What is the nature of the function n(φ) PhL 12.3.13

I think I have resolved these issues, see conclusions at the end; sections D.1 (a) and (b) now reflect what came out of this doc.

This function n(φ) appears in Appendix D, and by "what is the nature" I mean things like

(1) is it real?
(2) is it complex?
(e) if it is complex, does the phase of n(φ) vary with φ?

This is a nasty little question set that has till now been swept under the rug.

1. In Appendix D right now I have this expansion for the electric field

$$\mathbf{E}(r,\varphi,z,t) = e^{j(\omega t-\beta_d z)}\,[e^{jf(r,\omega)}\boldsymbol{E}(r,\omega)] \qquad (1.6.7)$$

$$\mathbf{E}(r,\varphi,z,t) = e^{j(\omega t-\beta_d z)}\,\mathbf{E}(r,\varphi)\ . \qquad (D.1.2)$$

I think I know that the phase f(r,ω) varies with r because that is what Chapter 2 showed. I can write

$$\mathbf{E}_{physical} = \mathrm{Re}\{\,\mathbf{E}(r,\varphi,z,t)\} = \cos[\omega t - \beta_d z + f(r,\omega)]\,\boldsymbol{E}(r,\omega)$$

where $\mathbf{E}_{physical}$ and $\boldsymbol{E}(r,\omega)$ are both real. This form may not be general enough if the components each have their own phase function. The point is that the cosine phase can be a function of r due to the swirling nature of activities inside the round wire. Recall in Chapter 2 that

$$\mathbf{E}(x,y,z,t) = e^{j(\omega t-kz)}\,\mathbf{E}(x,y) \qquad (2.1.1)$$

$$\mathbf{E}(x,y,z,t) = E_0\, e^{j\omega t}\,\frac{\mathrm{ber}_0[r\sqrt{\omega\mu\sigma}\,] + j\,\mathrm{bei}_0[r\sqrt{\omega\mu\sigma}\,]}{\mathrm{ber}_0[a\sqrt{\omega\mu\sigma}\,] + j\,\mathrm{bei}_0[a\sqrt{\omega\mu\sigma}\,]}\,\hat{\mathbf{z}} \qquad (2.3.14)$$

and here we see **proof** of the fact that there is some f(r,ω) which is a function of both r and ω, although it happens to be a function of $r\sqrt{\omega}$.

2. Now the question is this: How would you write an expansion for surface charge density n ? If we start off doing the analog of what was done above, we might write

$$n(a,\varphi,z,t) = e^{j(\omega t-\beta_d z)}\,[e^{jg(a,\omega)}n(a,\varphi,\omega)] \qquad (1.6.7)$$

$$n(a,\varphi,z,t) = e^{j(\omega t-\beta_d z)}\,n(a,\varphi,\omega)\ . \qquad (D.1.2)$$

This seems to be a reasonably general form for this expansion to take. I certainly allow φ dependence. Now it is all at r = a so delete that a and write

$$n(\varphi,z,t) = e^{j(\omega t-\beta_d z)}\,[e^{jg(\omega)}n(\varphi,\omega)] \qquad (1.6.7)$$

$$n(\varphi,z,t) = e^{j(\omega t-\beta_d z)}\, n(\varphi,\omega)\ . \qquad \text{(D.1.2)}$$

3. Now we have this very important boundary condition which "drives" the surface density

$$E_r(r=a,\varphi,z,t) = (j\omega/\sigma)\, n(\varphi,z,t)$$

Inserting our expansions we then get

$$[e^{jf(a,\omega)} E_r(a,\omega)] = (j\omega/\sigma)\, [e^{jg(\omega)} n(\varphi,\omega)]$$

What is wagging what here? If the $[e^{jg(\omega)} n(\varphi,\omega)]$ is prescribed, then that determines $[e^{jf(a,\omega)} E_r(a,\omega)]$. There is no way to "determine" $[e^{jg(\omega)} n(\varphi,\omega)]$ or g(ω). You prescribe it, and that then will determine the problem's solution $E_r(r,\omega)$.

4. What about for our coaxial cable central wire context? Would we not like in some sense to be able at some point to say that the surface charge density is REAL ? But it only has m = 0 which is a non-issue.

5. What about a simple twin lead with two round wires. There will exist some real n on the left wire of some sort. It will be a function of φ for sure. This is $n_{physical}(\varphi)$. How is this related to $E_{physical}$?

$$E_{r,physical}(r=a,\varphi,z,t) = (1/\sigma)\partial_t\, n_{physical}(\varphi,z,t) \qquad \text{.// BC in t domain} \qquad \text{(1)}$$

Right here in the time domain we have to say something about complex E_r and complex n. Let's define a complex E_r nd a complex n as follows

$$E_r(r,\varphi,z,t) = E_r'(r,\varphi,z,t) + j\, E_r''(r,\varphi,z,t) \qquad \text{// real and imag decomposition}$$
$$n(r,\varphi,z,t) = n'(r,\varphi,z,t) + j\ n''(r,\varphi,z,t) \qquad \text{// real and imag decomposition} \qquad \text{(2)}$$

