

The Hall Effect

1 Background

In this experiment, the Hall Effect will be used to study some of the physics of charge transport in metal and semiconductor samples.

In 1879 E. H. Hall observed that when an electrical current passes through a sample placed in a magnetic field, a potential proportional to the current and to the magnetic field is developed across the material in a direction perpendicular to both the current and to the magnetic field [1]. This effect is known as the Hall effect, and is the basis of many practical applications and devices such as magnetic field measurements, and position and motion detectors.

With the measurements he made, Hall was able to determine for the first time the sign of charge carriers in a conductor. Even today, Hall effect measurements continue to be a useful technique for characterizing the electrical transport properties of metals and semiconductors. Indeed, the failure of the simple model of metallic conductivity, which we discuss below, to account for many experimental measurements of the Hall effect has been one of the principal motivators leading to a better understanding of electronic properties of materials [5, pp. 58-62].

1.1 The simple theory of the Hall effect

Consider a conducting slab as shown in Fig. 1 with length L in the x direction, width W in the y direction and thickness T in the z direction.

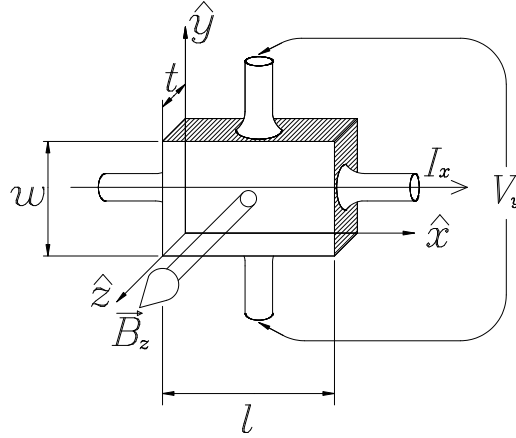


Figure 1: Geometry of fields and sample in Hall effect experiment.

Assume the conductor to have charge carrier of charge q (can be either positive or negative or both, but we take it to be of just one sign here), charge carrier number density n (i.e., number of carriers per unit volume), and charge carrier drift velocity v_x when a current I_x flows in the positive x direction. The drift velocity is an average velocity of the charge carriers over the volume of the conductor; each charge carrier may move in a seemingly random way within the conductor, but under the influence of applied fields there will be a net transport of carriers along the length of the conductor. The current I_x is the current density J_x times the cross-sectional area of the conductor

WT . The current density J_x is the charge density nq times the drift velocity v_x . In other words

$$I_x = J_x WT = nqv_x WT . \quad (1)$$

The current I_x is caused by the application of an electric field along the length of the conductor E_x . In the case where the current is directly proportional to the field, we say that the material obeys Ohm's law, which may be written

$$J_x = \sigma E_x , \quad (2)$$

where σ is the conductivity of the material in the conductor.

Now assume that the conductor is placed in a magnetic field perpendicular to the plane of the slab. The charge carriers will experience a Lorentz force $q\vec{v} \times \vec{B}$ that will deflect them toward one side of the slab. The result of this deflection is to cause an accumulation of charges along one side of the slab which creates a transverse electric field E_y that counteracts the force of the magnetic field. (Recall that the force of an electric field on a charge q is $q\vec{E}$.)

When steady state is reached, there will be no net flow of charge in the y direction, since the electrical and magnetic forces on the charge carriers in that direction must be balanced. Assuming these conditions, it is easy to show that

$$\underline{E_y = v_x B_z} , \quad (3)$$

where E_y is the electric field, called the Hall field, in the y direction and B_z the magnetic field in the z direction.

In an experiment, we measure the potential difference across the sample—the Hall voltage V_H —which is related to the Hall field by

$$V_H = - \int_0^W E_y dy = -E_y W . \quad (4)$$

Thus, from equations (1), (3) and (4) we obtain

$$V_H = - \left(\frac{1}{nq} \right) \frac{I_x B_z}{T} . \quad (5)$$

The term in parenthesis is known as the Hall coefficient:

$$R_H = \frac{1}{nq} . \quad (6)$$

It is positive if the charge carriers are positive, and negative if the charge carriers are negative. In practice, the polarity of V_H determines the sign of the charge carriers. Note that the SI units of the Hall coefficient are $[\text{m}^3/\text{C}]$ or more commonly stated $[\text{m}^3/\text{A}\cdot\text{s}]$.

