Hall Effect in Round Wire 1 PhL 3.23.14

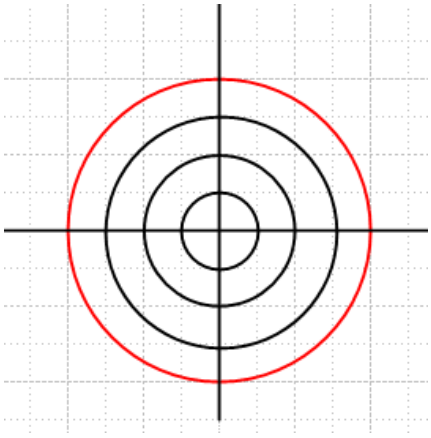
This was my first attempt at the "radial Hall effect" problem, and this attempt is summarized in Summary of Hall in Round Wire REVIEWED.doc. Today is 3/31/14.

Here I came up with (Plan B below) a very fancy ODE for η which when simplified gives the same result obtained now by a new method which needs no ODE at all. I was unaware of the Magnetic Ohm's Law when I wrote this so there were some seeming paradoxes.

Plan A below is interesting: It shows that you cannot explain the radial Hall effect in terms of a line charge down the middle and a surface charge outside. This concept gives $E_r \sim 1/r$ instead of $E_r \sim r$, and this leads to contradictions. This was my first approach to the radial Hall effect.

Soon after this I realized I could have Maple handle all the dimensions as well as the numbers! You can see below all my painful work with dimension bugs which I eventually got all fixed.

We have a round isolated conductor shown in red, and B are the mag field lines



Now (ignoring polarity), the Lorentz force points everywhere toward the center. Let's try to solve this problem in its Hall aspect. We assume [in this doc, e < 0]

$$J_z(r) = n(r)\, e\, v_z(r) \qquad (1) \qquad // \;\; \rho(r) = n(r)-n_0$$

Dim check: dim(LHS) = amp/m²
dim(RHS) = m⁻³ Cou m/sec = Cou/sec/m² = amp/m² OK

The Hall field is this

$$E_r(r) = v_z(r)\, B_\theta(r) \qquad (2)$$

Dim check: dim(LHS) = volt/m
dim(RHS) = m/sec*amp-henry/m² = m/sec*amp-ohm-sec/m²
= volt/m OK

just because at each point, we have to neutralize the q **v** x **B** force so there will then be no steady-state transverse currents. Finally, the divergence operator in this situation is

$$\text{div } \mathbf{E} = r^{-1}\partial_r(rE_r) + r^{-1}\partial_\theta E_\theta + \partial_z E_z \;\; = r^{-1}\partial_r(rE_r)$$

and then our third equation is Maxwell equation

$$r^{-1}\partial_r(rE_r) \;= \rho/\varepsilon_0 \qquad\qquad (3)$$

Dim check: dim(LHS) = volt/m^2

dim(RHS) = Cou/m^3 * m/farad = volt/m^2 OK

where ρ would then be "some kind" of charge inside the wire. Normally of course ρ = 0 and everything is balanced, electrons against lattice ions.

Note: What about curl **E** = 0 ?

$$\text{curl } \mathbf{E} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta E_z - \partial_z E_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z E_r - \partial_r E_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r\,]$$

or

$$\text{curl } \mathbf{E} = \hat{\mathbf{r}}\,[\,0 - 0] + \hat{\boldsymbol{\theta}}\,[0 - \partial_r E_z] + \hat{\mathbf{z}}\,[\,0 - 0\,] \;= -\,\partial_r E_z \hat{\boldsymbol{\theta}} \qquad [\text{ correct }]$$

This tells us that $\partial_r J_z(r) = 0$ and thus J_z = constant. Now at least I know where that is coming from! Of course it also comes from my Chapter 2 result in the low ω limit.

The following Plan A is interesting: It shows that you cannot explain the radial Hall effect in terms of a line charge down the middle and a surface charge outside. This concept gives $E_r \sim 1/r$ instead of $E_r \sim r$, and this leads to contradictions. This was my first approach to the radial Hall effect.

