Rejected App D BC argument PhL 12.1.13

I installed this briefly, but quickly removed it to this new location. I have a much better argument for why the wire surface is an equipotential, now explained below (D.1.2) in App D.

(c) Explanation of the Boundary Conditions

In order to justify the boundary conditions to be used below, we have to look ahead at the nature of solution for the E fields in the round wire. We are going to find that the field E_z is "large" while the two fields E_r and E_φ are both "small", where roughly small/large = $(a\beta_d) << 1$. In our transmission line analysis, we shall always be working in the "transmission line limit" which says that the wavelength of the wave going down the wire, $\lambda = 2\pi/\beta_d$, is large relative to all transverse dimensions of the transmission line, such as conductor thickness and conductor spacing. In this limit, it will of course be true that $(a\beta_d) << 1$ since a is less than the maximum transverse dimension of the transmission line cross section.

Given that E_φ is very small, we can make an approximation (an ansatz) that this field E_φ is exactly zero on the surface of the round wire. This may not be exactly true, but we assume it for our purposes and see where it leads. This is the nature of an "ansatz". When we make this assumption, the cross section of the transmission line may be regarded as a two dimensional potential theory problem, basically a capacitor problem where one conductor has potential V and the other -V, say. In such a potential problem, one always assumes that the electrostatic potential Φ is a constant on the surface of each conductor, and that is precisely what our ansatz says: $E_\varphi = -(\nabla\Phi)_\varphi = 0$, Φ = constant in the φ direction. Now when we solve the capacitor problem for potential Φ, that gives $\mathbf{E} = -\nabla\Phi$ in the dielectric between the conductors, and from that we may deduce $\mathbf{E} \bullet \hat{\mathbf{n}}$ at the surface of one of the conductors. For the round wire with a cylindrical coordinate system whose axis is aligned with the wire center, that field is E_r . Next, from this surface value of E_r (which will be proportional to V) we may compute the surface charge density $n(\varphi)$ on the round wire using (D.1.22) which says $E_r(r=a,\varphi) = (j\omega/\sigma)\, n(\varphi)$. For a "fat" twin lead transmission line for example we expect this to have a bulge in $n(\varphi)$ on the side of the wire facing the other wire, since that is what happens in such a capacitor. In any event, given $n(\varphi)$ we may compute the moments N_m of the surface charge using (D.1.6) and this then provides one "boundary condition" on our coefficients a_m and K_m which appear in all the field expressions we found above,

$$E_r(r=a,m) = (j\omega/\sigma)\, N_m \qquad \text{(D.2.18)}$$

But recall that, in order to carry out this entire process just described, we had to start with the assumption that $E_\varphi = 0$ on the conductor cross section surface, so that we could have a capacitor problem in the first place. According to the right equation of (D.1.3), if $E_\varphi(r=a,\varphi) = 0$, then $E_\varphi(r=a,m) = 0$, so that in fact we must have $E_r(a,m)$ being zero in all partial waves m. Thus our assumed ansatz condition was

$$E_\varphi(r=a,m) = 0 \qquad \text{(D.2.19)}$$

which is then a second boundary condition on a_m and K_m. Although (D.2.19) might not be exactly true, we know it is very close to being true. More importantly, we know that the above two conditions on a_m and K_m are consistent with each other, even though both boundary conditions might be slightly wrong. We

then expect them to give good values for constants a_m and K_m in the transmission line limit where the ansatz (D.2.19) is very reasonable.

There are three footnotes that one might add to the above discussion.

First, one might argue that the round wire surface is an equipotential because the surface is where the free charge lies, and if the surface were not an equipotential, the free charge would quickly adjust itself to make the surface be an equipotential.

Second, we note that the second boundary condition does not force $E_\varphi(r,m) = 0$ for $r < a$ inside the wire. In fact, there will be some small azimuthal "swirling" current inside the wire even if $E_\varphi(r=a,m) = 0$, and this is just a result of Maxwell's equations and their solutions above. One might make the alternative argument that the round wire surface is an equipotential since that is the way a line is driven at the source. For example, the center conductor of a coaxial cable plugs into a tiny driving cylinder (jack) in a BNC connector and this drives only the wire surface, and it does so in an azimuthally symmetric way so that one expects to have the wire surface be an equipotential at the driving point, and this equipotential surface then moves down the line as the wave progresses. But we know that things work just fine if the center conductor is driven instead by an abutting gold block, say, and such a driving mechanism would force the potential to be constant across the entire round face of the wire, which contradicts the fact that there must be those swirling currents in the wire interior. In this case, the swirling currents would in fact be zero at the driving gold block, but they would soon develop after a short transition distance in z. So it seems that the best way to understand (D.2.19) is in the self-consistent approximation sense outlined above.

Third, we have the complication that we don't really have a purely electrostatic situation, and the potential is in fact related to **E** by equation (1.3.1) which says $\mathbf{E} = -\nabla\Phi - \partial_t\mathbf{A}$. The rescue here comes by claiming that roughly $\mathbf{A} \approx A\hat{\mathbf{z}}$ so that the transverse components A_r and A_φ are very small. In this case, we then do get $\mathbf{E} \approx -\nabla\Phi$. The argument for $\mathbf{A} \approx A\hat{\mathbf{z}}$ is that **A** is driven by **J**, and **J** is mostly in the $\hat{\mathbf{z}}$ direction (by a wide margin).