9/20/14

This work has been installed verbatim into lines doc.

D.4 Computation of the B fields in the round wire

The **B** field components may be computed from the Maxwell curl **E** equation (1.1.2),

$$-\partial_t \mathbf{B} = \text{curl } \mathbf{E} \ . \qquad \text{Maxwell curl E equation} \qquad (1.1.2) \qquad \text{(D.4.1)}$$

In cylindrical coordinates one has from (D.1.14),

$$\text{curl } \mathbf{E} = \hat{\mathbf{r}} \left[r^{-1}\partial_\theta E_z - \partial_z E_\theta \right] + \hat{\boldsymbol{\theta}} \left[\partial_z E_r - \partial_r E_z\right] + \hat{\mathbf{z}} \left[r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r \right] \qquad \text{(D.4.2)}$$

where the fields are of the traveling wave form shown in (D.1.1) which we assume also for the **B** field. Thus, combining (D.1.1) with (D.1.3a), one has

$$\mathbf{E}(r,\theta,z,t) = e^{j(\omega t - kz)}\, \mathbf{E}(r,\theta) = e^{j(\omega t - kz)} \sum_{m=-\infty}^{\infty} \mathbf{E}(r,m)\, e^{jm\theta} \qquad \text{(D.4.3)}$$

$$\mathbf{B}(r,\theta,z,t) = e^{j(\omega t - kz)}\, \mathbf{B}(r,\theta) = e^{j(\omega t - kz)} \sum_{m=-\infty}^{\infty} \mathbf{B}(r,m)\, e^{jm\theta} \ . \qquad \text{(D.4.4)}$$

Inserting the three cylindrical components of the **E** expansion (D.4.3) into (D.4.2), one finds that these replacements may be made,

$$\partial_t \to +j\omega \qquad \partial_z \to -jk \qquad \partial_\theta \to +jm \ . \qquad \text{(D.4.5)}$$

Similarly, inserting the **B** expansion (D.4.4) into $-\partial_t \mathbf{B}$ one may replace $\partial_t \to +j\omega$. After doing this, both sides of (D.4.1) are expansions having the general form of (D.4.3) and one may then equate terms in the m sum [completeness of the $e^{jm\theta}$ on $(-\pi.\pi)$] to find that

$$-j\omega\mathbf{B}(r,m) = \hat{\mathbf{r}} \left[r^{-1}jmE_z + jkE_\theta \right] + \hat{\boldsymbol{\theta}} \left[-jkE_r - \partial_r E_z\right] + \hat{\mathbf{z}} \left[r^{-1}\partial_r(rE_\theta) - r^{-1}jmE_r \right] \qquad \text{(D.4.6)}$$

and this then gives the three partial wave components of the **B** field

$$B_r(r,m) = (j/\omega)\,[\text{curl } E]_r = (j/\omega)\,[r^{-1}jmE_z + jkE_\theta]$$

$$B_\theta(r,m) = (j/\omega)\,[\text{curl } E]_\theta = (j/\omega)[-jkE_r - \partial_r E_z]$$

$$B_z(r,m) = (j/\omega)\,[\text{curl } E]_z = (j/\omega)\,[r^{-1}\partial_r(rE_\theta) - r^{-1}jmE_r] \ . \qquad \text{(D.4.7)}$$

It is now a simple task to insert into these B_i expressions the E_i field components from box (D.2.33),

Second summary of the E field solutions : $R_{dc} = \frac{1}{\sigma\pi a^2}$ $\beta'^2 = \beta^2 - k^2$ *(D.2.33)*

$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\, (a\beta')\, f_m$ $f_m = [\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}]$ $x = \beta' r$

$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\, (ak)\, g_m$ $g_m = [\frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m-1}(x)}{J_{m-1}(x_a)}]$ $x_a = \beta' a$

$E_\theta(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\, (ak)\, h_m$ $h_m = [\frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m-1}(x)}{J_{m-1}(x_a)}]$