Exercise 1 *Work through the math to derive Eq. (5). Now consider that an electric current in the positive x direction can be created by positive charges moving positive along the x axis or negative charges moving negative along the x axis. Draw diagrams showing the electric and magnetic forces on the charges to convince yourself that, in spite of this fact, positive charge carriers will produce a positive Hall coefficient and negative charge carriers will produce a negative Hall coefficient.*

1.2 The Hall effect in metals and semiconductors

In order to understand some of the ideas involved in theory of the Hall effect in real materials, it is instructive to construct a more careful model for electric currents under electric and magnetic fields from a classical point of view. We imagine that the charge carriers move in a medium that offers some resistance. The resistance is due to scattering between the carriers and impurities in the material and between the carriers and vibrations of the material's atoms. Each charge carrier is accelerated by the applied fields but every so often it scatters and loses energy. If we assume that the average time between scattering events is τ , then we have, on average, a retarding force acting on the carriers of

$$\vec{F}_{\text{retard}} = -\frac{m\vec{v}}{\tau}, \quad (7)$$

where m is the mass of the carrier. So under the influence of applied electric and magnetic fields, Newton's second law reads

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) - \frac{m\vec{v}}{\tau}, \quad (8)$$

where the velocity $\vec{v}$ is taken to be an average over all of the carriers.

At steady state, the time derivative of $\vec{v}$ will vanish. Under the usual convention that $\vec{B}$ points along the z axis, we obtain the component equations for $\vec{v}$ by setting the left hand side of Eq. (8) to zero and rearranging:

$$\begin{aligned} v_x &= \frac{q\tau}{m}E_x + \frac{q\tau}{m}B_z v_y, \\ v_y &= \frac{q\tau}{m}E_y - \frac{q\tau}{m}B_z v_x, \\ v_z &= \frac{q\tau}{m}E_z. \end{aligned} \quad (9)$$

From Eq. (1) we have that $J_x = nqv_x$ (and correspondingly for y and z components). By solving the above equations for v_x , v_y and v_z in terms of the components of $\vec{E}$ and B_z we get

$$\begin{aligned} J_x &= \frac{\sigma}{1 + (\omega_c\tau)^2} (E_x + \omega_c\tau E_y), \\ J_y &= \frac{\sigma}{1 + (\omega_c\tau)^2} (E_y - \omega_c\tau E_x), \\ J_z &= \sigma E_z, \end{aligned} \quad (10)$$

where

$$\sigma \equiv \frac{nq^2\tau}{m}, \quad (11)$$

and

$$\omega_c \equiv \frac{qB_z}{m}. \quad (12)$$

In the expression for the conductivity σ it may seem that all we have done is to replace one unknown quantity $\vec{v}$ with another unknown quantity τ . But the parameter τ , called the relaxation time, is widely used in discussions of electronic transport in materials, and can be justified in a quantum-mechanical context via the Boltzmann transport equation [8, Ch. 7]

all OK

The angular frequency ω_c is known as the “cyclotron frequency”. It is the frequency of rotation of a charge in a magnetic field, and can be taken as a measure of the strength of the field. The combination $\omega_c\tau$ is used to characterize an experimental situation: if the magnetic field is weak and/or the relaxation time short, $\omega_c\tau \ll 1$ and our experiment is in the “weak-field limit”; alternately if $\omega_c\tau \gg 1$ the experiment is in the “strong-field limit”. A number of materials show strikingly different behavior between the weak- and strong-field limits; aluminum is one.

In our classical model of the Hall effect with a single type of charge carrier, however, there is no such crossover between the weak and strong field. This can be proved by working out the following exercises.

Exercise 2 Show that a classical charged particle of charge q , mass m and speed v would execute a circular orbit of angular frequency ω_c if it moves under the influence of a magnetic field $\vec{B}$. Assume that no other forces act on the particle.

Exercise 3 Solve equations 10 under the conditions that $J_y = J_z = 0$ to show two results: $J_x = \sigma E_x$ and $J_x = \sigma E_y/(\omega_c\tau)$. Define the hall coefficient as $R_H \equiv E_y/(BJ_x)$ and show that $R_H = 1/(nq)$. everything here is correct

Note that this model predicts two things: the Hall coefficient is independent of the magnetic field strength and that there is no dependence of the sample resistance on the magnetic field either. This second effect is called “magnetoresistance”, and it was this effect that Hall originally tried (and failed) to find [1, 5].

The classical theory of the Hall effect presented above assumes that the electric current is the result of many charge carriers moving independently of each other and responding to applied fields as classical particles. But we know that electrons are quantum particles, specifically fermions, and they have wavelike properties. Curiously, the act of changing the model from *classical* independent particles moving freely to *quantum* independent particles moving freely changes little in the results so far presented. The “free-electron quantum gas” model still predicts a hall coefficient of $1/nq$ and zero magnetoresistance [5].