We might try then to make these associations

$$E_{r,physical}(r,\varphi,z,t) = E_r'(r,\varphi,z,t)$$
$$n_{physical}(\varphi,z,t) = n'(r,\varphi,z,t) \qquad \text{(3)}$$

Then the physical boundary condition says

$$E_r'(r=a,\varphi,z,t) = (1/\sigma)\partial_t n'(r,\varphi,z,t) \qquad \text{(4)}$$

and we can then regard this as the real part of this complex equation, assuming σ is real,

$$E_r(r=a,\varphi,z,t) = (1/\sigma)\partial_t n(r,\varphi,z,t) \qquad \text{(5)}$$

We can now FT this complex equation to get this complex equation in ω space,

$$E_r(r=a,\varphi,z,\omega) = (j\omega/\sigma) n(r,\varphi,z,\omega) \qquad \text{(6)}$$

NOW suppose we make this ansatz assumptions for our complex fields

$$E_r(r,\varphi,z,t) = e^{j(\omega_1 t-\beta_d z)}\,[e^{jf(r,\varphi,\omega_1)}E_r(r,\varphi,\omega_1)] \quad \text{// ansatz form t domain}$$
$$n(\varphi,z,t) = e^{j(\omega_1 t-\beta_d z)}\,[e^{jg(\varphi,\omega_1)}n(\varphi,\omega_1)] \quad \text{// ansatz form for n in t domain} \qquad (7)$$

We can compute the FT of each of these to get

$$E_r(r,\varphi,z,\omega) = 2\pi\,\delta(\omega-\omega_1)e^{-j\beta_d z}\,[e^{jf(r,\varphi,\omega_1)}E_r(r,\varphi,\omega_1)] \quad \text{// ansatz form in } \omega \text{ domain}$$
$$n(\varphi,z,\omega) = 2\pi\,\delta(\omega-\omega_1)e^{j(-\beta_d z)}\,[e^{jg(\varphi,\omega_1)}n(\varphi,\omega 1)] \quad \text{// ansatz form for n in } \omega \text{ domain} \qquad (8)$$

Our boundary condition now reads

$$2\pi\,\delta(\omega-\omega_1)e^{-j\beta_d z}\,[e^{jf(a,\varphi,\omega_1)}E_r(a,\varphi,\omega 1)] = (j\omega/\sigma)\,2\pi\,\delta(\omega-\omega_1)e^{j(-\beta_d z)}\,[e^{jg(\varphi,\omega_1)}n(\varphi,\omega 1)]$$

or

$$[e^{jf(a,\varphi,\omega_1)}E_r(a,\varphi,\omega 1)] = (j\omega/\sigma)\,[e^{jg(\varphi,\omega_1)}n(\varphi,\omega 1)] \qquad (9)$$

which then says

$$f(a,\varphi,\omega_1) = g(\varphi,\omega_1) + \pi/2$$
$$E_r(a,\varphi,\omega_1) = (\omega/\sigma)\,n(\varphi,\omega_1) \qquad (10)$$

These equations must be true in order to meet the physical boundary condition (1).

Now looking at (7) let's define two new field objects which are ID'd by their argument sequence

$$E_r(r,\varphi,z,t) = e^{j(\omega_1 t-\beta_d z)}\,[e^{jf(r,\varphi,\omega_1)}E_r(r,\varphi,\omega_1)] \quad \text{// ansatz form t domain}$$
$$n(\varphi,z,t) = e^{j(\omega_1 t-\beta_d z)}\,[e^{jg(\varphi,\omega_1)}n(\varphi,\omega_1)] \quad \text{// ansatz form for n in t domain} \qquad (7)$$

$$E_r(r,\varphi,z,t) = e^{j(\omega_1 t-\beta_d z)}\,[E_r(r,\varphi,\omega_1)] \quad \text{// ansatz form t domain}$$
$$n(\varphi,z,t) = e^{j(\omega_1 t-\beta_d z)}\,[n(\varphi,\omega_1)] \quad \text{// ansatz form for n in t domain} \qquad (11)$$

$$E_r(r,\varphi,\omega_1) = e^{jf(r,\varphi,\omega_1)}E_r(r,\varphi,\omega_1)$$
$$n(\varphi,\omega_1) = e^{jg(\varphi,\omega_1)}n(\varphi,\omega_1) \qquad (12)$$

How are these two objects related? Well, we have

$$E_r(r=a,\varphi,\omega_1) = e^{jf(a,\varphi,\omega_1)}E_r(a,\varphi,\omega_1)$$

$$n(\varphi,\omega_1) = e^{jg(\varphi,\omega_1)}n(\varphi,\omega_1) = e^{jf(a,\varphi,\omega_1)}\,e^{-j\pi/2}\,n(\varphi,\omega_1)$$

$$= (-j)\,e^{jf(a,\varphi,\omega_1)}\,(\sigma/\omega)\,E_r(a,\varphi,\omega_1) = (\sigma/j\omega)\,E_r(r=a,\varphi,\omega_1)$$