We have carried out this task manually to obtain the following results (which will be verified below),

$$B_r(r,m) = -(1/4)(a/\omega)\,\eta_m\, I\, R_{dc}\,(\, r^{-1} m\, (\beta')\, f_m + k^2\, h_m\,)$$

$$B_\theta(r,m) = (j/4)(a/\omega)\,\eta_m\, I\, R_{dc}\,(\, k^2\, g_m - \beta'^2 f_m\,[\,(m/x) - J_{m+1}(x)/J_m(x)]\,)$$

$$B_z(r,m) = (j/4)(a/\omega)\,\eta_m\, I\, R_{dc}\,(k\,\beta'\, e_m)\,. \qquad \text{(D.4.8)}$$

Here the f_m, g_m and h_m are the same functions appearing in the box above, and the new function e_m is

$$e_m \equiv [\frac{J_m(x)}{J_{m+1}(x_a)} + \frac{J_m(x)}{J_{m-1}(x_a)}]\,. \qquad \text{(D.4.9)}$$

Notice that the E_i and B_i fields all have the common factor $[(1/4)\,\eta_m\, I\, R_{dc}\, a\,]$. For purposes of *verifying* the B_i expressions above, we shall set this factor to 1 everywhere to obtain these scaled fields,

$$E_z(r,m) = \beta'\, f_m$$
$$E_r(r,m) = j\,k\, g_m$$
$$E_\theta(r,m) = k\, h_m$$

$$B_r(r,m) = -(1/\omega)\,(\, r^{-1} m\, (\beta')\, f_m + k^2\, h_m\,)$$
$$B_\theta(r,m) = j\,(1/\omega)\,(\, k^2\, g_m - \beta'^2 f_m\,[\,(m/x) - J_{m+1}(x)/J_m(x)]\,)$$
$$B_z(r,m) = j\,(1/\omega)\,(k\,\beta'\, e_m)\,. \qquad \text{(D.4.10)}$$

To verify that $-j\omega\mathbf{B} = \text{curl}\,\mathbf{E}$ for the above set of fields, we shall check these three equations

$$-j\omega B_i = [\text{curl}\,\mathbf{E}]_i \qquad ? \qquad i = r,\theta.z \qquad \text{(D.4.11)}$$

where from above

$$[\text{curl}\,\mathbf{E}]_r = [r^{-1} jm E_z + jk E_\theta]$$
$$[\text{curl}\,\mathbf{E}]_\theta = [-jk E_r - \partial_r E_z]$$
$$[\text{curl}\,\mathbf{E}]_z = [r^{-1}\partial_r(r E_\theta) - r^{-1} jm E_r]\,. \qquad \text{(D.4.12)}$$

We start by entering these three curl expressions into Maple,

```
restart; alias(I=I,j=sqrt(-1),J=BesselJ):
```

Enter the curlE expressions

```
curlEr := (j*m/r)*Ez + j*k*Eth;
```

$$curlEr := \frac{j\,m\,Ez}{r} + j\,k\,Eth$$

```
curlEth := -j*k*Er - Diff(Ez,r);
```

$$curlEth := -j\,k\,Er - \left(\frac{\partial}{\partial r} Ez\right)$$

```
curlEz := (1/r)*( Diff(r*Eth,r) - j*m*Er);
```

$$curlEz := \frac{\left(\frac{\partial}{\partial r} r\,Eth\right) - j\,m\,Er}{r}$$

followed by the scaled B and E field expressions from (D.4.10) above,

Enter the B expressions

```
Br := -(1/omega)*( (m*bp*f/r) + k^2*h );
```

$$Br := -\frac{\frac{m\,bp\,f}{r} + k^2\,h}{\omega}$$

```
Bth := (j/omega)*(k^2*g - bp^2*f*(m/x - J(m+1,x)/J(m,x)));
```

$$Bth := \frac{j\left(k^2\,g - bp^2\,f\left(\frac{m}{x} - \frac{J(m+1,x)}{J(m,x)}\right)\right)}{\omega}$$

```
Bz := (j/omega)*k*bp*e;
```

$$Bz := \frac{j\,k\,bp\,e}{\omega}$$

Enter the E field expressions

```
Ez := bp*f;
```

$$Ez := bp\,f$$

```
Er := j*k*g;
```

$$Er := j\,k\,g$$

```
Eth := k*h;
```

$$Eth := k\,h$$

Next come the various supporting functions,

Enter the supporting functions

```
e :=  J(m,x)/J(m+1,xa) + J(m,x)/J(m-1,xa);
```

