N.8 Magnetic Ohm's Law for Arbitrary B

In Section N.4 we developed a Magnetic Ohm's Law for $\mathbf{B} = B\hat{\mathbf{z}}$. We now allow $\mathbf{B}(\mathbf{x})$ to point in a general direction with components B_1, B_2 and B_3 and we consider first a DC static situation. Equation (N.1.5) then says

$$\kappa \mathbf{v} = \mathbf{E} + \mathbf{v} \times \mathbf{B} \quad \text{where} \quad \kappa \equiv 1/\mu \tag{N.8.1}$$

or

$$\begin{aligned} \kappa v_1 &= E_1 + v_2 B_3 - v_3 B_2 \\ \kappa v_2 &= E_2 + v_3 B_1 - v_1 B_3 \\ \kappa v_3 &= E_3 + v_1 B_2 - v_2 B_1 \ . \end{aligned} \tag{N.8.2}$$

Notice that dim(κ) = dim(B) = Tesla. Maple solves this equation for the velocity components v_i :

```
> eq1 := kappa*v1 - B3*v2+B2*v3 = E1;
```

$$eq1 := \kappa\, v1 - B3\, v2 + B2\, v3 = E1$$

```
> eq2 :=  kappa*v2 - B1*v3+B3*v1 = E2;
```

$$eq2 := \kappa\, v2 - B1\, v3 + B3\, v1 = E2$$

```
> eq3 :=  kappa*v3 - B2*v1+B1*v2 = E3;
```

$$eq3 := \kappa\, v3 - B2\, v1 + B1\, v2 = E3$$

```
> f := solve({eq1,eq2,eq3},{v1,v2,v3});
```

$$f := \left\{ v3 = \frac{B2\,\kappa\,E1 - B1\,\kappa\,E2 + B1\,B3\,E1 + B2\,E2\,B3 + E3\,\kappa^2 + E3\,B3^2}{\kappa\,(B2^2 + B1^2 + \kappa^2 + B3^2)},\right.$$

$$v1 = \frac{-\kappa\,B2\,E3 + \kappa^2\,E1 + B1^2\,E1 + B1\,B2\,E2 + B1\,E3\,B3 + \kappa\,E2\,B3}{\kappa\,(B2^2 + B1^2 + \kappa^2 + B3^2)},$$

$$\left. v2 = \frac{B2\,B3\,E3 - E1\,B3\,\kappa + \kappa^2\,E2 + \kappa\,B1\,E3 + E2\,B2^2 + B1\,B2\,E1}{\kappa\,(B2^2 + B1^2 + \kappa^2 + B3^2)} \right\}$$

From (N.1.8) and (N.1.9) we know that $\mathbf{J} = nq\mathbf{v} = (\sigma/\mu)\mathbf{v} = \kappa\sigma\,\mathbf{v}$. Extracting the v_i from the above Maple solution and multiplying by κσ we get

```
J1 := kappa*sigma*V1;
```

$$J1 := \frac{\sigma\,(-\kappa\,B2\,E3 + \kappa^2\,E1 + B1^2\,E1 + B1\,B2\,E2 + B1\,E3\,B3 + \kappa\,E2\,B3)}{B2^2 + B1^2 + \kappa^2 + B3^2}$$

```
J2 := kappa*sigma*V2;
```

$$J2 := \frac{\sigma\,(B2\,B3\,E3 - E1\,B3\,\kappa + \kappa^2\,E2 + \kappa\,B1\,E3 + E2\,B2^2 + B1\,B2\,E1)}{B2^2 + B1^2 + \kappa^2 + B3^2}$$

```
J3 := kappa*sigma*V3;
```

$$J3 := \frac{\sigma\,(B2\,\kappa\,E1 - B1\,\kappa\,E2 + B1\,B3\,E1 + B2\,E2\,B3 + E3\,\kappa^2 + E3\,B3^2)}{B2^2 + B1^2 + \kappa^2 + B3^2}$$