The benefit of using a quantum approach becomes apparent when it is coupled with a more realistic model of solid matter, specifically, crystalline. In a crystal, the atoms are arranged in a periodic lattice. Electrons in the lattice feel the effect of a periodic potential on their motion. The strongest effect occurs for those electrons in the outer atomic orbitals—the “valence” electrons, and especially those valence electrons whose deBroglie wavelength is close to the spacing of the potential’s periodicity.

Within the periodic potential the allowed energies of the valence electrons are broken into into a series of energy bands with energy gaps between them. If the number of valence electrons per unit cell of the crystal is exactly enough to fill a band, the solid will be a poor conductor, since by symmetry at each energy there will be filled momentum states pointing in opposite directions. Conduction can occur only if an electron can jump a gap into an unoccupied state. If the gaps are large, the necessary energy may be too high, and the solid is an insulator. If the gap is small, then thermal energy may be enough to cause sufficient electrons to jump the gap, and the solid is called a semiconductor. On the other hand, if the number of valence electrons per unit cell is not enough to fill a band, then many unfilled momentum states lie within easy energy reach, and the solid is a good conductor—a metal.

However, the shape of the energy surfaces—the *band structure*—has a strong effect on the type of conduction that can occur. Electron states with energies near the “bottom” of a band—the lowest allowed energy in the band—behave like free electrons, except that their response to an applied field may be that of a heavier or lighter particle. It as if the electron mass m has been changed to an effective mass m^* . While this statement seems strange, stranger still are the responses of electrons with energies near the “top” of a band; these electrons act as if their mass is *negative*! In other words, they accelerate *oppositely* a free electron when acted upon by an applied field. Indeed, it is easier to imagine particle states of this type as acting like *positive* charges. Such states are called “hole” states.

Another way to think of these effects is that the electron states with wavenumbers lying near the band extremes are *diffracted* by the periodic lattice in the same way that light is diffracted by a grating: the motion of an electron wave is greatly altered, even reversed, analogous to the way that a grating can reflect light of a specific color at specific angles.

In a semiconductor the band gap is relatively small, and electrons may be excited by thermal energy to jump the gap. This process allows an electron-like state near the bottom of an upper band and a hole-like state near the top of a lower band to come into existence. Both states carry current, with the hole state acting as a positive charge. The number of these current-carrying states depends on the temperature in a roughly exponential way: the number is proportional to the Boltzmann factor $e^{-E_g/kT}$ where E_g is the energy of the gap.

The idea of simultaneous electron+hole states has yielded a useful model of the Hall effect called the “two-band model”. The mathematics works through just like in Exercise 3, except that the total current is the sum of contributions from the holes and the electrons. If we let each type of carrier have a Hall coefficient: R_e (for electrons) and R_h (for holes), as given by the Eq. (6) and we assume the conductivity $\sigma = \sigma_e + \sigma_h$, where σ_e and σ_h are given by Eq. (11), then we can derive an expression for the total Hall coefficient as

$$R_H = \frac{\sigma_h^2 R_h + \sigma_e^2 R_e}{(\sigma_h + \sigma_e)^2} . \quad (13)$$

In the above, the effective mass m^* is substituted for m and the charge q taken as positive for holes and negative for electrons. This equation follows from Eqs. (10) in the limit that $\omega_c \tau \ll 1$: the low-field limit.

In the case of semiconductors, it has become customary to separate out the carrier density n from the overall conductivity formula for σ , and define a new quantity called the mobility μ :

$$\mu = \left| \frac{q\tau}{m^*} \right| . \quad (14)$$

Thus, in our two-band model, the conductivity would look like

$$\sigma = \mu_e |q_e| n_e + \mu_h |q_h| n_h , \quad (15)$$

and the low-field Hall coefficient would look like

$$R_H = \frac{1}{|q|} \frac{n_h \mu_h^2 - n_e \mu_e^2}{(n_h \mu_h + n_e \mu_e)^2} , \quad (16)$$

where we have assumed that the charge is the same magnitude for both types of carriers.

Exercise 4 Show that Eq. (16) reduces to Eq. (6) in the case of one type of carrier. Then show that for this case, $\mu = |v_x|/E_x$. Thus, the mobility can be found from a Hall effect experiment: show that $\mu = \sigma R_{\text{Hall}}$.

For metals, the exact value of the charge carrier density and the sign of the Hall coefficient depend on the energy band structure of the particular metal. For alkali metals (Li, Na, K, etc.) and some of the transition metals (Cu, Ag, Au), the charge carriers are electrons (negative) and the charge carrier density is approximately one electron per atom. Thus, we can often use the simple result, Eq. (6) for this class of materials as an approximate result.

