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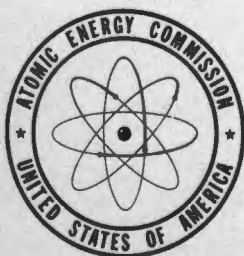
**MEASUREMENT OF MAGNETIC FIELD
GRADIENTS BY THE HALL EFFECT**

By
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MEASUREMENT OF MAGNETIC FIELD GRADIENTS BY THE HALL EFFECT *

by

Robert D. Redin and G. C. Danielson

ABSTRACT

A magnetic field gradient measuring device, which uses the Hall effect in germanium, has been constructed. The field sensitive element is a bar of germanium 1 mm by 1 mm by 12 mm with two sets of Hall leads attached 2 mm either side of its center. One hundred cycle alternating current flows in the long direction of the bar. In a magnetic field two 100 cycle Hall voltages are obtained. These voltages, which are proportional to the magnetic field strength at two points of the field 4 mm apart, are subtracted to give an output directly proportional to the magnetic field gradient. The instrument will also measure field strengths and relative gradients. The instrument was tested in a calculable magnetic field produced by step pole pieces. Gradients from 5 gauss/inch to 500 gauss/inch in magnetic fields below 5000 gauss were measured. The usefulness of the instrument is still uncertain owing to errors caused by excessive field dependence of the gradient voltage.

*This report is based on an M.S. thesis by R. D. Redin submitted December, 1955 to Iowa State College, Ames, Iowa. This work was done under contract with the Atomic Energy Commission.

I. INTRODUCTION

A. Purpose

The purpose of this work was to determine the feasibility of using the Hall effect to measure magnetic field gradients. Devices using the Hall effect in semiconductors to measure magnetic field strength have been described (14, 12). In this investigation an attempt was made to extend the principles used in these devices to obtain an instrument sensitive to magnetic field gradients.

The specific application for which the instrument was designed is the testing and alignment of large magnets used in high energy particle accelerators. Another possible application is checking the magnetic field gradients used in magnetic susceptibility measurements.

B. Requirements of the Instrument

The quantities of interest are the magnetic induction B , the field gradient dB/dx and the relative gradient $(dB/dx)/B$. Values of magnetic induction in general use range from 100 gauss to 20 kilogauss. Relative gradients of interest in high energy particle accelerators range from 0.001 cm^{-1} to 0.1 cm^{-1} . The small gradients may exist over a distance of less than 1 cm. These gradients would be due, for example, to irregularities in the surface of a pole piece. In some cases the fields may be varying with time. Maximum allowable errors desired for accelerator design work may be as low as 0.1 per cent for the value of the relative gradient.

Other factors to be considered are the weight and size of the probe, its sensitivity to temperature, humidity and shock and the extent of calibration necessary.

II. GENERAL THEORY AND REVIEW OF LITERATURE

A. Methods of Measuring Magnetic Field Gradients

Gradients can be measured by sampling the field in two or more places. This may be accomplished either by the use of one field sensitive element which is moved to measure the

field at different places at necessarily different times or by the use of two or more field sensitive elements which are stationary and which measure the field at two or more places simultaneously.

The most common method of field gradient measurement applies the first principle. A point by point plot of the field is made and the gradient is then obtained by differentiating the result either graphically or analytically. Thus, in this way, any field sensitive device which has a sufficiently small sensitive area and which will operate in a gradient can be used to measure gradients. This method is limited to stationary magnetic fields and is time consuming. There is also generally a loss of precision in the differentiating process.

Many schemes have been devised to measure magnetic field strengths. These will not be discussed here. It may, however, be mentioned that the most precise method of measuring field strengths, nuclear magnetic resonance, is at present not useful for measuring gradients since it requires a very uniform field to operate at all. Some work is being done on its use in very small gradients (1).

The process of moving the sensitive element through the field can be made very rapid by using some sort of vibrating device. This principle has been used in a gradient meter developed at Brookhaven National Laboratory (2). A small coil was mounted with its plane perpendicular to the field direction and was made to vibrate in this plane. The voltage induced in the coil was directly proportional to the gradient in the direction of vibration. An output independent of the frequency and amplitude of motion was obtained by using a reference coil in a reference gradient. A sensitivity of less than one gauss per centimeter and an accuracy of less than one per cent were claimed. Its use in time varying magnetic fields was limited by the frequency of vibration. Vibration of the probe mount was also a difficulty.

If the second method of measuring gradients is used, an output directly proportional to the gradient can be obtained by subtracting the outputs of the two sensitive elements. However, very careful matching of the characteristics of the two sensitive elements is required in order to obtain a difference output which is independent of the field strength.

One successful application of this second method involves the use of differential coils. Two small matched coils are mounted a few millimeters apart and their outputs connected so as to subtract. The design of such coils is discussed by Garrett (8).

The Hall effect gradient meter described in this report makes use of this second method of gradient measurement.

B. The Hall Effect

1. Simple theory

In the arrangement shown in Fig. 1, a semiconductor in the form of a long flat plate carries a current in the x-direction. A magnetic field is applied in the z-direction. Under these conditions an electric field appears in the y-direction. This field produces the Hall voltage V between the sides of the plate. The simple free electron theory leads to the following expression for the Hall voltage:

$$V = \frac{3\pi IB}{8 \text{ net}} 10^{-8} \quad (1)$$

where n is the number of free carriers per cm^3 , e is the electronic charge in coulombs, I is the current in amperes, B is the induction in gauss, t is the thickness in cm, and V is the Hall voltage in volts.

The proportionality between V and B is the basis for using the Hall effect to measure magnetic field strengths. Instruments using this principle are described by Pearson (14) and Mason, Hewitt and Wick (12). A commercial instrument is built by Dyna-Labs, Inc.

2. Factors affecting the choice of probe material

The sensitivity of the Hall voltage to the magnetic field may be defined as

$$k = \frac{3\pi I}{8n_{\text{et}}} 10^{-8}. \quad (2)$$

The amount of current which can be sent through the Hall plate will be limited by the amount of power which can be

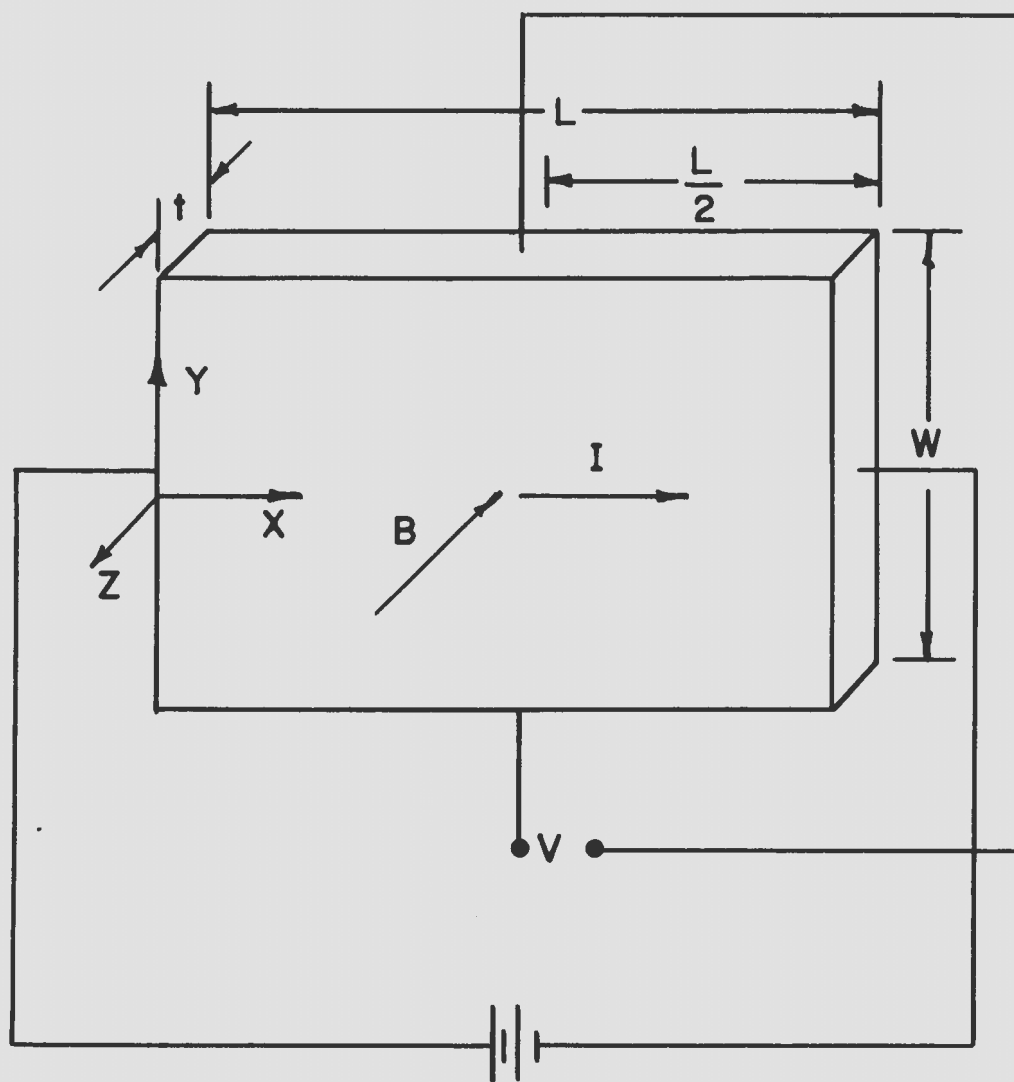


Fig. 1. Hall Effect Geometry

dissipated without undue heating. In terms of the power P we may write

$$k = \frac{3\pi}{8ne} \left[\frac{PW}{tL\rho} \right]^{1/2} 10^{-8} = \left[\frac{3\pi}{8tL} \frac{PW}{\mu|R|} \right]^{1/2} 10^{-8} \quad (3)$$

where W is the width of the plate in cm, L is the length of the plate in cm, P is the power input in watts, ρ is the resistivity in ohm-cm, $R = 3\pi/8ne$ is the Hall coefficient in cm³/coulomb, and $\mu = (ne\rho)^{-1}$ is the mobility in cm²/volt-sec. Values of the mobility and Hall coefficient at room temperature for typical materials are given in Table 1.