$$e := \frac{J(m,x)}{J(m+1,xa)} + \frac{J(m,x)}{J(m-1,xa)}$$

```
f := J(m,x)/J(m+1,xa) - J(m,x)/J(m-1,xa);
```

$$f := \frac{J(m,x)}{J(m+1,xa)} - \frac{J(m,x)}{J(m-1,xa)}$$

```
g := J(m+1,x)/J(m+1,xa) + J(m-1,x)/J(m-1,xa);
```

$$g := \frac{J(m+1,x)}{J(m+1,xa)} + \frac{J(m-1,x)}{J(m-1,xa)}$$

```
h := J(m+1,x)/J(m+1,xa) - J(m-1,x)/J(m-1,xa);
```

$$h := \frac{J(m+1,x)}{J(m+1,xa)} - \frac{J(m-1,x)}{J(m-1,xa)}$$

```
x := bp*r;
```

$$x := bp\, r$$

```
xa := bp*a;
```

$$xa := bp\, a$$

Taking the precautions noted in the "Maple Comment" below (D.2.21), we now verify (D.4.11) that $-j\omega B_i = [\text{curl } E]_i$:

```
-j*omega*Br - curlEr: simplify(%);
```

$$0$$

```
-j*omega*Bth - value(curlEth): simplify(%);
```

$$0$$

```
-j*omega*Bz - value(curlEz): simplify(%);
```

$$0$$

Here then is a summary of all the E and B field results in a single box

Summary of E and B fields inside a round wire (D.4.13)

$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\, (a\beta')\, f_m$ $\quad x = \beta' r$ $\quad x_a = \beta' a$ $\quad \beta'^2 = \beta^2 - k^2$

$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\, (ak)\, g_m$

$E_\theta(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\, (ak)\, h_m$

$B_z(r,m) = (j/4)\,(a/\omega)\,\eta_m\, I\, R_{dc}\, (k\,\beta'\, e_m)$

$B_r(r,m) = -(1/4)\,(a/\omega)\,\eta_m\, I\, R_{dc}\, (\, r^{-1} m\ \beta'\, f_m + k^2\ h_m\,)$

$B_\theta(r,m) = (j/4)\,(a/\omega)\,\eta_m\, I\, R_{dc}\, (\, k^2\, g_m - \beta'^2 f_m\, [\, (m/x) - J_{m+1}(x)/J_m(x)]\,)$

$$e_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} + \frac{J_m(x)}{J_{m-1}(x_a)}\right] \qquad g_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right] \qquad R_{dc} = \frac{1}{\sigma\pi a^2}$$

$$f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] \qquad h_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right]$$

[The above box is now

D.5 Verification that the E and B fields satisfy the Maxwell equations

We already know that the Maxwell curl E equation is satisfied, since it was used in the previous section to derive the B fields. As for the other three Maxwell equations, we expect to find that

div $\mathbf{B} = 0$ // since $\mathbf{B} = (1/j\omega)$ curl $\mathbf{E}$ so div $\mathbf{B} = (1/j\omega)$ div curl $\mathbf{E} = 0$

div $\mathbf{E} = 0$ // no free charge inside conductor

$$\text{curl } \mathbf{B} = \mu \mathbf{J} + \mu j\omega\varepsilon \mathbf{E} = \mu(\sigma + j\omega\varepsilon)\mathbf{E} = \mu(j\omega)(\varepsilon - j\sigma/\omega)\mathbf{E} = j\omega\mu\xi\mathbf{E} \qquad \text{(D.5.1)}$$

$= j(\beta^2/\omega)\mathbf{E}$. // see (1.5.1c)