Plan A. Assume that $n(r) = n_0$ so that ρ = 0 and also that J_z = constant. In this case, equation (3) says that $\partial_r(rE_r) = 0$ and then

$$rE_r(r) = K \qquad \Rightarrow \qquad E_r(r) = K / r$$

This is my "radial Hall effect" idea. This E field could be generated by a line charge along the axis of the cylinder. Recall that $\varphi = \ln(1/r) = -\ln(r)$ and $E_r = -\partial_r\varphi = 1/r$ in this situation. So let us assume that this is just one "surface" of the induced Hall surface charge, and the other polarity is on the outside surface of the conductor. We also have this equation combining the above with (2)

$$r\, v_z(r)\, B_\theta(r) = K \;.$$

If J_z is non-uniform, then we don't really know $B_\theta(r)$ so we are looking here at two unknown functions $v_z(r)$ and $B_\theta(r)$. But in Plan A we are assuming that J_z = constant. Equation (1) then says we must have $v_z(r)$ = constant. In this case, since J_z = constant, we know that $B_z(r) = Cr$ and then we have

$r\, v_z(r)\, B_\theta(r) = K \quad \rightarrow \quad r\, v_z\, Cr = K \quad \Rightarrow$ inconsistent!

$E_r(r) = v_z(r)\, B_\theta(r) \quad \rightarrow \quad K/r = v_z Cr \quad \Rightarrow$ inconsistent!

OK, my "line charge radial Hall effect theory" does not allow $v_z(r)$ = constant .

Plan B. Now assume only that J_z = constant. Let's now assume there *does* exist a charge density ρ. The electrons can shift radially relative to the ions. I think it is best to call this a free charge density and not a polarization charge density. So (3) then says

$$\partial_r(rE_r) = r\,\rho(r)/\varepsilon_0 \quad .$$

I think the best way to model this is to say that

$$\rho(r) = e\,[n(r)-n_0]$$

so we might have either an excess or a deficiency of electron density and this makes ρ. Here are our three equations again,

$J_z(r) = n(r)\, e\, v_z(r)$ (1) // $\rho(r) = n(r)-n_0$

$E_r(r) = v_z(r)\, B_\theta(r)$ (2)

$r^{-1}\partial_r(rE_r) = \rho/\varepsilon_0$ (3)

Since J_z = const we know that $B_\theta(r) = Cr$. Then we are saying from (1),

$n(r)\, v_z(r) = J_z/e$ J_z = constant

$B_\theta(r) = B_\theta(a)\,(r/a)$

From (2) we then have

$$E_r(r) = v_z(r)\, B_\theta(r) = \frac{J_z}{en(r)}\, B_\theta(a)\,(r/a) = [J_z\, B_\theta(a)/ea]\,[\,r / n(r)\,]$$

Dim check: dim(LHS) = volts/m

dim (RHS) = amp m^{-2} amp-henry-m^{-2} Cou^{-1} m^{-1} m m^3

= amp amp-henry Cou^{-1} m^{-1} = amp-henry sec^{-1} m^{-1} = amp-ohm-sec sec^{-1} m^{-1}

= amp-ohm m^{-1} = volt/m OK

Now I suspect that n(r) could deviate only very slightly from n_0, so I think it is best to define this variable as the one of interest, and which I suspect will be very small. The "fractional variation in n(r) " :

$\eta(r) \equiv [n(r)-n_0]/n_0 = [n(r)/n_0 - 1] \quad \Rightarrow \quad n(r) = n_0[1 + \eta(r)]$

$$\rho(r) = e\,[n(r)-n_0] = en_0\,\eta(r) \quad .$$

Then we have

$$E_r(r) = \frac{J_z\,B_\theta(a)}{ea}\,\frac{r}{n(r)} = \frac{J_z\,B_\theta(a)}{ean_0}\,\frac{r}{1+\eta(r)}$$

Dim check: LHS = volt/m
RHS = amp/m² amp-henry/m² * Cou⁻¹ m⁻¹ m³ m = amp/m amp-henry * Cou⁻¹
= Coul sec⁻¹/m amp-ohm-sec * Cou⁻¹ = 1/m amp-ohm = volt/m OK

Now planning ahead, we know that

$$\partial_r\,\frac{r^2}{1+\eta(r)} = \frac{(1+\eta)*2r - r^2\,\eta'}{(1+\eta)^2} = \frac{2r}{1+\eta} - \frac{1}{(1+\eta)^2}\,r^2\,\eta'(r) \qquad \text{Dim OK}$$

Then

$$\partial_r(rE_r) = \frac{J_z\,B_\theta(a)}{ean_0}\,\partial_r\,\frac{r^2}{1+\eta(r)} = \frac{J_z\,B_\theta(a)}{ean_0}\,\left[\frac{2r}{1+\eta} - \frac{1}{(1+\eta)^2}\,r^2\,\eta'(r)\right] \qquad \text{Dim OK}$$

