Old tail to Section D.9 (d) PhL 10.6.14

This tail is very short, and gets replaced with a much longer tail! Just saving off the old here.

Above are the **"modified" charge pumping boundary conditions** which replace (D.2.24) and (D.2.25) for a mildly conducting dielectric,

$$E_r(r=a,\theta) = (j\omega/\sigma)\, n(\theta) \qquad \textit{(D.2.24)}$$
$$E_r(r=a,m) = (j\omega/\sigma)\, N_m \ . \qquad \textit{(D.2.25)}$$

How then does $\sigma_d \neq 0$ alter the E field results summarized in box (D.2.33)? The rule is this:

$$N_m \rightarrow (\xi_d/\varepsilon_d)\, N_m \quad \text{everywhere} \ . \qquad (D.9.23)$$

For example, for the total current we have

$$I = 2\pi a N_0(\omega/k) \ \rightarrow \ I' = 2\pi a N_0(\omega/k)\,(\xi_d/\varepsilon_d) \ . \qquad (D.9.24)$$

If we now denote by I the actual current in the transmission line (I for G = 0, or I' for G>0) , then the E field solutions as expressed in (D.2.33) *do not change form*, and we may write

Second summary of the E field solutions : $R_{dc} = \dfrac{1}{\sigma\pi a^2}$ $\beta'^2 = \beta^2 - k^2$ *(D.2.33)*

conducting dielectric

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta')\, f_m \qquad f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] \qquad x = \beta' r$$

$$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\ (ak)\, g_m \qquad g_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right] \qquad x_a = \beta' a$$

$$E_\theta(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (ak)\ h_m \qquad h_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right]$$

From (D.1.8) we know from (D.2.31c) that $I = CV(\omega/k)$, so the effect of turning on G is,

$$I = CV(\omega/k) \ \rightarrow \ I' = CV(\omega/k)\,(\xi_d/\varepsilon_d) = C'V(\omega/k) \qquad (D.9.25)$$

where C' is the complex capacitance per length as in (1.5.19). Thus, the entire effect of a conducting dielectric results in $C \rightarrow C'$ so that $I = CV(\omega/k) \rightarrow C'V(\omega/k)$ in the field expressions above.