In cylindrical coordinates the curl and div operators are, from (D.1.14),

$$\text{curl } \mathbf{B} = \hat{\mathbf{r}}[r^{-1}\partial_\theta B_z - \partial_z B_\theta] + \hat{\boldsymbol{\theta}}[\partial_z B_r - \partial_r B_z] + \hat{\mathbf{z}}[r^{-1}\partial_r(rB_\theta) - r^{-1}\partial_\theta B_r]$$

$$\text{div } \mathbf{B} = r^{-1}\partial_r(rB_r) + r^{-1}\partial_\theta B_\theta + \partial_z B_z \qquad \text{(D.5.2)}$$

Using $\partial_z \rightarrow -jk$ and $\partial_\theta \rightarrow jm$ we can write these in m space as

$$\text{curl } \mathbf{B}(r,m) = \hat{\mathbf{r}}[r^{-1}jmB_z + jkB_\theta] + \hat{\boldsymbol{\theta}}[-jkB_r - \partial_r B_z] + \hat{\mathbf{z}}[r^{-1}\partial_r(rB_\theta) - r^{-1}jmB_r]$$

$$\text{div } \mathbf{B}(r,m) = r^{-1}\partial_r(rB_r) + r^{-1}jmB_\theta - jk\,B_z \,. \qquad \text{(D.5.3)}$$

We continue the Maple code of the previous section to verify that the other three Maxwell equations are satisfied:

Show that div B = 0

```
divB := (1/r)*diff(r*Br,r)+(1/r)*j*m*Bth-j*k*Bz: simplify(%);
```

0

Show that div E = 0

```
divE := (1/r)*diff(r*Er,r)+(1/r)*j*m*Eth-j*k*Ez: simplify(%);
```

0

Compute components of curl B

```
curlBr := (1/r)*j*m*Bz + j*k*Bth:
curlBth := -j*k*Br - diff(Bz,r):
curlBz := (1/r)*diff(r*Bth,r) - (1/r)*j*m*Br:
b := sqrt(bp^2+k^2):
```

set in beta here called b

One component at a time, show that the curl B Maxwell equation is satsified

```
curlBz - (j/omega)*b^2*Ez: simplify(%);
```

0

```
curlBr - (j/omega)*b^2*Er: simplify(%);
```

0

```
curlBth - (j/omega)*b^2*Eth: simplify(%);
```

0

The reader is again referred to the "Maple Comment" below (D.2.21). The expressions on the three last lines prior to simplification are quite complicated, for example

```
> curlBz - (j/omega)*b^2*Ez; simplify(%);
```

$$\left(\frac{j\left(k^2\left(\frac{J(m+1,bp\,r)}{J(m+1,bp\,a)}+\frac{J(m-1,bp\,r)}{J(m-1,bp\,a)}\right)-bp^2\left(\frac{J(m,bp\,r)}{J(m+1,bp\,a)}-\frac{J(m,bp\,r)}{J(m-1,bp\,a)}\right)\left(\frac{m}{bp\,r}-\frac{J(m+1,bp\,r)}{J(m,bp\,r)}\right)\right)}{\omega}+j\,r\left(\right.\right.$$

$$k^2\left(\frac{\left(J(m,bp\,r)-\frac{(m+1)\,J(m+1,bp\,r)}{bp\,r}\right)bp}{J(m+1,bp\,a)}+\frac{\left(-J(m,bp\,r)+\frac{(m-1)\,J(m-1,bp\,r)}{bp\,r}\right)bp}{J(m-1,bp\,a)}\right)$$

$$-bp^2\left(\frac{\left(-J(m+1,bp\,r)+\frac{m\,J(m,bp\,r)}{bp\,r}\right)bp}{J(m+1,bp\,a)}-\frac{\left(-J(m+1,bp\,r)+\frac{m\,J(m,bp\,r)}{bp\,r}\right)bp}{J(m-1,bp\,a)}\right)\left(\frac{m}{bp\,r}-\frac{J(m+1,bp\,r)}{J(m,bp\,r)}\right)$$

$$-bp^2\left(\frac{J(m,bp\,r)}{J(m+1,bp\,a)}-\frac{J(m,bp\,r)}{J(m-1,bp\,a)}\right)\left(-\frac{m}{bp\,r^2}-\frac{\left(J(m,bp\,r)-\frac{(m+1)\,J(m+1,bp\,r)}{bp\,r}\right)bp}{J(m,bp\,r)}+\frac{J(m+1,bp\,r)\left(-J(m+1,bp\,r)+\frac{m\,J(m,bp\,r)}{bp\,r}\right)bp}{J(m,bp\,r)^2}\right)\Bigg)/\omega$$