So we conclude that they have this relationship from the boundary condition:

$$n(\varphi,\omega_1) = (\sigma/j\omega)\, E_r(r=a,\varphi,\omega_1) \tag{13}$$

Now suppose we do partial wave expansions like this

$$E_r(r,\varphi) = \sum_{m=-\infty}^{\infty} E_r(r,m)\, e^{jm\varphi}$$

$$n(\varphi,\omega_1) = \sum_{m=-\infty}^{\infty} n(m)\, e^{jm\varphi} \tag{14}$$

Then in m space we would have

$$n(m) = (\sigma/j\omega)\, E_r(a,m) \tag{15}$$

Now it is a long way to Tipperary to get from n(m) back to the physical n, but let's try it.

$$n(\varphi,\omega_1) = \sum_{m=-\infty}^{\infty} n(m)\, e^{jm\varphi} \qquad \text{// step \#1}$$

$$n(\varphi,z,t) = e^{j(\omega 1 t-\beta_d z)}\, n(\varphi,\omega_1) \qquad \text{// step \#2}$$

$$n_{physical}(\varphi,z,t) = n'(r,\varphi,z,t) = \mathrm{Re}\{\, n(r,\varphi,z,t)\} \qquad \text{// step \#3}$$

Well, not so far after all. so

$$n_{physical}(\varphi,z,t) = \mathrm{Re}\ \{\, e^{j(\omega 1 t-\beta_d z)} \sum_{m=-\infty}^{\infty} n(m)\, e^{jm\varphi}\ \} \tag{16}$$

Question: In (13), can you make an ansatz that $n(\varphi,\omega_1)$ is real? That would mean $g(\varphi,\omega_1) = 0$ and that would in turn mean

$$E_r(r=a,\varphi,\omega_1) = e^{j\pi/2} E_r(a,\varphi,\omega_1) = j\, E_r(a,\varphi,\omega_1)$$

and that would mean that $E_r(r=a,\varphi,\omega_1)$ was pure imaginary. We would still have from (12)

$$E_r(r,\varphi,\omega_1) = e^{jf(r,\varphi,\omega 1)} E_r(r,\varphi,\omega_1) \qquad f(a,\varphi,\omega_1) = \pi/2 \tag{17}$$

This does not force $f(r,\varphi,\omega_1)$ to be $\pi/2$, just where r = a. Perhaps this is an OK ansatz.

What then are the implications of $n(\varphi,\omega_1)$ being real? $n(\varphi,z,t)$ is still $e^{j(\omega 1 t-\beta_d z)}\, n(\varphi,\omega_1)$ which is complex in the usual wave sense. But then $N_m = (1/2\pi) \int_{-\pi}^{\pi} d\varphi\, n(\varphi)\, e^{-jm\varphi}$ says

$n(-m) = n(m)^*$

and then you can get your one-sided expansion for n(φ). From (11) we would have

$n(\varphi,z,t) = e^{j(\omega_1 t - \beta_d z)}\ [n(\varphi,\omega_1)]$ // ansatz form for n in t domain (11)

$n_{physical}(\varphi,z,t) = Re\{\ n(\varphi,z,t)\}\ = \cos(\omega_1 t - \beta_d z)\ n(\varphi,\omega_1)$

which seems reasonable.

General Discussion of Appendix D.

Why do I even need to mention n ? The surface charge n is not used anywhere until I write down the boundary condition (D.2.26) which says $E_r(r=a,m) = (j\omega/\sigma)\ N_m$ which is (15) above.

So in all Appendix D, my only use of surface charge n and its moments is to write the constants in the solution in terms of the moments N_m. I could just leave the solution as in box (D.2.21) where the constants are called a_m and K_m. But after applying the two BC's, the fields are expressed in terms of the single unknown constant N_m for each partial wave, and in general we must regard this as complex.

Comment: There is no need to ever even mention the field ***E*** so then I don't have to face the fact that the phase function $f(r,\omega_1)$ is different for each field component. That only appears in my little working area above, not in Appendix D itself. So at least I don't have to worry about that.

Action Plan: I have now rewritten D.1 (a) and (b) where I "allow for the case" that n(φ) = real as an added ansatz. I don't say it is required. I have reviewed (a) and (b) several times and thing things are tuned up, finally. I am now finally HAPPY down to D.4 where I left off last time.

I think I am now done here, but first, let's reconsider our opening questions which were :

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(1),(2) We can assume n(φ) is real by the added ansatz now stated in D.1 (b). But nothing in appendix D requires this extra ansatz to be true! We end up with everything in terms of constants N_m . If we assume that n(φ) is real and that n(φ) is even, then it happens that N_m are real and $N_{-m} = N_m$.

(3) With the extra ansatz just mentioned where n(φ) = real, the phase of n(φ) then does NOT vary with angle φ. Without this ansatz, if the phase presumably COULD vary, but nothing in Appendix D depends on whether or not n(φ) has a φ dependent phase.

Ending this doc now and returning to the main lines edit log.