Table 1. Electrical Properties of Typical Materials

Material	(ohm-cm)	(cm ³ /coul)	(cm ² /volt-sec)	$\mu R $
Cu	$1.6(10)^{-6}$	$-5.5(10)^{-5}$	35	$4.2(10)^{-2}$
Bi	$1.2(10)^{-4}$	$-1.0(10)^{-2}$	9	$3.0(10)^{-1}$
Ge	10	$-2(10)^4$	3600	$8.5(10)^3$
InSb	$5(10)^{-3}$	$-3.5(10)^2$	$6(10)^4$	$4.6(10)^3$

The use of germanium should lead, therefore, to the highest sensitivity. It may be noted that if maximum power output rather than maximum voltage output is desired, the efficiency depends upon the square of the mobility. In this case InSb seems to be the preferable material. This is discussed by Saker, Cunnell and Edmond (16).

The number of free carriers, and hence the Hall coefficient, is temperature dependent. In the intrinsic region, which begins a little above room temperature for germanium and below room temperature for InSb, the Hall coefficient varies exponentially with the temperature. It is desirable, therefore, to avoid this temperature region. In this regard germanium is preferable to InSb since its Hall coefficient is very nearly independent of temperature in the vicinity of room temperature. The resistivity, however, is strongly temperature dependent, but this is much less important.

Experiment shows that the Hall coefficient of germanium is a function of the magnetic field strength. This field

dependence is much worse for p-type germanium than for n-type (21). The field dependence for n-type germanium is discussed by Mason, Hewitt, and Wick (12). Their measurements show that the minimum field dependence occurs if the Hall plate is cut so that the current is along a crystal axis and the magnetic field is applied along a second crystal axis. With this orientation the field dependence amounts to only 2 per cent at 20 kilogauss.

A thermoelectric effect which may interfere with the Hall effect is the Ettingshausen effect. The application of a magnetic field to a current-carrying semiconductor produces a temperature difference between the sides of the semiconductor. This may result in unbalanced thermal emf's at the Hall contacts. Since this effect is reversed by reversing the direction of the current and is slow in its response, it may be eliminated by using alternating current of relatively low frequency.

3. Shorting effect of current leads and effect of a non-uniform magnetic field

The Hall effect problem for a three-dimensional conductor of arbitrary shape can be formulated in the following way: We suppose that the conductor is carrying a current of density $\vec{J}$ in an external magnetic field of strength $\vec{B}$ and assume that (1) $\vec{B}$ is constant in time, (2) the field due to the current $\vec{J}$ is negligible, (3) the current $\vec{J}$ and the electric field $\vec{E}$ in the conductor are related by the equation

$$\vec{J} = \sum_j A_{1j}(\vec{B}) \vec{E}_j. \quad (4)$$

From Maxwell's equations we get

$$\text{div } \vec{B} = 0 \quad (5)$$

$$\text{curl } \vec{B} = 0 \quad (6)$$

$$\text{curl } \vec{E} = 0. \quad (7)$$

Because of Eq. 7 we may also take

$$\vec{E} = -\text{grad } \phi. \quad (8)$$

In the steady state condition

$$\text{div } \vec{J} = 0. \quad (9)$$

Combining Eq. 9 and Eq. 4 and using Eq. 8, we get

$$\begin{aligned} \text{div } \vec{J} &= \sum_i \frac{\partial J_i}{\partial x_i} = \sum_{ij} \left[A_{ij} \frac{\partial E_j}{\partial x_i} + \frac{\partial A_{ij}}{\partial x_i} E_j \right] \\ &= - \sum_{ij} \left[A_{ij} \frac{\partial^2 \phi}{\partial x_i \partial x_j} + \frac{\partial A_{ij}}{\partial x_i} \frac{\partial \phi}{\partial x_j} \right] = 0. \quad (10) \end{aligned}$$

The boundary conditions are:

- (1) the tangential component of $\vec{E}$ vanishes on perfectly conducting surfaces,
- (2) the normal component of $\vec{J}$ vanishes on insulated surfaces.

The form of the matrix A_{ij} must be determined from the properties of the medium. An expression for A_{ij} for germanium has been developed by Seitz (17) who extended Davis's (5) solution of Boltzmann's equation to arbitrary directions of the magnetic field and current with respect to the crystal axes. Seitz's expression for Eq. 4 is

$$\vec{J} = \sigma_0 \vec{E} + \alpha (\vec{E} \times \vec{H}) + \beta \vec{E} H^2 + \gamma \vec{H} (\vec{E} \cdot \vec{H}) + \delta \vec{T} \cdot \vec{E} \quad (11)$$

where $\vec{T}$ is a diagonal tensor with elements H_1^2, H_2^2, H_3^2 .

An even more general expression has been derived by Casimir (3). His derivation is based on the thermodynamics of irreversible processes and Onsager's principle of microscopic reversibility. His expression is

$$E_i = \sum_j \rho_{ij}(B) I_j + [\vec{r} \times \vec{R}(B)]_i \quad (12)$$

where the ρ_{ij} are even functions of B and the components of $\vec{R}$ are odd functions of B .

Using Eq. 12 and applying the symmetry properties of germanium, Mason, Hewitt and Wick (12) have shown that the Hall coefficient is a function of crystal orientation.

If one simply assumes that

$$\vec{J} = \sigma(\vec{E} + \vec{F}_H) \quad (13)$$

where $\vec{F}_H$ is the Hall force given by

$$\vec{F}_H = R\vec{J} \times \vec{B} \quad (14)$$

then this leads to

$$\frac{\vec{J}}{\sigma} = \frac{\vec{E} + \mu \vec{E} \times \vec{B} + \mu^2 \vec{B}(\vec{E} \cdot \vec{B})}{1 + \mu^2 B^2} \quad (15)$$

where $\mu = \sigma R$ is the mobility.

This agrees with Seitz's expression Eq. 11 to terms of order B^2 except for the term $\delta \vec{T} \cdot \vec{E}$.

The general problem in three dimensions has not been solved. The problem is reduced to two dimensions by assuming that the conductor is a very thin flat slab and that the magnetic field is perpendicular to the plane of the slab. Hence $\vec{J}$ and $\vec{E}$ are perpendicular to $\vec{B}$. It is generally further assumed that the medium is isotropic. If it is further assumed that the magnetic field is uniform and that the simple expression Eq. 15 holds, then Eq. 10 reduces to Laplace's equation.

This problem was solved for a rectangular slab by Isenberg, Russell and Greene (9) to determine the shorting effect of the current electrodes. The same problem was discussed by Volger (19).

The slightly more general problem of a slab of arbitrary shape with two current electrodes was solved by Frank (7). He points out that the solution depends on the condition

$$(\mu B)^2 \ll 1$$

and demonstrates that no particular advantage in sensitivity is obtained by using odd shaped materials. The case of a rectangular slab with four current terminals was investigated by Wick (20).

If the magnetic field is not uniform, Laplace's equation no longer holds in general. To first order in B it does hold and this case has been solved by Koppe and Bryon (10)

for a long narrow slab and a magnetic field which is a function only of the distance along the length of the slab and for which $B(x) \rightarrow 0$ as $|x| \rightarrow \infty$. The solution is in terms of an integral.

The case of a rectangular slab in a magnetic field which varies only in the direction of the length of the slab has been solved by Flanagan, Flinn and Averbach (6). Their solution was applied to the geometry of the probe used in the gradient meter described in this report.

A thin slab is located as shown in Fig. 1. The magnetic field is in the z-direction and is given by

$$B(x) = B(L/2) \phi(x). \quad (16)$$

The expression to be evaluated is

$$\frac{V_m}{V_H} = \frac{4}{\pi W} \sum_{n=1}^{\infty} \left\{ \int_0^L \phi(x') \sin \frac{n \pi x'}{L} dx' \right\} \frac{1}{n} \frac{\tanh n \frac{\pi W}{2L} \sin n \frac{\pi x}{L}}{L} \quad (17)$$

where V_m is the Hall voltage measured between the points $(x, \pm W/2)$ and

$$V_H = \frac{RIB(L/2)}{t}.$$

We assume a uniform gradient so that

$$\phi(x) = 1 + a(x - L/2). \quad (18)$$

Substituting Eq. 18 into Eq. 17 and evaluating the integrals, we get

$$\frac{V_m}{V_H} = \frac{8}{\pi^2} \frac{L}{W} \left[S_n - \text{odd} - a \frac{L}{2} S_n - \text{even} \right] \quad (19)$$

where

$$S_n = \sum_n \frac{1}{n^2} \tanh n \frac{\pi W}{2L} \sin n \frac{\pi x}{L}$$

and $S_n - \text{odd}$ means a sum over odd values of n and $S_n - \text{even}$ means a sum over even values of n . Eq. 19 was evaluated for $L = 1.23$ cm, $W = 0.10$ cm and five values of x/L , the Hall probe position. The series were evaluated through $n = 23$. The results are given in Table 2.

Table 2. Dependence of the Hall Voltage on Probe Position

x/L	V_m/V_H	
	Exact Theory	Simple Theory
1/2	1.01	1.00
0.36	1.02 - 0.28 aL/2	1.00 - 0.28 aL/2
1/3	1.01 - 0.33 aL/2	1.00 - 0.33 aL/2
1/4	1.01 - 0.50 aL/2	1.00 - 0.50 aL/2
1/6	1.00 - 0.66 aL/2	1.00 - 0.67 aL/2

If the simple theory held we would have

$$\frac{V_m}{V_H} = \phi(x) = 1 + a(x - L/2) = 1 + a \frac{L}{2} \left[\frac{2x}{L} - 1 \right]. \quad (20)$$

The values for the simple theory are also given in Table 2. It appears that the simple theory can be used with an error of less than 1 per cent even for probes as close to the end as L/6.