$$\left.\right)/r+\frac{j\,m\left(\frac{m\,bp\left(\frac{J(m,bp\,r)}{J(m+1,bp\,a)}-\frac{J(m,bp\,r)}{J(m-1,bp\,a)}\right)}{r}+k^2\left(\frac{J(m+1,bp\,r)}{J(m+1,bp\,a)}-\frac{J(m-1,bp\,r)}{J(m-1,bp\,a)}\right)\right)}{r\,\omega}-\frac{j\,(bp^2+k^2)\,bp\left(\frac{J(m,bp\,r)}{J(m+1,bp\,a)}-\frac{J(m,bp\,r)}{J(m-1,bp\,a)}\right)}{\omega}$$

$$0$$

No approximations were made in the E fields, the B fields, or in these Maxwell verifications.

D.6 The exact E and B fields for the m=0 partial wave

The m=0 partial wave is all there is for an axially symmetric problem like that considered in Chapter 2, where the round wire is imagined in isolation, but is operationally the central conductor of a coaxial cable with a very distant return cylinder (outer shield). Here is the reduction of box (D.4.13) for m = 0:

$$e_0 = \left[\frac{J_0(x)}{J_1(x_a)} + \frac{J_0(x)}{J_{-1}(x_a)}\right] = \left[\frac{J_0(x)}{J_1(x_a)} - \frac{J_0(x)}{J_1(x_a)}\right] = 0$$

$$f_0 = \left[\frac{J_0(x)}{J_1(x_a)} - \frac{J_0(x)}{J_{-1}(x_a)}\right] = \left[\frac{J_0(x)}{J_1(x_a)} + \frac{J_0(x)}{J_1(x_a)}\right] = 2\,\frac{J_0(x)}{J_1(x_a)}$$

$$g_0 = \left[\frac{J_1(x)}{J_1(x_a)} + \frac{J_{-1}(x)}{J_{-1}(x_a)}\right] = \left[\frac{J_1(x)}{J_1(x_a)} + \frac{J_1(x)}{J_1(x_a)}\right] = 2\,\frac{J_1(x)}{J_1(x_a)}$$

$$h_0 = \left[\frac{J_1(x)}{J_1(x_a)} - \frac{J_{-1}(x)}{J_{-1}(x_a)}\right] = \left[\frac{J_1(x)}{J_1(x_a)} - \frac{J_1(x)}{J_1(x_a)}\right] = 0 \qquad \text{(D.6.1)}$$

$$E_z(r,0) = (1/4)\,I\,R_{dc}\,(a\beta')\,f_0 = (1/4)\,I\,R_{dc}\,(a\beta')\,2\frac{J_0(x)}{J_1(x_a)}$$

$$E_r(r,0) = (j/4)\,I\,R_{dc}\,(ak)\,g_0 = (j/4)\,I\,R_{dc}\,(ak)\,2\frac{J_1(x)}{J_1(x_a)}$$

$$E_\theta(r,0) = (1/4)\,I\,R_{dc}\,(ak)\,h_0 = 0$$

$$B_z(r,0) = (j/4)\,(a/\omega)\,I\,R_{dc}\,(k\,\beta'\,e_0) = 0$$

$$B_r(r,0) = -\,(1/4)\,(a/\omega)\,I\,R_{dc}\,(k^2\,h_0) = 0$$

$$B_\theta(r,0) = (j/4)\,(a/\omega)\,I\,R_{dc}\,(\,k^2\,g_0 + \beta'^2 f_0\,[J_1(x)/J_0(x)]\,)$$

$$= (j/4)\,(a/\omega)\,I\,R_{dc}\,(\,k^2\,2\,\frac{J_1(x)}{J_1(x_a)} + \beta'^2\,2\,\frac{J_0(x)}{J_1(x_a)}\,[J_1(x)/J_0(x)]\,)$$

