Hall Effect Readings PhL 3.22.14

[Reviewed 5/6/14]. I am here looking for papers on Hall effect with a B field which varies over the sample. I did in fact find a good one I call FFA.

Motivation: I imagine a rectangular conductor on the left, and a round one on the right. The round one on the right (just a wire, say) creates a magnetic field within the rectangular conductor which is *non-uniform* going left to right. This means that the vxB Lorentz force has a different B going left to right, and that means that the required Hall E field to balance this vxB field will have to vary, provided v is the same going left to right. I want to know the solution to this kind of problem. Do you end up with v varying going left to right? Do you end up with a polarization charge distribution inside the conductor which allows v = constant and then the Hall E can vary going left to right? Somewhere, someone must have pondered this basic problem.

As noted in a related doc, I do now think there will be a $\rho \neq 0$ inside the sample (as there is in the radial Hall effect) and this causes the tiny Hall E_x field to vary just right with x so as to balance the varying Lorentz force. I was having trouble imagining how this E_x field could vary if it was *only* caused by surface charges on the faces of the sample! None of the papers below mentions this ρ detail.

Web searching does not show up any obvious sources. Here is one source that caught my attention: (1955 AEC Ames)

MEASUREMENT OF MAGNETIC FIELD GRADIENTS BY THE HALL EFFECT

By
Robert D. Redin
G. C. Danielson

If the magnetic field is not uniform, Laplace's equation no longer holds in general. To first order in B it does hold and this case has been solved by Koppe and Bryon (10)

for a long narrow slab and a magnetic field which is a function only of the distance along the length of the slab and for which $B(x) \rightarrow 0$ as $|x| \rightarrow \infty$. The solution is in terms of an integral.

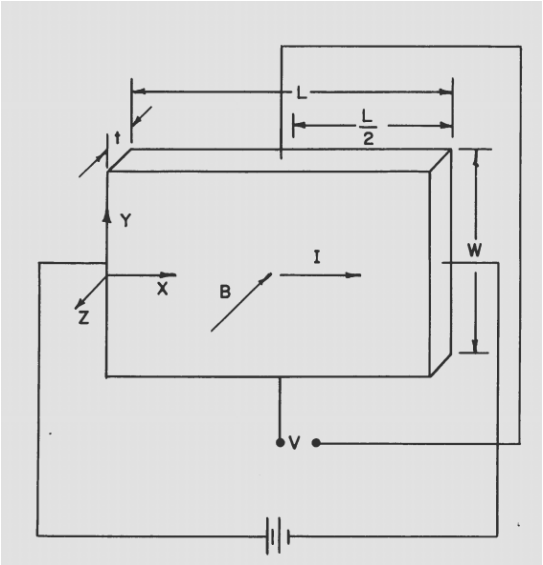
The case of a rectangular slab in a magnetic field which varies only in the direction of the length of the slab has been solved by Flanagan, Flinn and Averbach (6). Their solution was applied to the geometry of the probe used in the gradient meter described in this report.

10. Koppe, H. and Bryan, J. M. On the theory of the Hall effect. Can. J. Phys. 29: 274-284. 1951.

6. Flanagan, W. F., Flinn, P. A., Averbach, B. L. Shorting and field corrections in Hall measurements. Rev. Sci. Instr. 25: 593-595. 1954.

This last reference is FFA.

In their Hall geometry, B is in z, current is in x, and hall voltage is in y. These are traditional I think.



These authors are going to *measure* the Hall voltage as a function of x (left to right) which is the direction in which they assume $B_z(x)$ is varying. So they are on my wavelength at least! $V_m(x)$ is their measured voltage and V_H is a certain constant with voltage dimensions. They assume the gradient is linear and so B has the form $B(x) = BL/2\ \varphi(x)$ with $\varphi(x) = 1 + a(x-L/2)$, so a is the gradient strength. Their conclusion is this:

$$V_m(x)/V_H = (8/\pi^2)\,(L/H)\,[\ \Sigma_{n=odd}\ f_n(x) - (aL/2)\,\Sigma_{n=even}\ f_n(x)$$

where $f_n(x) = (1/n^2)\tanh(n\pi W/2L)\sin(n\pi x/L)$

They get this result from the reference FFA noted above. But then having done this fancy theory, they show that the obvious simple model of just doing the classic Hall thing at each x gives the same result within about 1% accuracy! This is an experimental paper, not a theory one, and they are just using the theory from this FFA source.