III. DESCRIPTION OF THE INSTRUMENT

A. General Principle

The field probe was a bar of germanium 1- by 1- by 12-mm with two sets of Hall leads attached about 2 mm either side of its center. One hundred cycle ac flowed in the long direction of the bar so that in a magnetic field two 100 cycle a-c Hall voltages were obtained which were proportional to the magnetic field strengths at two points of the field 4 mm apart. Each of these Hall voltages was fed through an isolation transformer and a phase shifting network and applied across a 100,000 ohm 10-turn Helipot. The voltages tapped off the two Helipots were connected in series and by proper phase adjustment were made to subtract. The difference voltage, which was proportional to the magnetic field gradient, was amplified and observed on a vacuum-tube voltmeter.

B. Probe Construction

1. Properties of germanium used

The field sensitive element was a 1- by 1- by 12-mm germanium bar. A number of these bars were obtained from the Motorola Company cut to size and with current leads attached. The bar used in the probe was one from the box labeled with the Motorola identification number C2A008. All the samples in this box were cut from a single wafer and resistivity measurements on a representative sample showed it to be homogeneous and to have a resistivity of 12.4 ohm-cm at room temperature. The Hall coefficient at room temperature was approximately $-5(10)^4$ cm³/coulomb.

2. Surface preparation

The only surface treatment given the bars by Motorola was an etching with a solution consisting of four parts HNO₃ and one part of HF, followed by a washing in distilled water. The only further treatment given the bars here was a rinsing in methyl alcohol to remove a protective layer of wax.

3. Attachment of Hall leads

A support for the germanium bar and its leads was made by cutting grooves in a piece of polystyrene. A groove 1 mm deep was cut for the bar and current leads. Grooves 1/2 mm deep were cut for the Hall leads. These grooves were accurately positioned and were used as guides when the Hall leads were being attached. The germanium bar was temporarily held in place by screws clamped against the current leads.

The Hall leads were attached by a microsoldering technique based on a method developed by W. H. Mitchell (13). This involved pressing a carefully tinned wire against the germanium surface and discharging a capacitor through the junction to melt the solder.

The wire for the leads was 5 mil copper. The solder was ordinary 60% Pb, 40% Sn. The wire was tinned by melting the solder on the end of a small electric soldering iron heated to just above the melting point of the solder. The wire was dipped into ZnCl - NH₃Cl flux and then into the melted solder and slowly pulled out. After a few attempts

it was usually possible to obtain a small cone of solder on the end of the wire. The base of the cone was no bigger in diameter than the diameter of the wire. Blowing on the wire, as it was pulled out of the melted solder, sometimes helped.

After all the wires were tinned they were attached one at a time by the microsoldering technique. A micromanipulator, similar to the one shown in the frontispiece of Shockley's book (18), was used to hold the wire. A low power microscope was also useful. With the aid of the micromanipulator the wire was moved into the groove and was pressed against the germanium surface with a slight pressure. The positive lead from the pulsing circuit was connected to the wire at the micromanipulator and the negative lead to a current lead. This gave a current flow in the forward direction for n-type germanium. A small drop of lactic acid was applied to the joint as a flux.

The pulsing circuit consisted of a 1000 microfarad capacitor and a relay actuated by a foot switch. The circuit is shown in Fig. 2. Pressing down on the foot switch connected the capacitor to a dc voltage supply. Releasing the foot switch connected the capacitor to the external circuit.

For the initial pulse the capacitor was charged to 35 volts. As this generally had no effect the voltage was increased in steps of 5 volts until the capacitor discharge melted the solder. The melting was observed through the microscope. The lactic acid also vaporized due to the heating of the germanium. The solder usually melted at a discharge voltage of 40 to 50 volts. At this point the pulsing circuit was replaced by a Simpson meter and the resistance of the junction was measured in each direction. If the forward and reverse resistances differed by more than one or two ohms, another pulse at the same voltage was applied across the junction and the resistance was measured again. This procedure was repeated (with perhaps a slight increase in voltage) until the resistance difference was reduced to a minimum. It was necessary to be careful not to increase the voltage more than about 5 volts above that voltage at which the solder melted or to pulse the sample an undue number of times. Otherwise there was danger of breaking the bar. Successful detachment of the wire from the micromanipulator was considered sufficient evidence for a mechanically secure junction.

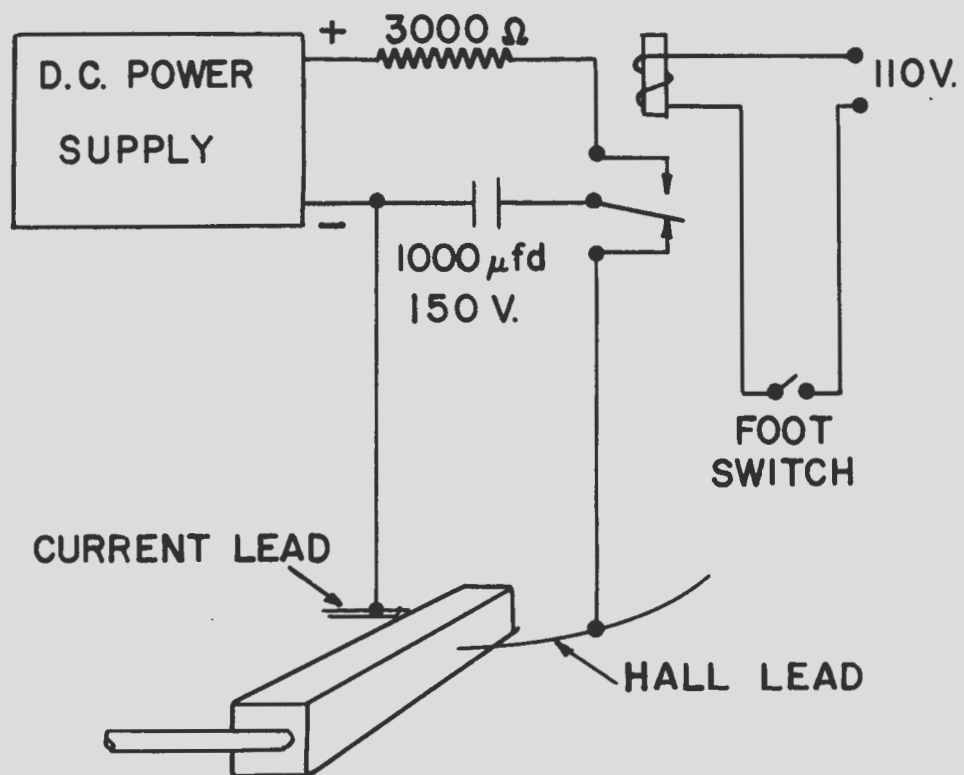


Fig. 2. Circuit Used to Attach Hall Leads.

The other probes were attached in the same way. When attaching several leads it was necessary to start at the center of the germanium bar and work toward the ends. Otherwise a lead previously attached would be melted off while attaching another one. When all the leads were on, Duco cement was poured over the germanium and wires to hold everything permanently in place. The dimensions of the completed probe are given in Fig. 3.

C. Circuit Description

1. Oscillator and power amplifier

A circuit diagram of the instrument is shown in Fig. 4. The 100-cycle oscillator, power amplifier and power supply were located on one chassis. This unit was designed primarily as a current source for a-c Hall effect measurements. The frequency could be varied over a range of a few cycles per second about 100 cycles per second to match the frequency of the narrow-band amplifier. Once the frequency was set it was rarely necessary to adjust it again. The amplitude of the output was continuously adjustable from zero to maximum. The output transformer T_3 was a UTC Type LS-56. The current for the probe was taken from a 1000 ohm secondary winding. A low impedance secondary winding was used to furnish a "bucking signal". This signal could be independently varied in amplitude and phase by the variable resistors R_7 , R_8 , R_9 , and R_{10} . This signal was fed to the amplifier where it was used to cancel out constant background signals.

2. Current measurement

A 10 ohm resistor was connected in series with the germanium probe to monitor the probe current. The voltage across this resistor could be connected to the vacuum-tube voltmeter by S_3 .

3. Probe balance resistors

It is very difficult to attach two Hall leads directly opposite each other as shown in Fig. 1. By using three leads as shown in Fig. 4, this difficulty was avoided. The probe balance resistors R_1 and R_2 were adjusted until there was no voltage at S_1 and S_2 in the absence of a magnetic field. This was then effectively the same as having two leads attached directly opposite each other.

