The E_φ Question PhL 12.12.13

The $E_\varphi = 0$ question is now at least addressed in Section D.8 with two proposed arguments why $E_\varphi = 0$ on the surface. Below I also ask questions that are now in the Reader Exercise at the end of App D, and I think the new picture I placed there goes a long way to answering the question of "where the charge comes from".

Question 1: I have addressed this question in several different docs, and the answer keeps wobbling back and forth. The question is this: in some partial wave m, can you claim that $E_\varphi(r=a,m) = 0$ based on some argument about quasi-static and the surface charge would just move around if not true? This is the same as the claim that the cross section surface is an equipotential surface, as I recall King claiming somewhere. I have argued this eagerly both ways.

A related question comes first:

Question 2: We know that the surface charge pattern moves down a transmission line. One can think of this as being due to the EM wave passing by, and "pulling up" charge on the surface to match the strong normal E field. The question is : Where does that surface charge come from?

When I say "surface charge", I refer to the function or pattern, not to an individual charged particle (electron). I know that each electron always does a very tiny orbit in situ, and the effect is like the ten in a bed and the end on said roll over. It is the density of free charges not the orbits that creates the surface pattern.

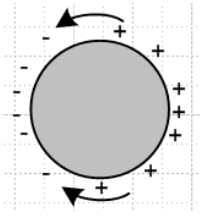
Theory 1 is that the surface charge is pumped up to the surface by the radial internal field E_r. One might imagine for m = 1 that on one side this E_r extracts plus charges and pushes them to the other side. That would work in the figure below, for example.

Theory 2 is that the E_z tangential charge causes the charge pattern. Here is how that might work,

t = 0	→	←	E_z pushes charge to the center
t = T/2	←	→	E_z now pulls that charge away

In this theory, it is the surface charge that is getting moved, whereas in the previous theory the charge is passing through the bulk interior. The arrows above would be a half λ

Theory 3 is that the E_φ tangential field causes the charge pattern. It moves the surface charge from one side to the other for m = 1 as I once drew in a picture



If theory 3 has some validity, then there might be some $E_\varphi \neq -$ at the surface and this of course relates to our main question.

Theory 4 is maybe that all three processes contribute to the surface charge pattern change.

So how do you determine where the charge comes from? I do know that the field pattern is this inside:

First summary of the E field solutions (D.2.21)

$$E_z(r,m) = -j\,(\beta'/\beta_d)\,\frac{K_m}{2}\,J_m(x) \qquad x = \beta' r \qquad \textit{(D.1.27)}$$

$$E_r(r,m) = a_m\,x^{-1}\,J_m(x) + \frac{K_m}{2}\,J_{m+1}(x)\ . \qquad \textit{(D.2.11)}$$

$$jE_\varphi(r,m) = -\,a_m\,x^{-1}\,J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x)\ . \qquad \textit{(D.2.15)}$$

There is a correlation between the three field components via the functions shown and the two unknown constants.

An **analogous problem** of interest is this: Imagine a charge q travelling at small velocity v a distance d above and parallel to the surface of a half space of conductor. This charge induces a known pattern on the conductor, and that pattern moves along with the charge. The math is a lot easier in this problem. We can ask the same question here? As you sit at a spot and wait for the charge pulse to go by, where does that charge come from? This charge in motion is a surface current in the z direction on the half space surface. It certainly seems in this case that the surface charge is just moving left to right along the surface and there is no pumping of charge to the surface from the interior. We are quasi-static so E = 0 inside the conductor we presume. Well, since the charge q is moving, we are NOT static, and there could be E_z along the surface and even E below the metal surface. If we oscillate the charge at a high rate, are things different?

Note that, as long as $E_r(a,m)$ is not zero,

$$\frac{E_r(r,m)}{E_r(a,m)} = \frac{a_m\,x^{-1}\,J_m(x) + \frac{K_m}{2}\,J_{m+1}(x)}{a_m\,x_a^{-1}\,J_m(x_a) + \frac{K_m}{2}\,J_{m+1}(x_a)}$$

I just have no idea what this looks like. The x is complex, and I don't know the constants.

I think I can relate $E_r(a.m)$ to the charge density of partial wave m through my N_m moment idea, so that does say that you cannot have both a_m and K_m vanish.