(N.8.3)

which is our new and very complicated tensor Magnetic Ohm's Law in the presence of an arbitrary **E** and **B** field. That is to say, we have **J** = Σ **E** where Σ is a 3x3 matrix which is a function of the B_i. If we could ignore the three B_i components (set them to zero in the above equations), the equations would reduce to the regular Ohm's Law $J_i = \sigma E_i$. This is in effect the case if $B_i << \kappa$ for all three components of **B**. So a condition for the tensor Ohm's Law reducing to the regular Ohm's law is this:

$B_i << \kappa$ $\kappa = 1/\mu$ $\mu = (q\tau/m)$ $\kappa = (m/q\tau)$ (N.8.4)

so we need then

$B_i << (m/q\tau)$. (N.8.5)

For copper, we compute $\kappa = m/q\tau$ using numbers from Section N.1,

```
me := 9.109e-31* kg;
```

$$me := .9109\ 10^{-30}\ kg$$

```
tau := 1e-14*sec;
```

$$\tau := .1\ 10^{-13}\ sec$$

```
kg := cou*volt*sec^2*m^2;
```

$$kg := cou\ volt\ sec^2\ m^2$$

```
kappa := me/(q*tau);
```

$$\kappa := -569.3125000\ volt\ sec\ m^2$$

Our conclusion is that "regular Ohm's Law" is applicable as long as $B_i << 569$ Tesla. Even the largest practical B fields are far below this number. From wiki:

Examples [edit]

Main article: Orders of magnitude (magnetic field)

- 31.869 μT (3.1×10^{-5} T) – strength of Earth's magnetic field at 0° latitude, 0° longitude
- 5 mT – the strength of a typical refrigerator magnet
- 0.3 T – the strength of solar sunspots
- 1.25 T – magnetic field intensity at the surface of a neodymium magnet
- 1 T to 2.4 T – coil gap of a typical loudspeaker magnet
- 1.5 T to 3 T – strength of medical magnetic resonance imaging systems in practice, experimentally up to 17 T[9]
- 4 T – strength of the superconducting magnet built around the CMS detector at CERN[10]
- 8 T – the strength of LHC magnets.
- 11.75 T – the strength of INUMAC magnets, largest MRI scanner.[11]
- 13 T – strength of the superconducting ITER magnet system [12]
- 16 T – magnetic field strength required to levitate a frog,[13] according to the 2000 Ig Nobel Prize in Physics.[14]

So even the Large Hadron Collider designers and frog levitators can use regular Ohm's Law (along with the writer of Section 1 and Appendix D of this document).

So why do we need to use the Magnetic Ohm's Law when dealing with the Hall Effect which has a relatively small B field? Recall (N.4.10),

$$J_x = \sigma\,(E_x + \omega_c\tau\, E_y)\,\frac{1}{1+(\omega_c\tau)^2} \qquad \omega_c \equiv (qB/m)$$

$$J_y = \sigma\,(E_y - \omega_c\tau\, E_x)\,\frac{1}{1+(\omega_c\tau)^2} \qquad \mathbf{B} = B\hat{\mathbf{z}}$$

$$J_z = \sigma E_z \ . \qquad \sigma = (nq^2\tau/m) \qquad (N.4.10)$$

In the Hall experiment of Fig N.1 we must have $J_y = 0$ and that means we cannot ignore the second term in the J_y expression above, even though it is much smaller than the first term. We get $E_y = \omega_c\tau\, E_x$ as in (N.5.3) which is the tiny delicate Hall field.

One can repeat the above analysis to get a tensor Magnetic Ohm's Law for a monochromatic AC situation by replacing $\kappa \rightarrow \kappa\,[\,1 + j\omega\tau\,]$ in (N.8.1), based on (N.1.5). As shown at the end of Section N.1, for copper we have $\omega\tau << 1$ for $f << 16{,}000$ GHz, so this κ replacement has a miniscule effect and our conclusions above still apply.