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first harmonic component of field gradient inhomogeneity is required with optimum coupling obtained for $\langle n \rangle = \frac{3}{4}$. It has been suggested³ that such a mechanism, existing for a very short time during the period of injection, is responsible for the damping of radial oscillations necessary if the electrons are to miss the structure of the electron gun from which they were injected. Its application to synchrotrons would, under optimum coupling conditions, make it possible to obtain wide output pulses of gamma rays or electrons with relatively small current pulses through a suitable array of conductors which are necessary to produce the coupling inhomogeneity. It is hoped to use such means to extract electrons from their phase stable orbits and enable them to strike an internal target in order to produce a beam of photons.

The scheme proposed for extraction of synchrotron beams by Clark, Getting, and Thomas,⁵ did not include the basic ideas of parametric excitation of radial oscillations. However, near $\langle n \rangle = \frac{3}{4}$ a first harmonic field inhomogeneity as considered by them would certainly produce parametric excitation.³

⁵ Clark, Getting, and Thomas, *Phys. Rev.* **70**, 562 (1946).

8. CONCLUSION

It has been shown that regenerative deflection forms one of a class of parametrically excited resonance phenomena. Numerical results are given for optimum coupling conditions, and a method of increasing the width of the extracted pulse, while at the same time increasing the efficiency of extraction by reducing the axial spread of the beam, is described. The coupling inhomogeneities are produced electro-dynamically. It is shown that phase stability of the particle motion is not seriously affected by passage of the particles in the deflector fields.

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Shorting and Field Corrections in Hall Measurements*

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A method is given for correcting the measured Hall voltage for shorting by the current contacts and for inhomogeneities in the applied magnetic field. This method is applied to measurements of the Hall coefficient in gold-silver alloys.

INTRODUCTION

HALL effect measurements in metallic solid solutions are in progress in order to investigate the electronic changes on alloy formation. These measurements are being made on thin alloy foils which were fabricated by rolling and sintering together foils of the pure metals and then subjecting them to a diffusion heat treatment with intermediate rollings to assure a homogeneous composition. These foils are $2 \times 3 \times$ approximately 0.0005 inch thick and of the geometry shown in Fig. 1.

Although the Hall voltage is a result of the surface charge established on the conductor by the interaction of the magnetic field with the charge carriers, the voltage one measures is not directly the Hall voltage, but one which has been modified by the geometry of the sample and the method of measurement. In our

measurements the Hall current was shorted by the heavy current leads extending the entire width of the specimen, and the applied magnetic field was inhomogeneous.

A correction has been described for the shorting effect of the current contracts, but the solution assumed a homogeneous magnetic field.¹ This paper describes a method of making the shorting correction for a

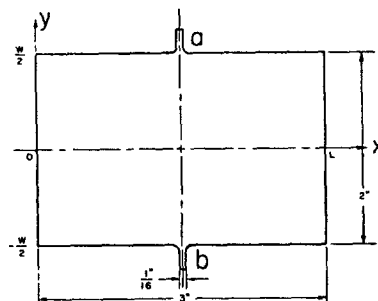


FIG. 1. Hall specimen geometry.

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¹ Isenberg, Russell, and Greene, *Rev. Sci. Instr.* **19**, 685 (1948).

specimen in a non-uniform field and gives the results of Hall measurements in a series of gold-silver alloys.

SHORTING CORRECTION

Inside the specimen Laplace's equation is satisfied. Assuming the main current through the sample to be constant (i.e., the Hall current is negligible) one can disregard its effect on the final field distribution. Considering just the effect of the Hall field, the related Laplacian equation may be written as:

$$\nabla^2 V(x,y) = 0, \quad (1)$$

where V represents the voltage contribution from the Hall effect only, and x and y are directions shown in Fig. 1. Assuming the variation in the applied magnetic field to be in the x direction only (this has been found to be true for our case), with the magnet geometry shown in the inset of Fig. 2, the Hall field at any distance x along the direction of the current density, J , can be represented by

$$E_H(x) = R_0 J B(x), \quad (2)$$

where R_0 is the Hall coefficient, and $B(x)$ is the magnetic induction at any point x . By introducing the relation,

$$B(x) = B(L/2)\phi(x), \quad (3)$$

where $\phi(x)$ is normalized to unity at $x = L/2$, the center of the specimen, and representing V_H by the expression

$$V_H/w = R_0 J B(L/2), \quad (4)$$

where V_H is the true Hall voltage and w the width of the foil, the field established by the surface charge along the edges of the sample can be represented by

$$R_0 J B(x) = \frac{V_H}{w} \phi(x). \quad (5)$$

Since this field is in the y direction, the boundary condition along the edges of the specimen becomes

$$\pm \frac{V(x, \pm w/2)}{y} = \mp \frac{V_{\text{Hall}}}{w} \phi(x). \quad (6a)$$