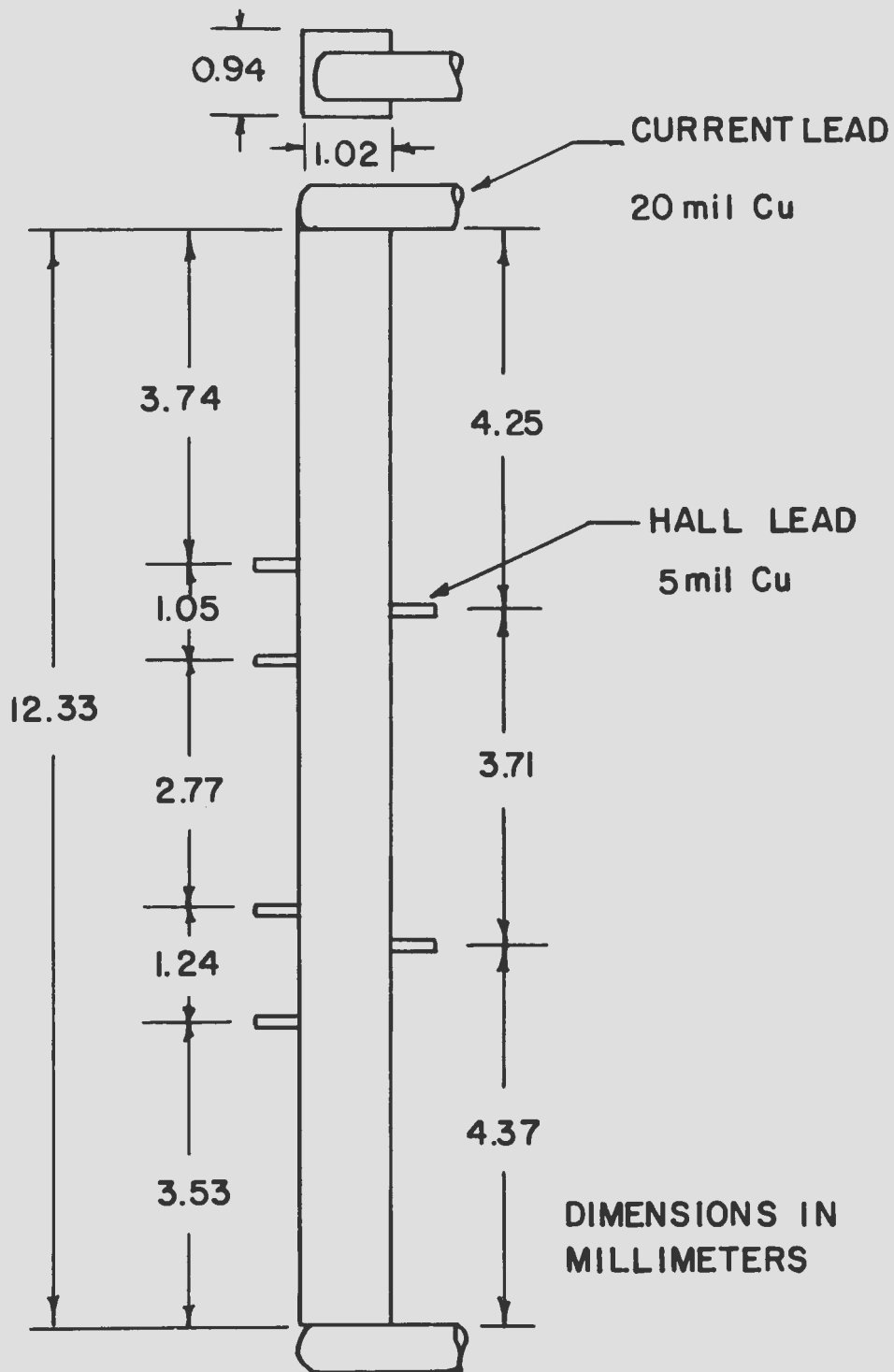


Fig. 3. Probe Dimensions.

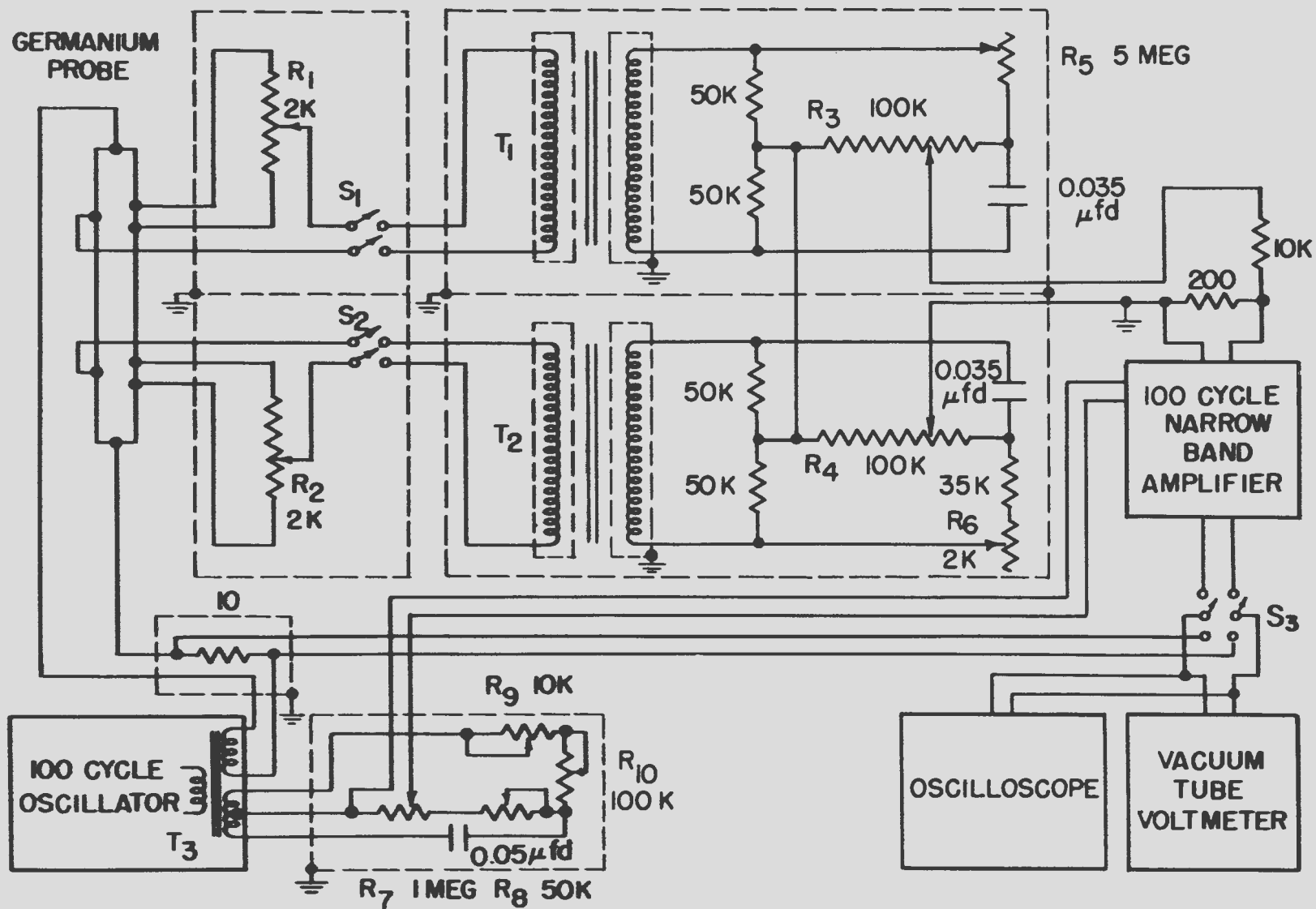


Fig. 4. Circuit of Hall Effect Magnetic Field Gradient Meter.

The values of the resistances of R_1 and R_2 were quite critical. If they were made too large an appreciable amount of the Hall voltage was lost across them. If they were made too small they tended to short a portion of the germanium bar. This led to a distorted Hall signal. The final value chosen for both R_1 and R_2 was 2000 ohms. Because of the large potential gradient along the germanium bar, their adjustment was critical even though 10-turn Helipot were used. Some sort of vernier adjustment was needed. Since they needed to be varied over only a small range they could have been partly replaced by fixed resistors.

Switches S_1 and S_2 were used to shut off either channel. They were used when adjusting R_1 and R_2 and when the field strength was being measured.

4. Isolation transformers

The General Radio Type 578B isolation transformers T_1 and T_2 had electrostatically shielded windings. The electrostatic shielding was found to be essential for reducing background signals to a minimum. Such signals were apparently caused by capacitive feed through between the transformer windings. The transformers had a turns ratio of 1000 to 4000 and were used as step up transformers. The primary impedance was approximately 2200 ohms.

5. Phase balance

The phase shifting circuits were standard. R_5 varied the phase of one channel and was used as a coarse phase control. It was a one-turn carbon potentiometer. R_6 varied the phase of the other channel and was used as a fine phase control. It was a 10-turn Helipot. This phase balance was essential since the two signals to be subtracted had to be exactly 180° out of phase.

6. Gain adjustment

R_3 and R_4 provided for compensation of any difference in gain of the two channels and for any differences in the Hall coefficient at the two points on the germanium bar. They were both 100,000 ohm 10-turn Helipot. One of them was always kept at maximum so that the overall gain would be constant. The relative gradient could also be measured by proper adjustment of these potentiometers. This is dis-

cussed below. The transformers T_1 and T_2 were connected so that the voltages taken off R_3 and R_4 were in opposition and subtracted.

7. Amplifier

The amplifier was a 100 cycle narrow-band amplifier. It was also designed to be used for ac Hall effect measurements. The band pass was only a few cycles and the maximum voltage gain was approximately 10^6 . Since the primary of the input transformer had an impedance of 200 ohms, the 50 to 1 voltage divider was added to the input for impedance matching. The only ground in the system was at the amplifier input. The bucking voltage was fed into the amplifier through a separate input and was subtracted from the main input electronically. The bucking voltage could be used to cancel out constant background signals.

8. Output

The output of the amplifier was fed through S_3 into a Hewlett-Packard vacuum-tube voltmeter, Model 400A.³ The output could also be viewed on a Dumont Type 304HR oscilloscope. This was useful when making the phase and gain adjustments. It also served to check polarity.

IV. TESTS OF THE INSTRUMENT

A. Preliminary Tests

1. Signal distortion and contact rectification

When the amplifier input was observed during the balancing of the probes with R_1 and R_2 , it was observed that at best balance a badly distorted signal still remained. The distortion was caused, principally, by a large second harmonic which was approximately one or two millivolts in amplitude. Many tests were made to determine the source of this distortion. Direct current measurements showed that the voltage V at S_1 was given approximately by the relation $V = aI + bI^2$ where I is the current through the germanium and a and b are constants. The linear term could be balanced out with R_1 and the quadratic term would account for the frequency doubling observed. When 30 microsecond current pulses were used and the probes were at best balance,

it was observed that the output pulse did not change polarity when the polarity of the current pulse was reversed. This indicated that the effect was practically instantaneous.