Dim check: LHS = volt/m
RHS = amp/m² * amp-henry/m² * Cou⁻¹ m⁻¹ m³ m =
= amp/m² * amp-henry * Cou⁻¹ m = amp/m² * amp-ohm-sec * Cou⁻¹ m
= Coul/sec/m² * volt-sec * Cou⁻¹ = m⁻² * volt = volt/m OK

But equation (3) says

$$\partial_r(rE_r) = r\rho(r)/\varepsilon_0 = (r/\varepsilon_0)\,\rho(r) = (r/\varepsilon_0)\,en_0\,\eta(r) = (en_0/\varepsilon_0)\,r\,\eta(r)$$

dim check: LHS = volt/m
RHS = Cou m⁻³ m/Farad *m = volt/m OK

Equating our two expressions for $\partial_r(rE_r)$ we then obtain this differential equation

$$\frac{J_z\,B_\theta(a)}{ean_0}\,\left[\frac{2r}{1+\eta} - \frac{1}{(1+\eta)^2}\,r^2\,\eta'(r)\right] = (en_0/\varepsilon_0)\,r\eta(r)$$

dim check: LHS = volts/m² m
RHS = Cou m⁻³ m/farad * m = Cou m⁻¹ 1/farad = volt/m OK

We can move all the constant stuff to the right side

$$\left[\frac{2r}{1+\eta} - \frac{1}{(1+\eta)^2}\,r^2\,\eta'(r)\right] = \frac{ean_0}{J_zB_\theta(a)}\,(en_0/\varepsilon_0)\,r\,\eta(r) = \frac{e^2an_0^2}{\varepsilon_0J_zB_\theta(a)}\,r\,\eta(r)$$

Now define

$$\beta \equiv \frac{e^2 a n_0^2}{\varepsilon_0 J_z B_\theta(a)}$$

so our ODE is then

$$\left[\frac{2r}{1+\eta} - \frac{1}{(1+\eta)^2} r^2 \eta'(r)\right] = \beta\, r\, \eta(r) \qquad \text{where} \quad \beta \equiv \frac{e^2 a n_0^2}{\varepsilon_0 J_z B_\theta(a)}$$

or

$$\left[\frac{2}{1+\eta} - \frac{1}{(1+\eta)^2} r\, \eta'(r)\right] = \beta\ \eta(r) \qquad \text{where} \quad \beta \equiv \frac{e^2 a n_0^2}{\varepsilon_0 J_z B_\theta(a)}$$

Notice that each term is dimensionless, and therefore we should have dim(β) = 1.

Dimension check: dim(β) = Cou² m m⁻⁶ / [farad/m * amp/m² * amp-henry/m²]
= Cou² m m⁻⁶ * m/farad * m²/amp * m²-amp⁻¹ henry⁻¹
= Cou² * 1/farad * 1/amp *amp⁻¹ henry⁻¹ = Cou² * 1/farad * amp⁻² henry⁻¹
= amp² sec² 1/farad * amp⁻² henry⁻¹ = sec² 1/farad henry⁻¹
= sec² / [farad-henry] = sec² / [farad-ohm-sec] = sec / [farad-ohm] = sec/sec = 1

Let's work on this constant β. We know that

$$2\pi a\, H_\theta(a) = I \quad \Rightarrow \quad H_\theta(a) = I/(2\pi a) \quad \Rightarrow \quad B_\theta(a) = \mu_0 I/(2\pi a)$$

Dimension check: dimB = amp-henry/m²
dim[$\mu_0 I/(2\pi a)$] = henry/m * amp m⁻¹ = amp henry/m² OK

$$J_z = I/(\pi a^2)$$

Dimension check: dimJ_1 = amp/m²
dim(RHS) = amp/m² OK

$$\beta = \frac{e^2 a n_0^2}{\varepsilon_0 J_z B_\theta(a)} * \frac{J_z \pi a^2}{I} * \frac{2\pi a B_\theta(a)}{\mu_0 I} = \frac{e^2 a n_0^2}{\varepsilon} * \frac{\pi a^2}{I} * \frac{2\pi a}{\mu_0 I} = \frac{2\pi^2 e^2 a^4 n_0^2}{\varepsilon_0 \mu_0 I^2} = 2\pi^2 (e a^2 n_0 c/I)^2$$

Dimension check:
dim [$ea^2 n_0 c/I$] = Cou -m² m⁻³ m/sec / Cou/sec =1
dim(β) = 1