So the upshot is that there is some Hall $E_y(x)$ that just follows the field strength $B_z(x)$ of their "simple model". I am looking for someone to tell me the charge density associated with this $E_y(x)$ field, but these authors are not interested in that theoretical question. [Magnetoresistance may be something to look into]

17. Seitz, F. Note on the theory of resistance of a cubic semiconductor in a magnetic field. Phys. Rev. 79: 372-375. 1950.

19. Volger, J. Note on the Hall potential across an inhomogeneous conductor. Phys. Rev. 79: 1023-1024. 1950.

I guess the idea is that in the simple model approach, they are assuming a uniform J_x and maybe this is slightly modified in the fancy theory? Can I find any of these references?

Koppe Bryan: It does show in the Canadian phys journal, but of course I am blocked.

Koppe Bryan "on the theory of the Hall effect". I go to Marriott, Jim gets me in, but they only have since 2001.

I then try the FFA reference while logged on, and I have it! Also I got the Seitz reference. I log off by closing all the windows.

I have FFA as Hall,pdf, it is about the "shorting effect" of the end caps.

FFA: right off the bat, they just assume $J_x = J$ = constant, all done. Then the Hall voltage does vary with x,

$$E_H(x) = R_0 J B(x), \qquad (2)$$

where R_0 is the Hall coefficient, and $B(x)$ is the magnetic induction at any point x. By introducing the relation,

$$B(x) = B(L/2)\phi(x), \qquad (3)$$

where $\phi(x)$ is normalized to unity at $x = L/2$, the center of the specimen, and representing V_H by the expression

$$V_H/w = R_0 J B(L/2), \qquad (4)$$

where V_H is the true Hall voltage and w the width of the foil, the field established by the surface charge along the edges of the sample can be represented by

$$R_0 J B(x) = \frac{V_H}{w}\phi(x). \qquad (5)$$

Inside the specimen Laplace's equation is satisfied. Assuming the main current through the sample to be constant (i.e., the Hall current is negligible) one can disregard its effect on the final field distribution. Considering just the effect of the Hall field, the related Laplacian equation may be written as:

$$\nabla^2 V(x,y) = 0, \qquad (1)$$

So OK, V(x,y) is the electrostatic potential inside the specimen, fine. Remember the current is J_x and the Hall direction is y, so this is just Laplace and so they are assuming therefore no free or polarization charge density inside the metal! I am sort of claiming this is wrong, but let's keep going, maybe not.

What are the boundary conditions on V? They claim one is this

$$\pm \frac{V(x,\pm w/2)}{y} = \mp \frac{V_{\text{Hall}}}{w} \phi(x). \qquad (6a)$$

This makes no sense to me at all. Why would you divide the potential at (x,y) = (x,w/2) by y ?

OK, these guys are assuming that V = 0 all along the two ends of the sample where the current is applied, as if those contacts did not have their own Hall effect? "heavy contacts". That then sets V = 0 on the two ends and then you have a Laplace Dirichlet problem which yields their fancy integral. The claim is that these end contacts short out the Hall voltage. The title of the paper is "shorting correction".

So this paper basically leaves my question totally unanswered! Too bad, nice try. It does at least claim without proof that J is a constant even though B(x) varies. The shorting situation is irrelevant for me.

I think I will just have to roll my own solution to this problem.

Again, the conclusion is that at DC, J_z really is perfectly uniform.

This FFA paper did two things for me:

(1) it claimed that J_z is uniform even if B has a gradient (pushing me in that direction)
(2) it shows the Hall voltage $E_H(x)$ just tracking B(x), as one would guess is the solution