What is A and φ for the Round Wire of Chapter 2 ? PhL 10.19.13

Comments 12/5/13. This seems to be the first time I ever tried this calculation, notice early date. Even now this subject is up in the air. I have something in Appendix B, but have not "gotten to it" yet. Remember that Chapter 2 is E and B field oriented, there is no potential. There ought to be a way to get that same result using potential theory, you would think. I later tried this in my "illogical" uniform J Helmholtz treatment. The King method requires a prescribed current, and that just doesn't work well with this problem where one of the "outputs" of the calculation is the current J and its skin effect.

This is something I never computed because that section was done using fields only. Remember that we always have gauge freedom for the potentials.

In Chapter 2 I assume that

$$\mathbf{E} = E\hat{\mathbf{z}} = E(r)\,\hat{\mathbf{z}}$$
$$\mathbf{B} = B\hat{\boldsymbol{\theta}} = B(r)\,\hat{\boldsymbol{\theta}}$$

Now let's look at our defining relations:

$$\mathbf{B} = \text{curl}\,\mathbf{A} \qquad \mathbf{E} = -\,\text{grad}\,\varphi - \partial_t\mathbf{A}\ . \tag{1.3.1}$$

We than have

$$B(r) = [\text{curl}\,\mathbf{A}]\bullet\hat{\boldsymbol{\theta}} = \partial_z A_r - \partial_r A_z$$

$$E(r) = -\partial_z\varphi(r) - \partial_t A_z(r) = -\partial_z\varphi(r) - j\omega\, A_z(r)$$

We are allowed to pick any gauge we want, so perhaps

$$\text{div}\,\mathbf{A} = 0 = \partial_z A_z + (1/r)\partial_r(rA_r) + (1/r)\partial_\theta A_\theta$$

In addition to the above, we also have

$$[\text{curl}\,\mathbf{A}]\bullet\hat{\mathbf{r}} = 0 \quad \Rightarrow \quad (1/r)\,\partial_\theta A_z - \partial_z A_\theta = 0$$

$$[\text{curl}\,\mathbf{A}]\bullet\hat{\mathbf{z}} = 0 \quad \Rightarrow \quad (1/r)\partial_r(rA_\theta) - (1/r)\partial_\theta A_r = 0$$

$$[-\,\text{grad}\,\varphi - \partial_t\mathbf{A}]\bullet\hat{\mathbf{r}} = 0 \quad \Rightarrow \quad -\partial_r\varphi - \partial_t A_r = 0 \quad \Rightarrow \quad -\partial_r\varphi - j\omega A_r = 0$$

$$[-\,\text{grad}\,\varphi - \partial_t\mathbf{A}]\bullet\hat{\boldsymbol{\theta}} = 0 \quad \Rightarrow \quad (1/r)\partial_\theta\varphi - \partial_t A_\theta = 0 \quad \Rightarrow \quad (1/r)\partial_\theta\varphi - j\omega A_\theta = 0$$

So here is a summary of all the equations in the ω domain

$(1/r)\,\partial_{\theta}A_z = \partial_z A_{\theta}$ 1

$\partial_r(rA_{\theta}) = \partial_{\theta}A_r$ 2

$\partial_r\varphi = -j\omega A_r$ 3

$\partial_{\theta}\varphi = rj\omega A_{\theta}$ 4

$\partial_z A_r - \partial_r A_z = B(r)$ 5

$\partial_z\varphi(r) + j\omega\, A_z(r) = -E(r)$ 6

$\partial_z A_z + (1/r)\partial_r(rA_r) + (1/r)\partial_{\theta}A_{\theta}$ = any function you want 7

One starting point is to require that $\partial_{\theta}\varphi = 0$ as a symmetry condition, which from 4 says $A_{\theta} = 0$. The equations then simplify to this smaller set

$(1/r)\,\partial_{\theta}A_z = 0$ 1

$\partial_{\theta}A_r = 0$ 2

$\partial_r\varphi = -j\omega A_r$ 3

$\partial_{\theta}\varphi = 0$ 4

$\partial_z A_r - \partial_r A_z = B(r)$ 5

$\partial_z\varphi + j\omega\, A_z = -E(r)$ 6

$\partial_z A_z + (1/r)\partial_r(rA_r)$ = any function you want 7

Now 1 says that $A_z = A_z(r,z)$.
Now 2 says that $A_r = A_r(r,z)$ and this is true for all functions.

Question: In item 7, why can't I select "any function" to be $(1/r)\partial_r(rA_r)$. Then the gauge condition says

$\partial_z A_z = 0$ => $A_z = A_z(r)$ only

The equations of interest are then

$\partial_r\varphi(r,z) = -j\omega A_r(r,z)$ 3

$\partial_z A_r(r,z) - \partial_r A_z(r) = B(r)$ 5

$$\partial_z\varphi(r,z) + j\omega A_z(r) = -E(r) \qquad 6$$

Suppose I use 3 to eliminate A_r. then

$$\partial_z[-\partial_r\varphi(r,z)/j\omega] - \partial_r A_z(r) = B(r) \qquad 5$$

$$\partial_z\varphi(r,z) + j\omega A_z(r) = -E(r) \qquad 6$$

or

$$(1/j\omega)[\partial_z\partial_r\varphi(r,z)] - \partial_r A_z(r) = B(r) \qquad 5$$

$$\partial_z\varphi(r,z) + j\omega A_z(r) = -E(r) \qquad 6$$

Now use 6 to eliminate $\partial_z\varphi$ to get

$$(1/j\omega)[\partial_r\{ -E(r) - j\omega A_z(r)\}] - \partial_r A_z(r) = B(r) \qquad 5$$

or

$$(1/j\omega)[\partial_r\{ E(r) + j\omega A_z(r)\}] + \partial_r A_z(r) = -B(r)$$

or

$$[\partial_r\{ E(r) + j\omega A_z(r)\}] + j\omega\partial_r A_z(r) = -j\omega B(r)$$

or

$$\partial_r E(r) + j\omega \partial_r A_z(r) + j\omega\partial_r A_z(r) = -j\omega B(r)$$

or

$$\partial_r E(r) + 2j\omega \partial_r A_z(r) = -j\omega B(r)$$

or

$$2j\omega \partial_r A_z(r) = -j\omega B(r) - \partial_r E(r) \equiv F(r)$$

But in my round wire solution (2.1.9) I found that

$$\partial_r E(r) = j\omega B(r) . \qquad (2.1.9)$$

so therefore

$$2j\omega \partial_r A_z(r) = -\partial_r E(r) - \partial_r E(r) \equiv F(r) = -2\partial_r E(r)$$

This tells me that

$$j\omega \partial_r A_z(r) = -\partial_r E(r)$$

$$j\omega A_z(r) = -E(r) + \text{constant}$$

Now look again at

$$\mathbf{B} = \text{curl } \mathbf{A} \qquad \mathbf{E} = -\text{ grad } \varphi - \partial_t \mathbf{A} \ . \tag{1.3.1}$$

You can always shift A by a constant and not affect the E and B fields. This is in addition to the gauge choice you might make which involves only derivatives of A. So write

$$j\omega A_z(0) = -E(0) + \text{constant} = 0$$

So we set the constant to $E(0) = E(a)/ J_0(\beta a)$. Then we have

$$j\omega A_z(r) = -E(r) + E(0)$$

$$A_z(0) = 0.$$

So there is my Big Solution for $A_z(r)$.

$$A_z(r) = -(1/j\omega)\,[E(r) - E(0)\,] = (j/\omega)\,[E(r) - E(0)]$$

STOP. I want to see someone else do this problem!