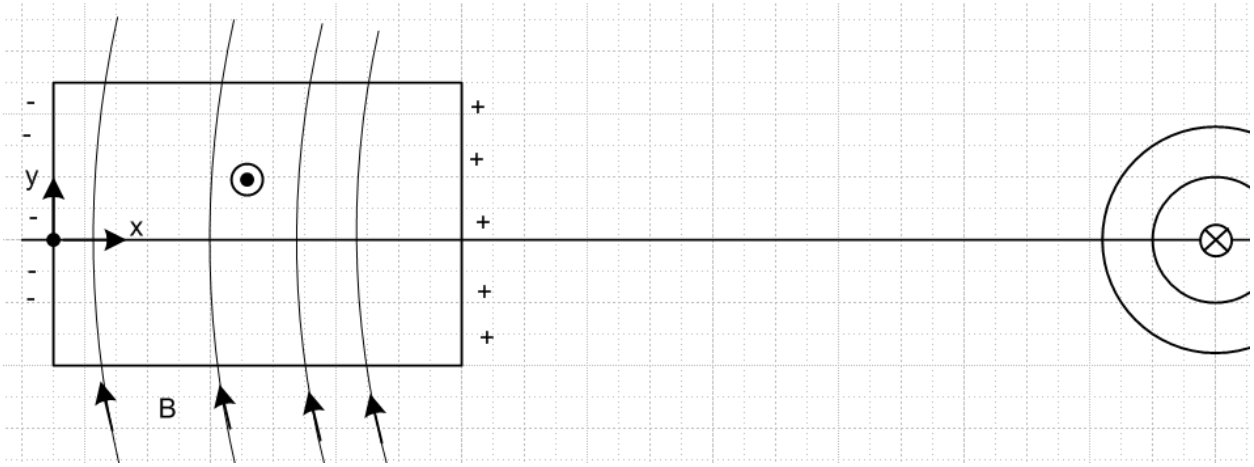
The Hall Effect with a non-uniform B field PhL 3.22.14

(Review 5/6/14). Here I am wondering if the DC Hall Effect with a non-uniform B field can cause J_z to be non-uniform in a conductor (a rectangular conductor in Problem 1). This was written well before I started pondering eddy currents. I really did think at the time that a non-uniform J_z might be possible at DC, but I no longer think that, despite the comments of a lone author. This led me to the subject of the radial Hall effect, and how J_z can be uniform in a round wire in the presence of this radial Hall effect. Even if there were some DC anomaly due to the Hall effect, it would probably be unmeasurable and total negligible compared to the AC eddy current effects of Appendix P and of lines doc!

Having said all this, in Problem 1 below there will be some Hall effect because the external B field will deflect current carriers. Probably if you just consider v_z constant you get $E_x(x) = v_z B_z(x)$ and so the Hall field E_x does vary with x as one would expect. Now that I know about the radial Hall effect, I would guess there is some tiny charge density ρ inside the sample which allows $E_x(x)$ to balance the different Lorentz force strength at different x locations, and the perfect uniformity of J_z is not affected. I never really completed this little problem.

Problem 1.

Here is the physical scenario (everything is DC, no time variations) :



A wire on the right creates a magnetic field which is thus non-uniform in the x direction going across a rectangular conductor on the left. This left-side conductor carries a uniform J_z out of the plane of paper when the right-side B field is turned off. We wonder if this J_z stays uniform when the field is turned on?

To simplify, we imagine that the B field lines are in the y direction, even though we know this violates the condition div **B** = 0. It is just an approximation to simplify things.

The current density on the left is assumed to have this form:

$$J_z(x) = n(x)\, e\, v_z(x) \quad = \rho_e(x)\, v_z(x) \qquad\qquad J_z(x) \;= \sigma\, E_z(x)$$

where n(x) is the density of electrons, e the electron charge which we take to be positive again for further simplification, and $v_z(x)$ is the small drift velocity. We assume again for simplicity that the B field has this form

$B_z(x) = B_0 + ax$ // linear shape

Based on simple Hall effect models presented in "hall effect 1.pdf" we expect to see a "Hall electric field" appear inside the conductor which cancels the Lorentz force q**v** x **B**

$E_x(x) = v_z(x)\, B_z(x)$

All we know at this point is that $B_z(x)$ varies linearly with x. This equation has two unknown functions in it, $E_x(x)$ and $v_z(x)$, so we need more information to solve the problem.