Let's evaluate this β thing to see the ball park using Maple:

```
I := 1;           # amps

e := 1.602e-19;   # electron charge

c := 3e8;         # light in m/sec

a := 1e-2;        # 1 cm radius wire

n0 := 8.5e28;

beta := evalf(2*Pi^2*(e*a^2*n0*c/I)^2);
```

$$I := 1$$

$$e := .1602\ 10^{-18}$$

$$c := .3\ 10^{9}$$

$$a := .01$$

$$n0 := .85\ 10^{29}$$

$$\beta := .3294087458\ 10^{31}$$

So $\beta = 0.32 \times 10^{31}$. And here is our ODE

$$[\frac{2}{1+\eta} - \frac{1}{(1+\eta)^2} r\,\eta'(r)\,] = \beta\ \eta(r) \qquad \text{where} \quad \beta \equiv \frac{e^2 a n_0^2}{\varepsilon_0 J_z B_\theta(a)} = 0.32 \times 10^{31}$$

OK, mult through

$$[\frac{2}{1+\eta} - \frac{1}{(1+\eta)^2} r\,\eta'(r)\,] = \beta\ \eta(r)$$

$$2(1+\eta) - r\,\eta' = \beta\,\eta\,(1+\eta)^2$$

$$r\,\eta' + \beta\,\eta\,(1+\eta)^2 - 2(1+\eta) = 0$$

$$r\,\eta' + (1+\eta)[\,\beta\,\eta\,(1+\eta) - 2] = 0$$

Right here I think we are forced to have (since β is so large)

$$[\,\beta\,\eta\,(1+\eta) - 2] \approx 0 \quad \Rightarrow \quad \beta\,\eta \approx 2 \quad \Rightarrow \quad \eta = 2/\beta \approx \textit{very}\ \text{small}$$

OK, if we know that $\eta << 1$, then we can set $1+\eta = 1$ to get

$$r\,\eta' + [\,\beta\,\eta - 2] = 0$$

$$r\,\eta' + \beta\,\eta - 2 = 0 \qquad \text{// a pretty simple ODE!}$$

```
eq1 := r*diff(eta(r),r) + b*eta(r) - 2 = 0;
```

$$eq1 := r\left(\frac{\partial}{\partial r}\eta(r)\right) + b\,\eta(r) - 2 = 0$$

```
dsolve(eq1);
```

$$\eta(r) = 2\frac{1}{b} + r^{(-b)}_C1$$

OK, this says

$$\eta(r) = C_1\, r^{-\beta} + 2/\beta$$

I verified this solution on scratch. Now we need a boundary condition. I want the total charge over the cross section to be 0, so that says

$$\int_0^a r dr \int_0^{2\pi} d\theta\, \rho(r)\, dz = 0$$

$$\int_0^a r dr \int_0^{2\pi} d\theta\, en_0\, \eta(r)\, dz = 0$$

$$\int_0^a r dr\, \eta(r) = 0$$

But we have trouble now because the $r^{-\beta}$ term in $\eta(r)$ diverges violently at r = 0. This seems to say that we must have $C_1 = 0$ and then we get $\eta(r) = 2/\beta$. But since this is always positive, it cannot integrate to 0. [OK, fix this up by having a surface charge of opposite sign on the wire surface, Hall-like]

Perturbation Attempt: How about a perturbation theory solution:

$$\eta(r) = [2/\beta + \varepsilon\, u(r)] \qquad 1+\eta = [1 + 2/\beta + \varepsilon\, u(r)] \qquad \eta' = \varepsilon u'$$

Then

$$r\,\eta' + \beta\,\eta\,(1+\eta)^2 - 2(1+\eta) = 0$$

$$r\,\varepsilon u' + \beta\,[2/\beta + \varepsilon\, u(r)]\,[1 + 2/\beta + \varepsilon\, u(r)]^2 - 2\,[1 + 2/\beta + \varepsilon\, u(r)] = 0$$

$$r\,\varepsilon u' + \beta\,[2/\beta + \varepsilon\, u(r)]\,[1 + \varepsilon\, u(r)]^2 - 2\,[1 + \varepsilon\, u(r)] = 0$$

$$r\,\varepsilon u' + [2 + \varepsilon\beta\, u]\,[1 + \varepsilon\, u]^2 - 2\,[1 + \varepsilon\, u] = 0$$

```
(2+e*b*u)*(1+e*u)^2 - 2*(1+e*u);
```

$$(2+e\,b\,u)(1+e\,u)^2 - 2 - 2\,e\,u$$

```
expand(%):
collect(%,e);
```

$$e^3 b\,u^3 + (2u^2 + 2\,b\,u^2)\,e^2 + (2u + b\,u)\,e$$

So we then have

$$r\,\varepsilon u' + \varepsilon\,(2+\beta)u + \text{order}(\varepsilon^2) = 0$$