The following explanation of this effect is proposed: Since it is known that the Hall probe-germanium junctions rectify an alternating current, each junction might act as a variable resistor which, in effect, would move the tap on the balancing resistor back and forth as the current changed direction. Since the effective motion would always be reversed when the direction of the current was reversed, the symmetric behavior (represented by the quadratic term) would be explained. The magnitude of the effect varied from probe to probe. In the probe used for the tests described later, the effect was quite small (less than one millivolt).

As long as measurements were made using the narrow band amplifier, the effect caused no trouble since only the 100 cycle component was passed by the amplifier. If it were necessary, however, to measure time varying magnetic fields, a wide band amplifier would be necessary and this effect would then seriously limit the performance of the instrument.

Another effect which prohibited the use of a wide band amplifier was 60 cycle pick up. This occurred mostly at the transformers T_1 and T_2 which apparently needed better magnetic shielding.

2. Magnetoresistance

A decrease in current through the germanium probes with increasing magnetic field has been noticed in all probes made up to the present time. For the probe used for most of these experiments the decrease amounted to 2 per cent at 7500 gauss and 5 per cent at 10 kilogauss. Since no gradient measurements were made above 5 kilogauss, no special attempt was made to maintain a constant current.

3. Amplifier linearity

A check of the linearity of the amplifier and balancing circuits was made by removing the germanium probe and simulating a Hall voltage by a 1 ohm resistor carrying a measurable current. Within the experimental error of a

few per cent, the gain was constant over the range of input voltages 0 - 440 microvolts. The overall voltage gain was $4.5 (10)^4$.

4. Field dependence of the Hall coefficient

The Hall voltage as a function of magnetic field was measured at the amplifier input. The amplifier was not needed since the voltages were large enough to be read directly on the vacuum-tube voltmeter. A rough fit to a cubic equation led to the expressions

$$\begin{aligned} V_1 &= 0.57 (10)^{-3} [B + 2.6(10)^{-6}B^2 - 7.9(10)^{-10}B^3] \\ V_2 &= 0.60 (10)^{-3} [B + 4.2(10)^{-6}B^2 - 10.0(10)^{-10}B^3] \end{aligned} \quad (21)$$

for V_1 and V_2 in volts and B in gauss. V_1 is the voltage with S_1 closed and V_2 is the voltage with S_2 closed.

The major portion of the cubic term is due to the decrease in current caused by the magnetoresistance effect. The remainder of the cubic term can be attributed to a change in the Hall coefficient. The origin of the quadratic term is somewhat uncertain. It may be partly due to induced voltages caused by probe and lead vibration, but more likely it is due to the cross-magnetoresistance terms discussed by Mason, Hewitt and Wick (12). Owing to the anisotropy of germanium, these arise when the current and magnetic field are not along crystal axes.

5. Crystal orientation

The germanium bar used in the probe was obtained without specifying the crystal orientation. Since, as pointed out above, an improper orientation can lead to a field dependent Hall coefficient, it was of interest to locate the crystal axes to see if this was a possible cause of the observed field dependence. It would have been somewhat difficult to make x-ray measurements on the actual bar used. Hence the directions of the crystallographic axes of another bar from the same lot were determined. A series of back-reflection Laue pictures were taken to locate the axes of threefold symmetry. From these measurements the directions of the crystal axes were determined.

With reference to a set of rectangular coordinates oriented perpendicular to the 100, 010 and 001 crystal planes, the length of the bar (direction of current) was parallel to the vector

$$\vec{A} = 0.734\hat{i} + 0.256\hat{j} + 0.630\hat{k}.$$

A vector perpendicular to a side surface (Hall voltage or magnetic field direction) was given by

$$\vec{B} = -0.613\hat{i} + 0.593\hat{j} + 0.465\hat{k}.$$

It therefore appears that the crystals were cut with a random orientation and this fact probably caused the quadratic term in the Hall effect.

6. Field dependence of the difference voltage

In a uniform field the two Hall voltages should exactly cancel each other and there should be no difference voltage at any field strength. Measurements made in a uniform field, however, show a very strong field-dependent difference voltage. This can be explained if it is assumed that the two Hall voltages are not exactly the same function of the magnetic field. The voltages at S_1 and S_2 can be represented by

$$\begin{aligned} V_1 &= C a_0 (B_1 + a_1 B_1^2 + b_1 B_1^3) \\ V_2 &= a_0 (B_2 + a_2 B_2^2 + b_2 B_2^3) \end{aligned} \quad (22)$$

where C is a parameter representing the adjustable gain of one channel. The difference voltage is then

$$\begin{aligned} \Delta V = V_1 - V_2 &= a_0 \left[C(1 + 2a_1 B_2 + 3b_1 B_2^2) \Delta B + (C - 1) B_2 \right. \\ &\quad \left. + (C a_1 - a_2) B_2^2 + (C b_1 - b_2) B_2^3 \right]. \end{aligned} \quad (23)$$

Therefore as long as $a_1 \neq a_2$ and $b_1 \neq b_2$ there will be a difference voltage in a uniform field ($\Delta B = 0$). The behavior of this difference voltage will depend on the magnitudes of the differences in the coefficients and the value of the parameter C . Only one quantitative measurement of this difference voltage was made. The phase and amplitude controls were set for zero output at 4700 gauss and data were taken for only one direction of the magnetic

field. The best fit to the experimental points was obtained with the quadratic expression

$$\Delta V = 1.20 + 1.02(10)^{-6}B(4700 - B). \quad (24)$$

The constant term is due to improper setting of the zero point. This result does not show the cubic dependence indicated by Eqs. 22 and 23. The reason for this is not clear.

B. Tests with Step Pole Pieces

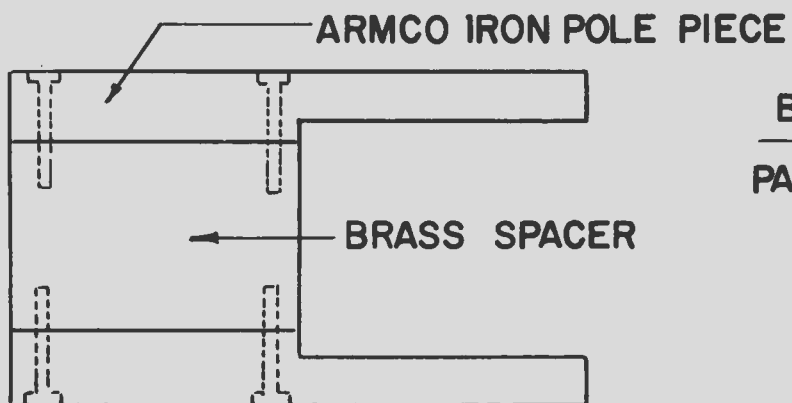
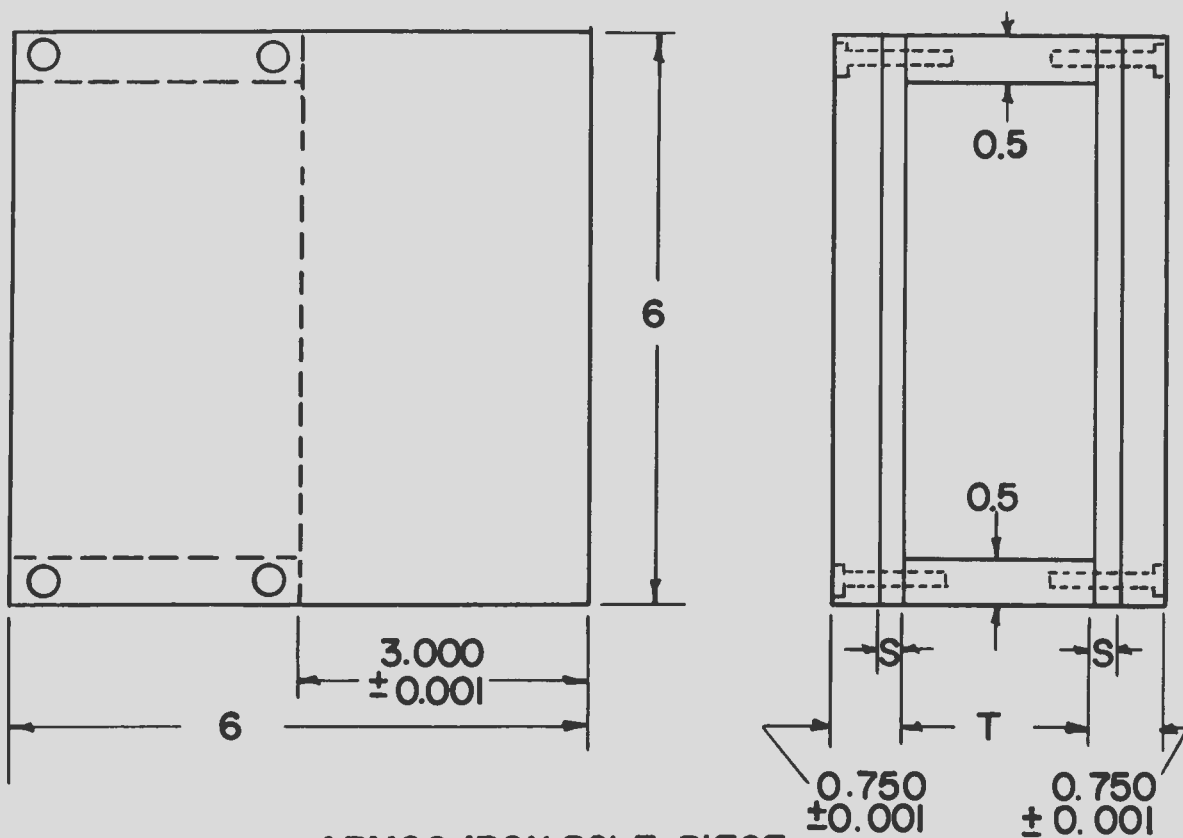
1. Object

The object of these tests was to determine the performance of the instrument in regard to useful range, sensitivity, and accuracy. This was done by measuring the gradient of a calculable magnetic field and comparing the experimental and theoretical values.

