This is just the G.1 repair and it was installed on 10.31.13. I had some kind of math error in here before it got repaired.

G.1 Evaluation of the integral in 4.1 (9).

(a) The first claim is that the integral on the left below has the form shown on the right:

$$I(\mathbf{x}) = \int_{-\infty}^{\infty} dz' \, (z' - z)^n \, \frac{e^{-j\beta R}}{R} = s^n \, f_n(\beta s) \qquad \text{(G.1)}$$

where

$$R = \sqrt{(z'-z)^2 + s^2} \quad \text{and} \quad s = \sqrt{(x-x')^2 + (y-y')^2} \quad .$$

To show this, the first step is to define $z'' \equiv z'-z$ to get

$$I(\mathbf{x}) = \int_{-\infty}^{\infty} dz'' \, (z'')^n \, \frac{e^{-j\beta R}}{R} \qquad R = \sqrt{(z'')^2 + s^2} \qquad \text{(G.2)}$$

which shows that $I(\mathbf{x})$ is independent of z so we can think of (G.2) as (G.1) with $z = 0$.

Next define $x \equiv z''/s$ to get

$$dz'' \, (z'')^n = s^n \, dx \, x^n$$

$$R = \sqrt{(xs)^2 + s^2} = s\sqrt{x^2 + 1}$$

so that

$$I(\mathbf{x}) = s^n \int_{-\infty}^{\infty} dx \, x^n \, \frac{\exp(-js\beta\sqrt{x^2+1})}{\sqrt{x^2+1}} = s^n \, f_n(s\beta)$$

which then confirms the claimed form in (G.1).

(b) Our task is then to compute this integral

$$f_n(\alpha) = \int_{-\infty}^{\infty} dx \, x^n \, \frac{\exp(-j\alpha\sqrt{x^2+1})}{\sqrt{x^2+1}}$$

For odd integer n, the integral clearly vanishes since the integrand is then an odd function of x. For even n we replace the integral with double its positive side value and replace n = 2m with m = 1,2,3 ... to list off the non-vanishing integrals.

$$f_{2m}(\alpha) = 2\int_0^\infty dx\, x^{2m}\ \frac{\exp(-j\alpha\sqrt{x^2+1})}{\sqrt{x^2+1}} \qquad m = 0,1,2...$$

Finally, change to $y = \sqrt{x^2+1}$ which says $x^2 = y^2 = 1$ and so $xdx = ydy$. Then

$$dx\, x^{2m} = (y/x)dy\, x^{2m} = y\, dy\, x^{2m-1} = y\, dy\, (y^2-1)^{m-1/2}$$

and so

$$f_{2m}(\alpha) = 2\int_1^\infty dy\, (y^2-1)^{m-1/2}\, e^{-j\alpha y}$$

Finally we have something we can find in the standard tables [GR 3.387 3 with $\nu = m+1/2$] shows that the result is:

$$f_{2m}(\alpha) = (2/\sqrt{\pi}\,)\,(2/j\alpha)^m\,\Gamma(m+1/2)\,K_m(j\alpha) \qquad (G.5)$$

where $K_m(z)$ is one of the modified Bessel functions. Using the standard properties of the Gamma function $\Gamma(z)$ we can set

$$\Gamma(m+1/2) = (\sqrt{\pi}\,/2^m)\ (2m-1)!! \qquad \text{Spiegel 16.6} \qquad (G.6)$$

to get the final result,

$$f_{2m}(\alpha) = (2/\sqrt{\pi}\,)\,(2/j\alpha)^m\ (\sqrt{\pi}\,/2^m)\ (2m-1)!!\ K_m(j\alpha)$$

$$= 2\,(2m-1)!!\,K_m(j\alpha)\,/(j\alpha)^m$$

and then restoring $2m = n$ we have

$$f_n(\alpha) = 2\,(n-1)!!\,K_{n/2}(j\alpha)\,/(j\alpha)^{n/2} \qquad \text{for n even}$$
$$f_n(\alpha) = 0 \qquad \text{for n odd} \qquad (G.7)$$

The conclusion is then that

$$I(\mathbf{x}) = \int_{-\infty}^{\infty} dz'\,(z'-z)^n\ \frac{e^{-j\beta R}}{R} = s^n\, f_n(\beta s) = \begin{cases} 0 & \text{for odd n} \\ 2s^n(n-1)!!\,K_{n/2}(j\beta s)\,/(j\beta s)^{n/2} & \text{for n even} \end{cases} \qquad (G.8)$$