So the first order correction ODE is then

$$r\,u' + (2+\beta)u = 0$$

or

$$r\,u' + \beta u = 0 \qquad (*)$$

```
eq1 := r*diff(u(r),r) + b*u(r) = 0;
```

$$eq1 := r\left(\frac{\partial}{\partial r}u(r)\right) + b\,u(r) = 0$$

```
dsolve(eq1);
```

$$u(r) = _C1\, r^{(-b)}$$

So our first order solution is this problem child

$$u(r) = C_1\, r^{-\beta}$$

and then we are back to our original solution. Let's then look at the ε^2 situation:

$$\eta(r) = [2/\beta + \varepsilon\,u(r) + \varepsilon^2 v(r)] \qquad 1+\eta = [1 + 2/\beta + \varepsilon\,u(r) + \varepsilon^2 v(r)]$$

$$\eta(r) = [2/\beta + \varepsilon\,u + \varepsilon^2 v] \qquad 1+\eta = [1 + 2/\beta + \varepsilon\,u + \varepsilon^2 v] \approx [1 + \varepsilon\,u + \varepsilon^2 v]$$

Then we get

$$r\,\eta' + \beta\,\eta\,(1+\eta)^2 - 2(1+\eta) = 0$$

Write this stuff as $\eta = x$ so

$$x = [2/\beta + \varepsilon\,u + \varepsilon^2 v]$$

$$r\,x' + \beta\,x\,(1+x)^2 - 2(1+x) = 0$$

Here is what Maple says:

```
> restart;
> f := r*diff(x(r),r) + b*x(r)*(1+x(r))^2 - 2*(1+x(r));
```

$$f := r\left(\frac{\partial}{\partial r}x(r)\right) + b\,x(r)\,(1+x(r))^2 - 2 - 2\,x(r)$$

```
> x := r -> (2/b) + e*u(r) + e^2 * v(r);
```

$$x := r \to 2\frac{1}{b} + e\,u(r) + e^2\,v(r)$$

```
> f;
```

$$r\left(e\left(\frac{\partial}{\partial r}u(r)\right) + e^2\left(\frac{\partial}{\partial r}v(r)\right)\right)$$
$$+ b\left(2\frac{1}{b} + e\,u(r) + e^2\,v(r)\right)\left(1 + 2\frac{1}{b} + e\,u(r) + e^2\,v(r)\right)^2 - 2 - 4\frac{1}{b} - 2\,e\,u(r)$$
$$- 2\,e^2\,v(r)$$

```
> collect(expand(f),e);
```

$$b\,e^6\,v(r)^3 + 3\,b\,e^5\,u(r)\,v(r)^2 + (6\,v(r)^2 + 3\,b\,u(r)^2\,v(r) + 2\,b\,v(r)^2)\,e^4$$
$$+ (12\,u(r)\,v(r) + b\,u(r)^3 + 4\,b\,u(r)\,v(r))\,e^3$$
$$+ \left(2\,b\,u(r)^2 + 6\,u(r)^2 + r\left(\frac{\partial}{\partial r}v(r)\right) + b\,v(r) + 6\,v(r) + 12\frac{v(r)}{b}\right)e^2$$
$$+ \left(r\left(\frac{\partial}{\partial r}u(r)\right) + 6\,u(r) + b\,u(r) + 12\frac{u(r)}{b}\right)e + 4\frac{1}{b} + 8\frac{1}{b^2}$$

The last terms for large b vanish so OK.

The order ε equation is this

ru' +6u + bu +12u/b = 0 => ru' +bu = 0 agrees with *

Now what do we do for order e^2 ? It says this

$2bu^2 + 6u^2 + rv' + bv + 6v + 12v/b = 0$

$rv' + bv + 6v + 12v/b = -2bu^2 - 6u^2$

$rv' + bv = -2bu^2$

But I had to set $C_1 = 0$ and that made u = 0 and then this v equation is the same as the u equation, and we have the same problem again! So perturbation is not working.