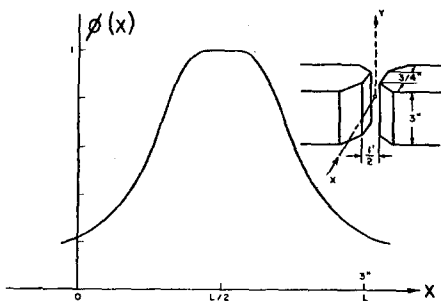


FIG. 2. Experimentally determined magnetic field shape function, $\phi(x)$, and magnet geometry.

By symmetry one can assume that

$$V(x,0) = 0, \quad (6b)$$

and consequently the shorting effect of the current contacts at the ends of the specimen imposes the conditions

$$V(0,y) = V(L,y) = 0. \quad (6c)$$

Solving Eq. (1) with the boundary conditions (6a,b,c), one obtains:

$$V(x,y) = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left\{ \int_0^L \frac{V_H}{w} \phi(x) \sin\left(\frac{n\pi x}{L}\right) dx \right\} \times \frac{\sinh(n\pi y/L)}{\cosh(n\pi w/2L)} \sin\left(\frac{n\pi x}{2L}\right). \quad (7)$$

Since the voltage is measured between points a and b (see Fig. 1), the measured Hall voltage, V_m is given by:

$$V_m = 2V(L/2, w/2) = - \sum_{n=\text{odd}}^{\infty} \frac{4}{\pi} \left\{ (-1)^{(n-1)/2} \frac{1}{n} \tanh\left(\frac{n\pi w}{2L}\right) \times \int_0^L \frac{V_H}{w} \phi(x) \sin\left(\frac{n\pi x}{L}\right) dx \right\}. \quad (8)$$

This solution assumes complete shorting at the ends, and a surface charge proportional to the magnetic field $B(x)$ along the sides. At the corners these assumptions are in conflict, but in practice the error introduced is small if the length-to-width ratio is at least $\frac{3}{2}$. This is especially true in the present case, where $\phi(x)$ has a very low value at both ends. The integral can be evaluated numerically after $\phi(x)$ has been determined, and the summation is then straightforward.

A rotating coil fluxmeter was used to obtain $\phi(x)$, which is shown in Fig. 2, and which was found to be independent of the magnitude of the magnetic field. This resulted in a constant correction term for all fields. The integral in Eq. (8) was evaluated numerically as a summation making use of Lipson-Beevers strips,² and $\phi(x)$ was assumed to vary slowly enough so that this integral was well represented by a summation whose increments were 1/30th of the total length of the sample. Thus,

$$\int_0^L \phi(x) \sin\left(\frac{n\pi x}{L}\right) dx = \sum_{h=\text{even}}^{60} \phi_h \sin\left(\frac{n\pi h}{60}\right) \left(\frac{L}{30}\right), \quad (9)$$

² H. Lipson and C. A. Beevers, Proc. Phys. Soc. (London) 48, 772 (1936).

with $\phi_h = \phi(x_i)$, where $x_i = hL/60$. Substituting Eq. (9) into (8):

$$f = \frac{V_m}{V_H} = \frac{2L}{15\pi w} \sum_{n=\text{odd}}^{\infty} \left\{ (-1)^{(n-1)/2} \frac{1}{n} \tanh\left(\frac{n\pi w}{2L}\right) \times \sum_{\substack{h=0 \\ \text{even}}}^{60} \phi_h \sin\left(\frac{n\pi h}{60}\right) \right\}. \quad (10)$$

For a length to width ratio of $\frac{3}{2}$ (see Fig. 1) and for the field shape shown in Fig. 2, the correction becomes $V_m/V_H = 0.743$. In practice, Eq. (10) was found to converge fast enough so that 15 terms gave sufficient accuracy. V_m refers to the measured voltage between points a and b , and V_H refers to the Hall voltage which would be present in the absence of shorting and inhomogeneity.

