quad$ (D.2.33)

$$E_z(r,m) = (1/4)\ \eta_m\ I\ R_{dc}\ (a\beta')\left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] \qquad a = \text{radius} \qquad \eta_m \equiv \frac{N_m}{N_0}$$

$$E_r(r,m) = (j/4)\ \eta_m\ I\ R_{dc}\ (a\beta_d)\left[\frac{2mx^{-1}J_m(x)}{J_{m-1}(x_a)} + \frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m+1}(x)}{J_{m-1}(x_a)}\right] \qquad x = \beta' r$$

$$E_\varphi(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta_d)\ \left[\ -\frac{2mx^{-1}J_m(x)}{J_{m-1}(x_a)} + \frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m+1}(x)}{J_{m-1}(x_a)}\right] \qquad x_a = \beta' a$$

My next collection however goes back to the coefficients, it is (D.4.9), and this is the box that I used to do the m = 0 reductions:

$$E_z(r,m) = -\,j\,(\beta'/\beta_d)\,\frac{K_m}{2}\,J_m(x) \qquad x = \beta' r \qquad \beta'^2 = \beta^2 - \beta_d^2$$

$$E_r(r,m) = a_m\, x^{-1}\, J_m(x) + \frac{K_m}{2}\, J_{m+1}(x)\ .$$

$$jE_\varphi(r,m) = -\,a_m\, x^{-1}\, J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x) \qquad \textit{(D.2.21)}$$

$$B_z(r,m) = (\beta'/\omega)\left(\frac{K_m}{2} + \frac{a_m}{m}\right)J_m(x)$$

$$B_r(r,m) = j(\beta'/\omega)\left\{ + \left(m\frac{K_m}{2}\frac{\beta'}{\beta_d} - \frac{\beta_d}{\beta'}\,a_m\right) x^{-1}J_m(x) + \frac{\beta_d}{\beta'}\left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x)\right\}$$

$$B_\varphi(r,m) = (\beta'/\omega)\left\{ - \left(m\frac{K_m}{2}\frac{\beta'}{\beta_d} - \frac{\beta_d}{\beta'}\,a_m\right) x^{-1}J_m(x) + \frac{K_m}{2}\left(\frac{\beta_d}{\beta'} + \frac{\beta'}{\beta_d}\right) J_{m+1}(x)\right\}, \qquad \textit{(D.4.8)}$$

The key facts are:

$$\left(\frac{K_m}{2} + \frac{a_m}{m}\right)\Big|^{m=0} = (j\omega/2\sigma)\, N_0 \left[\frac{1}{J_1(x_a)} + \frac{1}{J_{-1}(x_a)}\right] = (j\omega/2\sigma)\, N_0 \left[\frac{1}{J_1(x_a)} - \frac{1}{J_1(x_a)}\right] = 0$$

$$\frac{K_0}{2} = (j\omega/\sigma)\, N_0\, \frac{1}{J_1(x_a)} =$$

Here then is my corrected (D.6.1)

Summary of E and B fields inside a round wire (m = 0 only) (D.6.1)

$$E_z(r,0) = -\,j\,(\beta'/\beta_d)\,\frac{K_0}{2}\,J_0(x) \qquad \text{// large} \qquad a_0 = 0 \qquad x = \beta' r \qquad \beta'^2 = \beta^2 - \beta_d^2$$

$$E_r(r,0) = \frac{K_0}{2}\,J_1(x)\ . \qquad \text{// small} \qquad \frac{K_0}{2} = (j/2)\,(a\beta_d)\, I\, R_{dc}\,\frac{1}{J_1(x_a)}$$

$$jE_\varphi(r,0) = 0 \qquad \text{// small} \qquad R_{dc} = \frac{1}{\sigma\pi a^2}$$

$$B_z(r,0) = 0 \qquad \text{// small} \qquad \sim \beta'$$

$B_r(r,0) = 0$ // very small $\sim \beta_d$

$B_\phi(r,0) = (\beta'/\omega)\left(\frac{\beta_d}{\beta'} + \frac{\beta'}{\beta_d}\right)\frac{K_0}{2} J_1(x)$ // large $\sim \beta' (\beta'/\beta_d)$

Ouch, I have not one but three mistakes in this box.

I did a long round of edits just now in lines doc, see edit log, and I included the above fixes.