2. Step pole pieces

A calculable field was produced by special step pole pieces fitted to a Consolidated 23-104A electromagnet. The pole pieces consisted of two parallel plates of Armco iron 6- by 6- by 3/4-inches. Half of the surface of each plate was milled down to a prescribed depth to form a step along the center line. The step poles are shown in Fig. 5. Measurements were made in the median plane and along a line perpendicular to the step and to the axis of the magnet. The field decreased slightly as the step was passed due to the increased spacing between the pole faces. A peak in the field gradient occurred at the step. The peak had a width of one to two inches. The gradient as a function of position along the median plane was calculated from the known width of the pole face gap and the known height of the step.

Fig. 6 shows the pole pieces mounted in the magnet. Brass spacers provided the proper separation. The movable poles of the magnet were used to clamp the complete assembly in place. The field probe was fastened to an aluminum framework which allowed it to be moved through the field. A meter stick was used to determine the position of the probe.



ALL DIMENSIONS
IN INCHES

BRASS SPACERS

PAIR NO.	T
1	1.000 ± 0.001
2	$1.500 \pm "$
3	$2.000 \pm "$
4	$2.500 \pm "$

POLE PIECES

PAIR NO.	S
1	0.0050 ± 0.0001
2	0.050 ± 0.001
3	0.250 ± 0.001

Fig. 5. Step Pole Pieces.

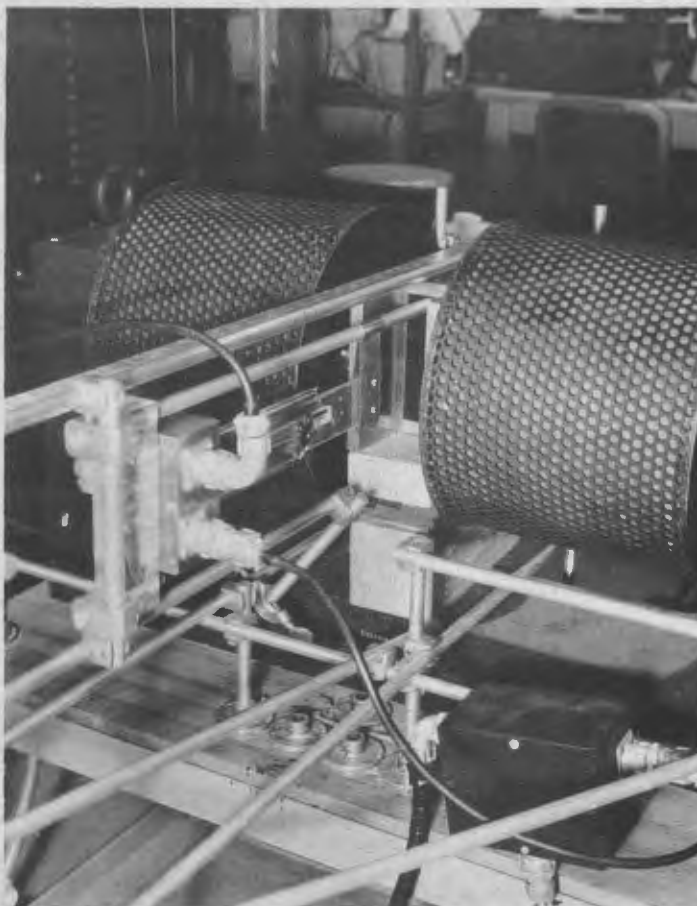


Fig. 6. Arrangement Used for Measuring Gradient Produced by Step Pole Pieces.

The step pole pieces can be seen between the magnet coils. A guide bar for the probe holder passes between the pole faces. The germanium field sensitive element can be seen to the right of the upper connector just below a strip of black tape. The upper cable carries the Hall voltage from one probe set to the balancing resistors. The probe current is supplied through the lower cable which is connected to the current monitoring resistor located in the black box in the lower right hand corner of the picture.

3. Procedure

With the probe well away from the magnet and the magnet off, the current through the probe was adjusted to 2 ma. The probe balance potentiometers R_1 and R_2 were then adjusted until a minimum output from each Hall probe set had been obtained. R_1 was adjusted with S_2 open and R_2 was adjusted with S_1 open. This is the standard three probe method of adjusting two Hall leads to the same potential. This adjustment did not reduce the output to zero because of a small amount of voltage pickup somewhere in the system. In order to reduce the output to zero, a small amount of "bucking voltage" was added through a separate input to the amplifier (with both S_1 and S_2 closed). The probe was then moved into the magnet as near as possible to a position of zero gradient (as established from the theoretical gradient calculations) and the magnetic field strength was adjusted to the value at which measurements were to be made. The amplitudes and phases of the two Hall signals were then adjusted (R_3, R_4, R_5, R_6) so that they exactly cancelled each other and the output again was zero. Readings of output vs. probe position were then made as the probe was moved through the gap. The entire procedure was repeated after each run.

4. Results

(a) Method of analysis. The calculated quantity with which the experimental measurements were compared is $G = (dB/dx)/B_0$ where B_0 is the field that would exist well away from the step if there were no edge effects. The quantity B_0 was calculated with the aid of the theoretical formulas and a measurement of the field at the step. Field measurements were made either directly by a rotating coil gaussmeter or by the use of only one Hall voltage channel which had been previously calibrated with the rotating coil gaussmeter.

In terms of the output difference voltage ΔV , the gradient G is given by

$$G = K \Delta V / B_0 \quad (25)$$

where

$$K = t / (gRI \Delta x), \quad (26)$$

and t is the thickness of the germanium bar, R is the Hall coefficient, I is the current through the bar, Δx is the probe spacing, and g is the gain of the system. The calibration constant K was computed from the maximum value of G calculated theoretically, the measured values of B_0 , and the measured values of the maximum ΔV . The values for K , obtained by substitution in Eq. 25, are given in Table 3.

Table 3. Computed Values of the Calibration Constant K
(gauss/volt-inch)

Peak gradient (inches ⁻¹)	(approx.) B_0 - gauss			
	1000	2000	3300	4100
0.063	16.4	15.8	14.5	12.7
0.141	14.5	13.6	13.6	12.4

Runs were also made at a peak gradient of 0.007 inches⁻¹. However, uncertainty in the zero position and distortion due to edge effects made calculation of K impractical.

(b) Graphs. Fig. 7 shows the gradient obtained with a step (S) of 0.250 inches and a gap (T) of 2 inches. The value of K used for all the experimental points on this graph was 14.5 gauss/volt-inch. The large peak in the center is due to the step. The large gradients at either end are due to edge effects. The curve for the lowest value of B_0 fits the theoretical curve at the peak gradient because of the calibration constant used. The deviations of the other experimental points are probably due to saturation effects and to an uncertainty in the position of the point of zero gradient (see section IV B5).

Fig. 8 shows the gradient obtained with a step of 0.050 inches and a gap of 1.5 inches. The value of K used for all experimental points on this graph was 16.4 gauss/volt-inch.

Fig. 9 shows the gradient obtained with a step of 0.005 inches and a gap of 1.5 inches. The value of K used for all experimental points on this graph was 14.5 gauss/volt-inch. These measurements were made with the magnetic field reversed as compared to its direction for the runs of Fig. 7 and Fig. 8.