It should be noted that in the case of ferromagnetics some portions of the sample may not be above saturation because of the variation in magnetic field with

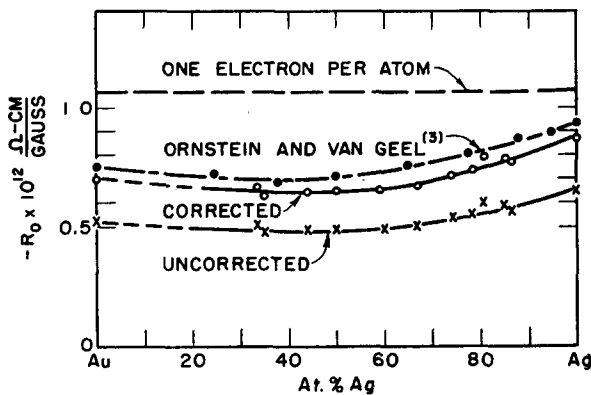


FIG. 3. Hall constants for gold-silver alloys.

distance along the sample. This introduces an error because of an additional contribution to the slope of the V_m vs B curves due to the variation of the magnetization with B .³ For this reason, this correction is only applicable to nonferromagnetic alloys, although the error introduced for ferromagnetics is probably small under conditions where the samples are easily saturated.

HALL MEASUREMENTS IN GOLD-SILVER ALLOYS

For the measurements reported here a main current of 2.5 amperes at 250 cycles per second was supplied by a class B push-pull power amplifier which had sufficient feedback to make the output amplitude insensitive to line voltage changes. In measuring the Hall voltage, a microphone transformer with a high input impedance was used to isolate the specimen from the measuring system. The transverse voltage was fed

³ A. I. Schindler and E. M. Pugh, Phys. Rev. **89**, 295 (1953).

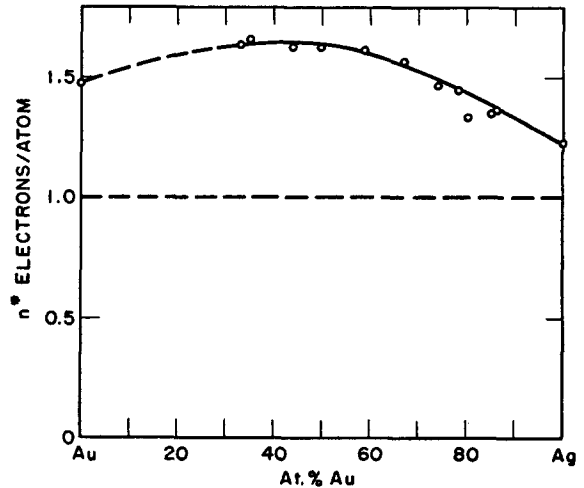


FIG. 4. Effective electron concentration in gold-silver alloys.

into a two-stage amplifier which had a bridge-T network peaked at the main current frequency, and the resultant voltage was measured with a vacuum tube voltmeter. The amplification was determined by means of a standard resistance placed in series with the specimen, and voltages were measured as a function of $B(L/2)$.

For these experiments the Hall constant was given by the relation:

$$R_0 = mR_s d / V_s f, \quad (11)$$

where m is the slope of the V_m vs $B(L/2)$ curve, R_s is the standard calibrating resistance, d is the thickness of the sample, V_s is the voltage obtained with the standard resistance, and f is the correction term, V_m/V_H . The sign of the effect was determined by observing the phase of the measured signal with respect to the input signal by means of Lissajous patterns, and comparing the phase change with that observed on pure gold, which is known to have a negative sign. Measurements were made at room temperature with magnetic fields varying from $\pm 11\,000$ gauss. The slopes obtained for the two directions of the field were averaged to minimize the Nernst and Richi-Leduc effects, and the Ettinghausen effect was absent because of the ac method used.

The uncorrected Hall constants for Ag-Au alloys, together with their corrected values, are shown in Fig. 3 along with values published by Ornstein and van Geel.⁴ Ornstein and van Geel did not apply a correction term, but this term would have been small for their geometry. The effective number of conduction electrons per atom calculated on the basis of free electrons[†] is given in Fig. 4. Similar measurements are being made on Au-Cu, Au-Ni, Cu-Ni, Cu-Pt, and Ag-Pd, and the effect of ordering is also being investigated.

⁴ L. S. Ornstein and W. Ch. van Geel, Z. Physik **72**, 488 (1931).

[†] $n = v/R_0 q_0 N_0$, where n = number of free conduction electrons per atoms; v = molar volume; R_0 = Hall coefficient; q = charge per electron; N_0 = Avogadro's number.