Maybe div **E** = 0 will help, where we assume there is no free *or* polarization charge inside the conductor. This may not be right for polarization charge, but let's assume if for the moment. This says

$\partial_x E_x(x) + \partial_y E_y(x) + \partial_z E_z(x) = 0$

We simplify again and assume that $E_y(x) \equiv 0$. Then $\partial_z E_z(x) = 0$ since we assumed E_z depends only on x. Then we get

$\partial_x E_x(x) = 0$

or

$\partial_x [v_z(x)\, B_z(x)] = 0$

which tells us that

$v_z(x)\, B_z(x) = K$, some constant // $= E_x(x)$

This has serious implications if true. It says we know how $v_z(x)$ varies,

$v_z(x) = K / B_z(x)$

But then

$E_x(x) = v_z(x)\, B_z(x) = K$

and somewhat to our surprise, we find that the Hall field $E_x(x)$ is a constant and can therefore be created by simple surface charges on the left and right as shown (although I have not bothered to determine which surface is actually positive in our particular case).

Meanwhile, we then have

$J_z(x) = n(x)\, e\, v_z(x) = n(x)\, e\, K / B_z(x)$

But once again, we have two unknown functions here, n(x) and $J_z(x)$. It seems possible that the density of electrons n(x) might vary with x, since the initial Lorentz deflections could have done this. This would imply a polarization charge density $\rho_{pol}(x)$ in the metal. But I assumed there was no such $\rho_{pol}(x)$ above

when I assumed that div **E** = 0 . To be consistent with this assumption, I have to say n(x) = n which is the at-rest electron carrier density, and then we obtain

$$J_z(x) = n\, e\, K / B_z(x)$$

and the conclusion in this theory pathway is that $J_z(x)$ does in fact very in inverse proportion to how B varies! Interestingly, J_z is then a maximum where $B_z(x)$ is a minimum. The reason is vB = K, so where B is larger, we need v to be smaller. This would be an "inverse proximity effect" .

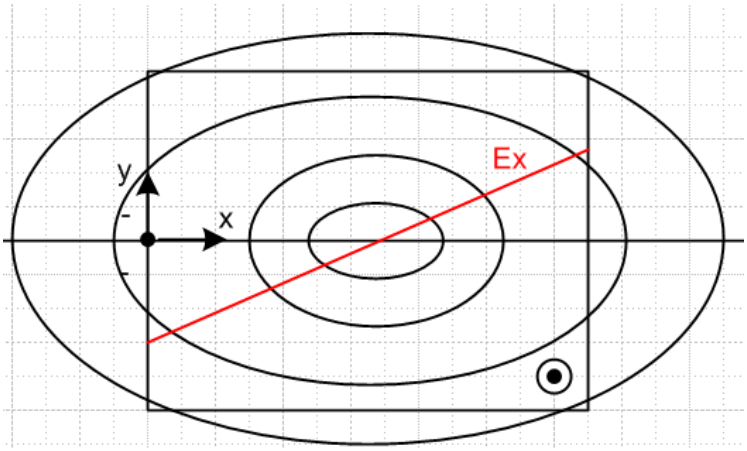
[But this is obviously a wrong conclusion, and conflicts with eddy current analysis and intuition, so one or more of my "simplifications" is not correct.]

I suspect this model is wrong, but I don't yet have a good reason. Here are some questions:

(1) Does an electron see the B field created by the other electrons flowing in the rectangular conductor? I think the answer is yes it does. [it is yes] So I have not included this in the discussion above.

Problem 2.

This question brings up a simple problem which is this:



This conductor is somehow in isolation with a DC J_z flowing in it. Is this J_z uniform?? It seems clear that the B field generated by this current will have B lines something like what I have drawn, and there will be a gradient of the B field inside the conductor. Dramatically, on opposite sides of one of the ellipses the Lorentz force will be oppositely pointed! Thus, the Hall field E_x must be positive on one side and negative on the other side, with a symmetrical but negated shape. Let us now examine this problem a bit! Can we make this still be a simple 1D problem? Not really, because the field lines close within the conductor. So this brings us to

[I think at this point I went on to ponder the "radial Hall effect"]