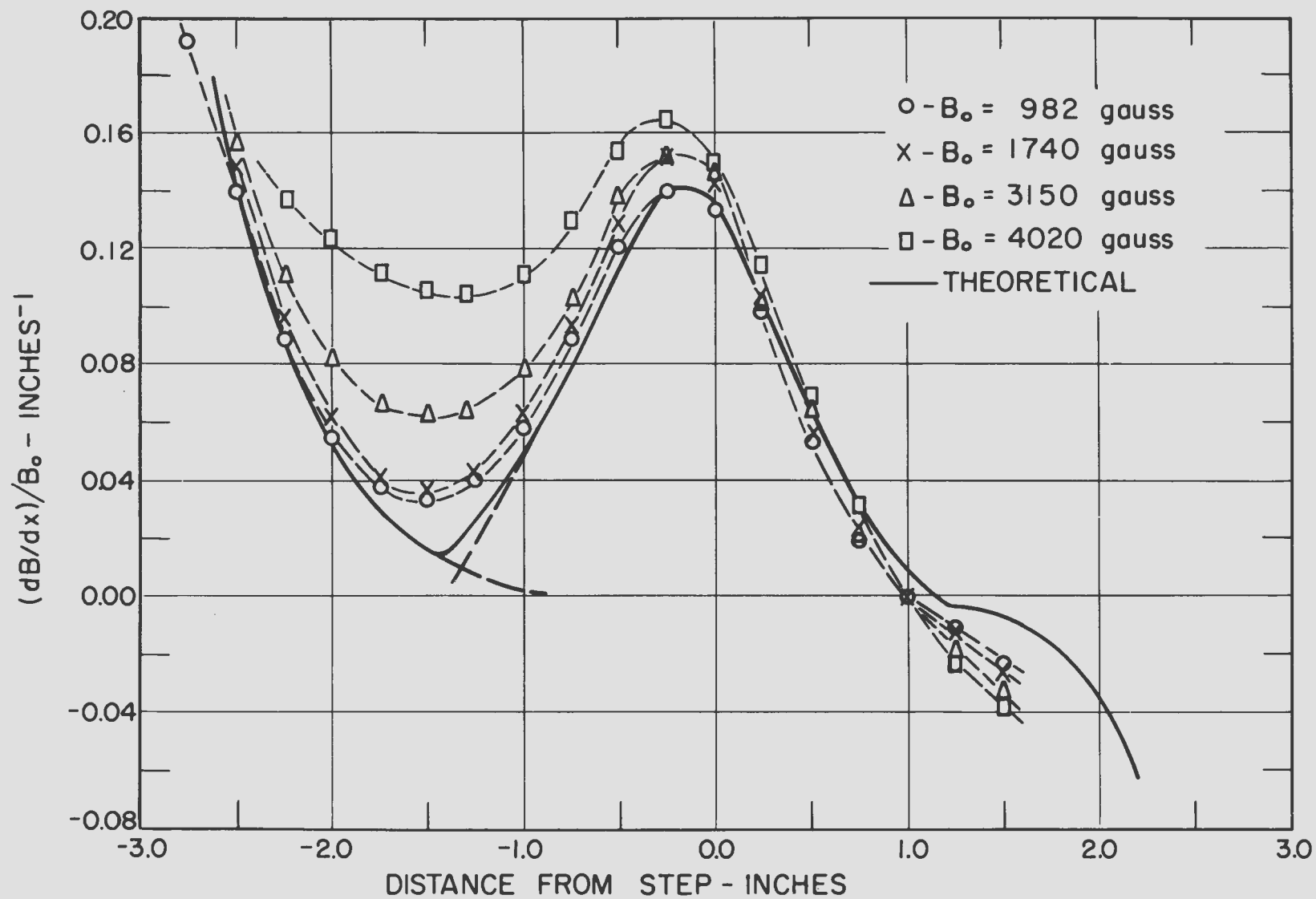


Fig. 7. Gradient Produced by Step Pole Pieces.
Pole face spacing for positive distances 2.000 inches.
Pole face spacing for negative distances 2.500 inches.

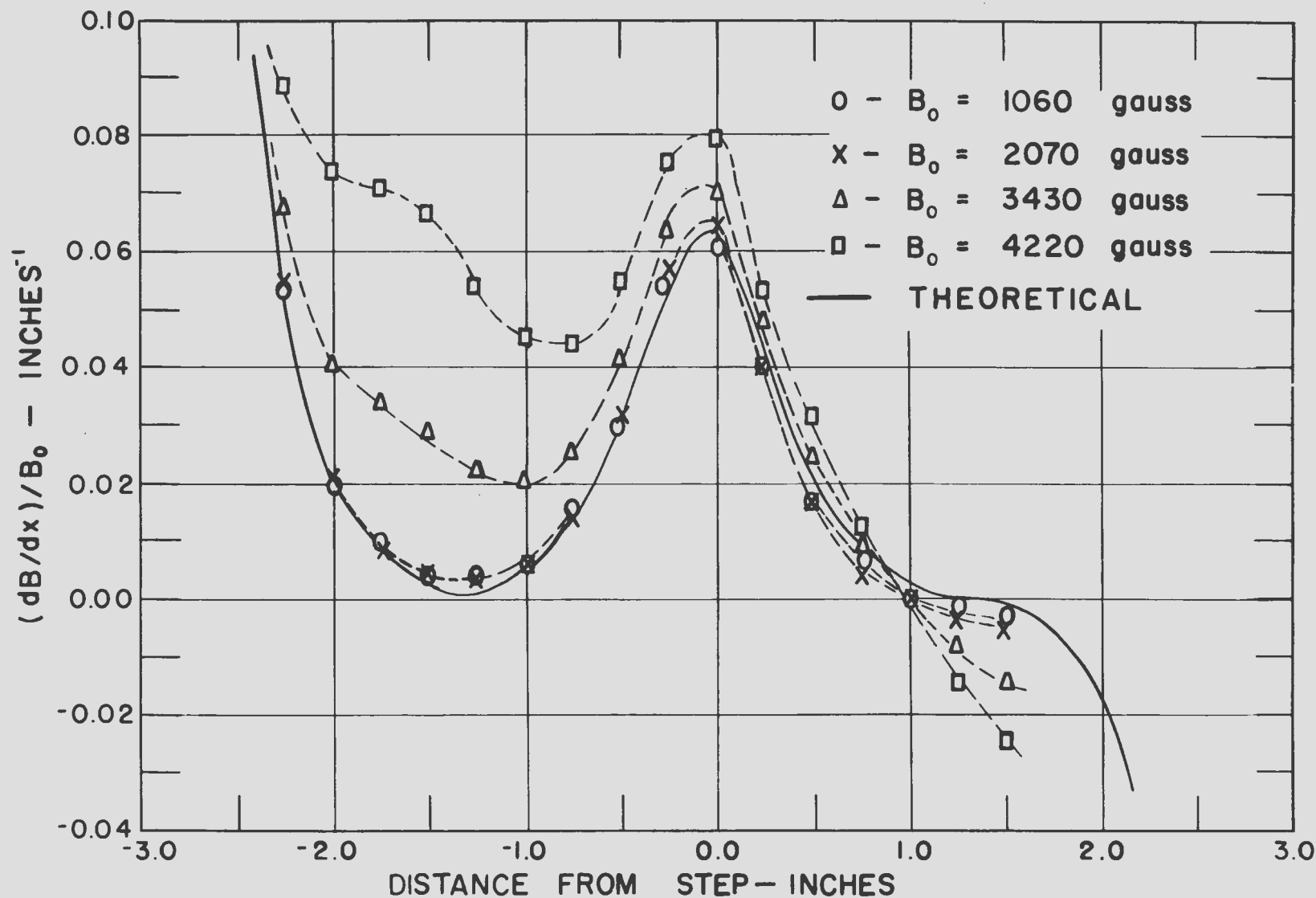


Fig. 8. Gradient Produced by Step Pole Pieces.
Pole face spacing for positive distances 1.500 inches.
Pole face spacing for negative distances 1.600 inches.

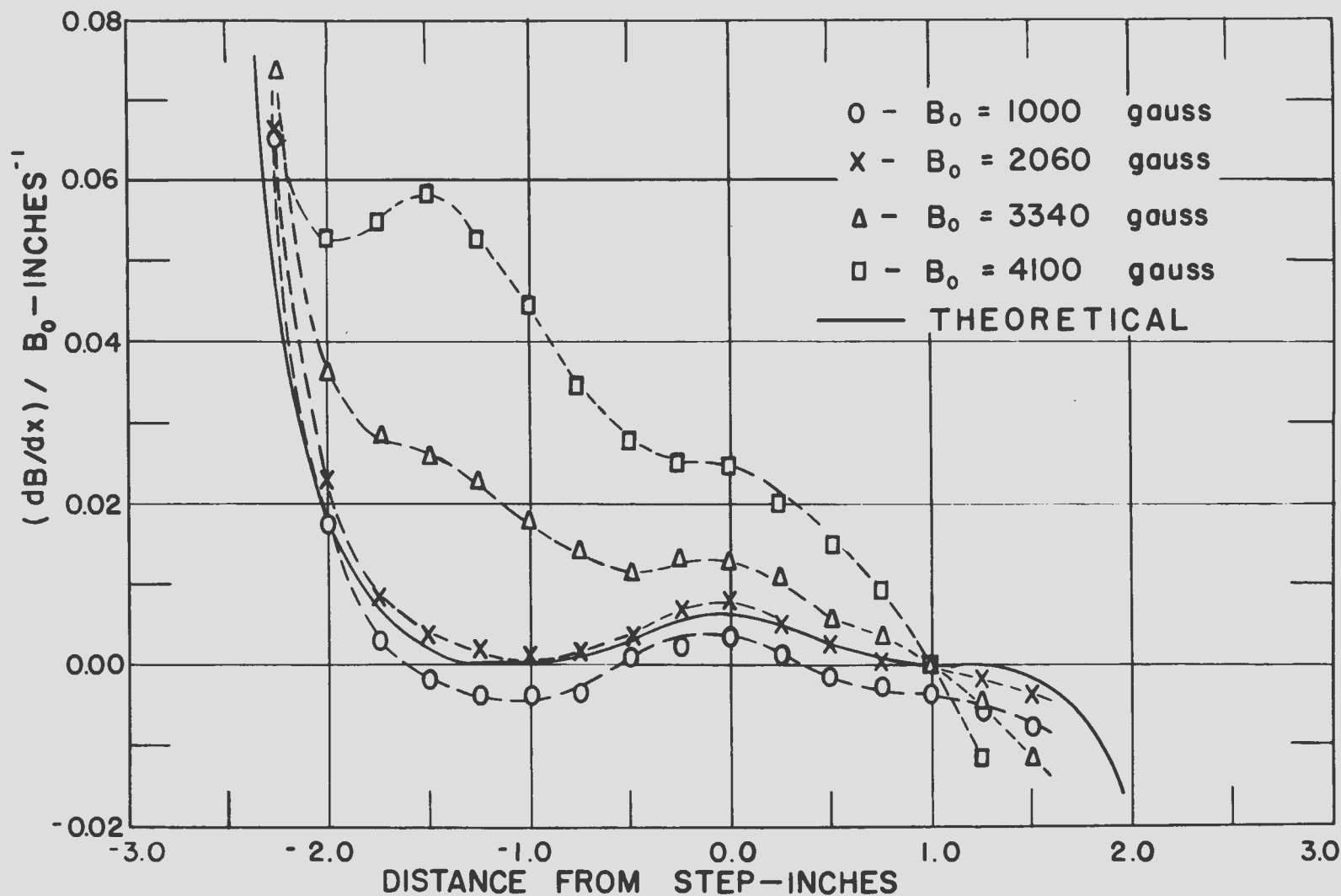


Fig. 9. Gradient Produced by Step Pole Pieces.
Pole face spacing for positive distances 1.500 inches.
Pole face spacing for negative distances 1.510 inches.