Resume with Different Interpretation. Suppose we go with this as the solution

$\eta(r) = 2/\beta$.

Thus we have

$\rho(r) = e\,[n(r)-n_0] = en_0\,\eta(r) = en_0 2/\beta = \rho$ = a constant !

Evaluate this:

$$\rho = en_0 2/\beta = en_0 2^* [2\pi^2 (ea^2 n_0 c/I)^2]^{-1} = 2en_0 * \frac{1}{2\pi^2 (ea^2 n_0 c/I)^2} = \frac{I^2}{\pi^2 e\, a^4 n_0 c^2}$$

Dim check" LHS = cou/m^3

RHS = $amp^2\ cou^{-1}\ m^{-4}\ m^3\ sec^2\ m^{-2} = amp^2\ cou^{-1}\ m^{-3}\ sec^2$

$= amp^2\ cou^{-1}\ m^{-3}\ sec^2 = cou^2\ sec^{-2}\ cou^{-1}\ m^{-3}\ sec^2 = cou\ m^{-3}$ OK

Maple says that $\rho = 0.8 \times 10^{-20}$ cou/m^3.

OK, then what is the total charge inside the wire slice of thickness dz ?

$Q/dz = \rho\,\pi a^2 = 0.26 \times 10^{-23}$ Coulombs/meter

For neutrality, we must then have -Q/dz Coulombs/m on the outside surface! A Hall charge. So

$Q_{surface} = -\rho\,\pi a^2 = -0.26 \times 10^{-23}$ Coulombs/meter

What is the surface charge per meter?

$\sigma = Q/2\pi a = 0.41 \times 10^{-22}$ $Coul/m^2$

I wonder about div E = 0 near the surface, where we would write

$$\int_V \rho\, dV = \int_S \varepsilon\, \mathbf{E} \bullet d\mathbf{S}$$

Using the obvious gaussian box, we shrink it and it only picks up the surface charge and that then creates a radial Hall-created E field outside the wire. Wrong! True that box only has the surface charge, but this will be accounted for by the E_r on the inside, leaving $E_r = 0$ on the outside. This has to be because outside the ρ charge appears as a line charge and so does the equal and opposite surface charge, so outside we have to have $E_r \equiv 0$.

So what is my solution to this problem? In Plan B I use div E = 0, Lorentz balance, and $J_z = nev_z$ and nothing else! I find that:

$\rho(r) = en_0\,\eta(r) = \rho = en_0\,(2/\beta)$ where $\beta \approx 10^{31}$

$Q_s = -\rho\,\pi a^2 = -en_0\,(2/\beta)\,\pi a^2$

$n = Q_s/2\pi a = -\rho\, \pi a^2/2\pi a = -\rho\, a/2 = -\, en_0\, (2/\beta)\, a/2$ linear surface charge

$$E_r(r) = \frac{J_z\, B_\theta(a)}{ean_0}\, r \qquad B_\theta(a) = \mu_0 I/(2\pi a) \qquad J_z = I/(\pi a^2)$$

Inserting these expressions gives

$$E_r(r) = \frac{\mu_0 I^2}{2\pi^2 a^4 en_0}\, r = \frac{\mu_0 I^2}{2\pi^2 a^3 en_0}\, (r/a)$$

Dimension check

$$\text{henry-m}^{-1}\ \text{amp}^2\ \text{m}^{-3}\ \text{cou}^{-1}\ \text{m}^3 = \text{henry-m}^{-1}\ \text{amp amp cou}^{-1} = \text{volt-sec-m}^{-1}\text{sec}^{-1} = \text{volt/m}$$

Paradox: If Ohm's Law holds that $J_r = \sigma E_r$, we get $J_r(a) \neq 0$ which cannot be correct. However, from my Ohm's Law doc, we do know that Ohm's Law in the presence of a B field reads

$$J_r = \sigma_0 (E_r + \tau\omega_c E_\theta) \frac{1}{1+(\tau\omega_c)^2} \qquad \sigma_0 = (n|e|^2\tau/m)$$

$$J_\theta = \sigma_0 (E_\theta - \tau\omega_c E_r) \frac{1}{1+(\tau\omega_c)^2} \qquad \omega_c(r) = -|e|\, B(r)/m$$

$$J_z = \sigma_0 E_z$$

We could kill off $J_r(a)$ by requiring there be some $E_\theta(a) \neq 0$, but that violates equipotential surface. Also, at low B we still have $J_r = \sigma E_r$. So <u>something is wrong with my solution</u>!