$$= (j/4)\,(a/\omega)\,I\,R_{dc}\,(\,k^2\,2\,\frac{J_1(x)}{J_1(x_a)} + \beta'^2\,2\,\frac{J_1(x)}{J_1(x_a)}\,)$$

$$= (j/4)\,(a/\omega)\,I\,R_{dc}\,(\,k^2 + \beta'^2\,)\,2\,\frac{J_1(x)}{J_1(x_a)}$$

$$= (j/4)\,(a/\omega)\,I\,R_{dc}\,\beta^2\,2\,\frac{J_1(x)}{J_1(x_a)} \qquad // \ \beta'^2 = \beta^2 - k^2 \qquad \text{(D.6.2)}$$

The results then are

Summary of E and B fields inside a round wire (m = 0 only) (D.6.3)

$$E_z(r,0) = (1/2)\,I\,R_{dc}\,(a\beta')\,\frac{J_0(x)}{J_1(x_a)} \qquad B_z(r,0) = 0 \qquad x = \beta' r \qquad x_a = \beta' a$$

$$E_r(r,0) = (j/2)\,I\,R_{dc}\,(ak)\,\frac{J_1(x)}{J_1(x_a)} \qquad B_r(r,0) = 0 \qquad R_{dc} = \frac{1}{\sigma\pi a^2}$$

$$E_\theta(r,0) = 0 \qquad B_\theta(r,0) = (j/2)\,(a/\omega)\,I\,R_{dc}\,\beta^2\,\frac{J_1(x)}{J_1(x_a)}$$

For a low-loss transmission line $k \approx \beta_{d0} = \omega/v_d$. As implied by the comments below (D.2.2), in this situation one has $k << |\beta|$ and thus also $k << |\beta'|$. Ignoring the difference between J_0 and J_1 in scale, (D.6.3) shows that in this situation $|\,E_r / E_z\,| \sim |\,k/\beta'\,| << 1$ so the E_r field is very small and E_z is the main electric field. For such a transmission line one has,

$$E_z(r,0) = (1/2)\,I\,R_{dc}\,(a\beta)\,\frac{J_0(x)}{J_1(x_a)} = (\omega/\beta)\,[(1/2)\,(a\beta^2/\omega)\,I\,R_{dc}]\,\frac{J_0(x)}{J_1(x_a)}$$

$$B_\theta(r,0) \approx (j/2)\,(a/\omega)\,I\,R_{dc}\,\beta^2\,\frac{J_1(x)}{J_1(x_a)} = j\,[(1/2)\,(a\beta^2/\omega)\,I\,R_{dc}]\,\frac{J_1(x)}{J_1(x_a)} \qquad \text{(D.6.4)}$$

where on the right we have rewritten the expressions in a seemingly obscure manner. As shown in (2.2.3), $\beta^2/\omega \approx -j\mu\sigma$ so that

$$[(1/2)\,(a\beta^2/\omega)\,I\,R_{dc}] = (1/2)\,a\,(-j\mu\sigma)\,I\,\frac{1}{\sigma\pi a^2} = -j\,\frac{\mu I}{2\pi a}\,. \qquad \text{(D.6.5)}$$

Since this is the bracket appearing in both field expressions in (D.6.4), we find that

$$E_z(r,0) = (\omega/\beta)\,[-j\,\frac{\mu I}{2\pi a}\,]\,\frac{J_0(x)}{J_1(x_a)} = -j\,(\omega/\beta)\,\frac{\mu I}{2\pi a}\,\frac{J_0(x)}{J_1(x_a)}$$

$$B_\theta(r,0) = j\,[-j\,\frac{\mu I}{2\pi a}\,]\,\frac{J_1(x)}{J_1(x_a)} = \frac{\mu I}{2\pi a}\,\frac{J_1(x)}{J_1(x_a)}\,. \qquad \text{(D.6.6)}$$

These results are in agreement with E(r) and B(r) shown in summary box (2.2.30) from the Chapter 2 calculation where we assumed $\mathbf{E} = E(r)\,\hat{\mathbf{z}}$ and $\mathbf{B} = B(r)\,\hat{\boldsymbol{\theta}}$.