5. Errors

(a) Edge effects. At the edges of the pole faces there are very large gradients. The complicated geometry at the edge makes an exact calculation of these effects very difficult. As a first approximation, however, it was assumed that the edge could be considered as the corner of a very thick semi-infinite pole piece. The gradient for this geometry was easily calculated and it is plotted on the graphs as part of the theoretical curve. The general agreement with the measured points at low field strengths seems to justify the assumptions made.

(b) Saturation effects. Saturation at the corner of the step would tend to decrease the peak gradient slightly, but actually an apparent increase in the peak gradient with field strength was observed. There also appeared to be an additional gradient between that due to the edge and that due to the step. The following explanation is proposed. The poles of the magnet consist of two cylinders each with a radius of two inches. The pole pieces are square plates 6 inches by 6 inches. The area of the face outside the poles is therefore $36 - \pi 4 = 23.4$ inches². If the square plates are $3/4$ inch thick, all the flux leaving this area must pass through a circular band of area $2 \pi (2)(3/4) = 9.42$ inches². The flux density in this band will therefore be approximately 2.5 times that at the surface. Thus, for a surface flux density of 4000 gauss, the flux density in the band would be about 10,000 gauss and saturation effects would be large. This would cause the field to fall off before the edge is reached and hence give rise to the observed extra gradient.

(c) Effect of the field dependent term. Eq. 23 may be written

$$\Delta V = \Delta B / (K \Delta x) + a_0 \left[(C-1)B + (Ca_1-a_2)B^2 + (Cb_1-b_2)B^3 \right] \quad (27)$$

where

$$1/K = a_0 \left[C(1 + 2a_1B + 3b_1B^2) \right] \Delta x.$$

To obtain a rough idea of the effect of this field dependence, the second term in Eq. 27 may be replaced by the experimental result given in Eq. 24. Eq. 24 may also be

made slightly more general by dropping the constant term and inserting the parameter B' . We then have

$$\Delta V = \Delta B/K \Delta x + 1.02(10)^{-6} B(B'-B). \quad (28)$$

Neglecting the field dependence of K and multiplying Eq. 28 by K/B_0 , we get

$$G' = G + 1.02(10)^{-6} KB(B'-B)/B_0 \quad (29)$$

where $G' = K \Delta V/B_0$ and $G = (\Delta B / \Delta x)/B_0$.

If $B' = B_0$ and $B' - B = kB_0$, then

$$G' = G + 1.02(10)^{-6} Kk(1-k)B_0. \quad (30)$$

Thus it is seen that the error in the gradient is directly proportional to B_0 and reaches a maximum when $k = 1/2$, that is, when the field has fallen to half its initial value. For the step with a maximum G of $0.141 \text{ inches}^{-1}$ the field changes by $0.2B_0$ in passing over the step. For $B_0 = 4020$ gauss the second term in Eq. 30 amounts to $0.0095 \text{ inches}^{-1}$ which is 6.7 per cent of the peak gradient. For the step with a maximum G of $0.063 \text{ inches}^{-1}$ the field changes by $0.06B_0$ in passing over the step. For $B_0 = 4220$ gauss the second term in Eq. 30 amounts to $0.0040 \text{ inches}^{-1}$ or 6.3 per cent of the peak value. This effect is therefore significant at the higher field strengths.

In the event that these quadratic terms cannot be removed, it would be possible to prepare calibration curves giving the appropriate correction in terms of the field strength at which the gradient is being measured. This, of course, would require field measurements along with the gradient measurements.

(d) Uncertainty in the position of zero gradient.

Failure to properly adjust the instrument to give zero output in a zero gradient may lead to a constant error in the measured gradient. An error of this type may be clearly seen in Figure 9 for the points for $B_0 = 1000$ gauss. Because of the field dependence discussed above, it was necessary to adjust the zero point with the step pole pieces in place. Due to the edge effect there was then only one position of the probe at which the gradient was actually zero. This position had to be determined from the theoreti-

cal curve and the results show that the choice was not always correct. The apparent increase of the peak gradient and the decrease of K with increasing field strength are probably due to an incorrect choice of the position of zero gradient. If it is assumed that the position of zero gradient moved toward the step due to the extra gradient caused by saturation as the field increased, then the plotted curves should be corrected by displacing them downward. In this way all the curves may be matched at the peak gradient and the resulting fit with the theoretical curve is not unreasonable. Until the field dependence can be eliminated it would seem necessary to have a separate magnet available providing uniform fields at the desired field strengths.

(e) Other errors. The magnetoresistance effect caused the current I through the probe to decrease and hence K to increase with increasing field strength. This effect is probably 1 per cent or less at the field strengths used.

A glance at the graphs suggests that the measured position of the probe with respect to the step may have been slightly in error.

The noise level at the output was 0.2 volts. This has the most effect at low fields and small gradients. For example, at 1032 gauss at the peak gradient of 0.063 inches⁻¹ the value of ΔV was 4.1 volts. In calculating K the value $4.1 \pm 0.2 - 0.0 \pm 0.2 = 4.1 \pm 0.4$ volts was used. There is therefore a 10 per cent uncertainty in the value $K = 16.4$ gauss/volt-inch due to noise alone. The measurement of B_0 adds another 2 per cent. At 982 gauss and a peak gradient of 0.141 inches⁻¹ the value of ΔV was 9.6 volts. In this case noise caused an uncertainty of 4.2 per cent when K had a value of 14.5 gauss/volt-inch. This may be part of the reason for the difference between these two values of K .

6. Conclusions

The sensitivity of the instrument may be taken as $1/K = 1/14.5 = 0.069$ volts/(gauss/inch). The noise level of 0.2 volts corresponds to an absolute gradient of 2.9 gauss/inch or 1.14 gauss/cm. The maximum absolute gradient measured was theoretically 567 gauss/inch and experimentally 669 gauss/inch, the error being 18 per cent. The large error is, as explained above, due to the field dependent term in

the difference voltage and the resultant inability to correctly zero the instrument in an unknown field. This should be considered as a maximum error since the plotted results show that once the zero is properly set the agreement is much better.

IV. DISCUSSION

The major limitation of the instrument at present is its field dependence. Because of this field dependence it is not possible to balance the instrument at one field strength and use it at another. Hence a uniform field must be available at every field strength at which gradients are to be measured. In addition the maximum gradient that can be measured is limited to about 500 gauss/inch, since a large gradient is equivalent to a large change in the field strength with a change in position.

The extra field dependent terms in the Hall effect formula, which lead to this field dependence, also lead to a nonlinearity in response. Hence the sensitivity is field dependent. Corrections could be made by calibration but simultaneous field measurements would be necessary in order to use the calibration.

It is expected that these field dependence difficulties can be reduced by making the probe out of a properly oriented germanium crystal. If field dependence still proves to be troublesome, there remains the possibility of reorienting the probe so as to avoid large Hall voltages.

Neglecting the magnetic field due to the probe current the curl of the field to be measured is zero. Hence, $\partial B_z / \partial x = \partial B_x / \partial z$. The measurements described above were made with the field in the median plane in the z-direction and with the probe oriented so as to measure $\partial B_z / \partial x$. If, instead, the probe were oriented so as to measure $\partial B_x / \partial z$, then the main field would be parallel to the probe current and, as long as measurements were made in the median plane, neither Hall voltage would be larger than the difference voltage. The field dependent terms should, therefore, be very small.

Another source of non-linearity is the change in current due to the magnetoresistance effect. This non-linearity

might be avoided by using some type of constant current source. The simple method of placing a large resistor in series with the probe was tried and proved to be quite effective.

The minimum gradient which can be measured is about 5 gauss/inch. This limit is set by the gain of the system and the noise level. Perhaps the easiest way to increase the sensitivity would be to use a thinner germanium bar in the probe. The present bar is 1 mm thick. With care it should be possible to make a bar only 0.1 or 0.2 mm thick. This would increase the sensitivity by a factor of 5 to 10. Another possibility is to increase the probe current. Large currents without excessive heating may be obtained by using current pulses. Tests showed that the germanium bar of the dimensions used in the probe would stand 165 volt, 10 μ sec pulses at a repetition rate of 1000 pulses per second without heating. The peak pulse current was 0.11 amperes which is 55 times the current used with 100 cycle sine waves. Some tendency for hole injection was noted at these large currents. This would lead to quite erratic behavior. The instrument would have to be redesigned to handle the Hall voltage pulses.

The instrument cannot be used in time varying magnetic fields not only because of the field dependence already discussed but also because a narrow-band amplifier was used. A wide band amplifier would mean a greater noise level.

There is no lower limit to the field strengths at which the instrument will operate as long as the gradient is at least 5 gauss/inch. In fact, the lower the field strength the better it will operate. No gradient measurements have been made in field strengths above 5000 gauss. Presumably all the difficulties discussed above would become worse at higher field strengths.

The advantages of the instrument are its small, stationary probe and the fact that it is direct reading. It will follow gradients which exist over a space of one inch. The mounting can be simple and light since it need only support the germanium bar and leads. If only one channel is used, the field strength may be measured.

The relative gradient may be read from the amplitude adjustment dials by adjusting one of them to give zero output. Since, when reading the gradient, both channels have the same gain,

$$\Delta V = k_1 H_1 - k_1 H_2 = k_1 \Delta H. \quad (31)$$

If now the gain of one channel is changed to give zero output

$$0 = k_2 H_1 - k_1 H_2 = k_2 H_1 - k_2 H_2 + k_2 H_2 - k_1 H_2 = k_2 \Delta H - \Delta k H_2$$

or

$$\frac{\Delta k}{k_2} = \frac{\Delta H}{H_2}. \quad (32)$$

The relative gradient is then obtained by dividing by the probe spacing. The k values may be obtained directly from the dial reading of the amplitude control since only relative gains are required. This measurement is also independent of probe current.

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