For semiconductors, the band structure can give rise to both negative (electron-like) or positive (“holes”, or absence of electrons) effective charge carriers, and the electrical conductivity is determined by the density and mobility of both kinds. We need to often use the more complex form of R_H , Eq. (16) for these materials. In any case, at room temperatures, the charge carrier density of semiconductors is much smaller than that of metals, and thus the magnitude of V_{Hall} is much larger for a given I_x , B_z and film thickness, t . This difference will become very clear when you take measurements on the samples in the lab.

Exercise 5 In a semiconductor the carrier density is strongly dependent on the temperature, mostly proportional to the Boltzmann factor of the energy gap between the valence and conduction band, as noted above. How would the Hall voltage, for a fixed I_x , depend on temperature for semiconductors? Discuss this question qualitatively and quantitatively.

In recent times, the study of the Hall effect in thin films at low temperatures and high magnetic fields has demonstrated the existence of the quantum Hall effect. This effect is most apparent in very small structures and at very cold temperatures. Under these conditions one can distinguish the quantization of the energy levels, as the small structures force the energy levels to split and the cold temperatures allow the low-lying levels to become filled. In this case, the Hall resistance (defined as V_{Hall}/I_x) does not increase linearly with B_z , but exhibits steps as B_z increases. The magnitude of the Hall resistance at the steps is $h/(e^2i)$, where i is an integer. The Hall conductance, the reciprocal of the Hall resistance, thus increases in integer steps, and the quantity e^2/h now defines the basic unit of conductance. In 1985 Klaus von Klitzing was awarded the Nobel prize for the discovery of the integer quantum Hall effect.

More recently, a fractional form of the quantum Hall effect has been discovered, where the conductance steps are in rational fractions of the basic e^2/h unit. It is generally believed that this peculiar effect is due to interactions among the charge carriers, requiring a quantum theory going beyond the effects of confinement.

2 Procedure

There are three different samples for which you will determine the sign and density of the charge carriers. Some specifics for the samples are given in Table 1.

A schematic for the circuit is shown in Fig. 2.

Mount the probe into the clamp attached to the stand so that it can slip between the magnet pole pieces. Make sure that the probe is well aligned between the poles: it should be perfectly vertical with the plane of the paddle perpendicular to the table and centered both vertically and horizontally between the poles.

very thin to enhance the Hall voltage

Material	Thickness t (m)	Width w (mm)	Length ℓ (mm)	Resistance (Ω)
Au	$1.37 \pm 0.16 \times 10^{-7}$	12.7 ± 0.05	30.5 ± 0.05	0.770 ± 0.006
Al	$2.34 \pm 0.17 \times 10^{-7}$	12.7 ± 0.05	30.5 ± 0.05	0.872 ± 0.018
InAs	$1.26 \pm 0.02 \times 10^{-4}$	0.692 ± 0.004	1.542 ± 0.005	1.3 ± 0.3

Table 1: Characteristics of the Hall effect probes. The thicknesses of the metal samples were determined optically by comparing fringes created by thin-film interference. (See *Optics, 2nd edition*, by E. Hecht, pp. 381,382 for an explanation of this method.) The length and width were measured with calipers. The dimensions of the of the InAs probe were determined by the use of a measuring microscope. The resistance of the probes was measured along the length ℓ of the sample by a the four-point-probe method using a low frequency AC excitation and lock-in amplifiers.

Connect the probe wires according to color following the diagram. Note that the voltage measurement wires are attached to a dual banana-plug connector that can fit into the DMM sockets. Note that the meter which measures current should use the *white* and *black* connection points, while the meter that measures voltage should use the *red* and *black* connection points. *Do NOT use the connection points marked SENSE Ω 4 WIRE.*

Turn on the meters and the power supply. Set the voltage measurement meter to measure DC volts by pressing the “DCV” button, and the current measurement meter to measure DC amps by pressing the “DCI” button.

Several precautions must be followed:

- **Do not exceed 200 mA current** through either of the two metal samples (Au and Al).
- **Do not exceed 100 mA current** through the InAs sample.
- Turn down the current and voltage settings on the HP 6181B to ZERO before connecting or disconnecting any of the Hall samples. Because the current source attempts to maintain the same current regardless of load resistance, a sudden increase in the resistance (say, by disconnecting the wires) can cause a spike in the output voltage.
- The HP6181B should be operated in the constant current mode: be sure that the current is being limited by the current set knob and not the voltage set knob. You can tell if the current is being limited by the voltage knob if the light near that knob is on. Test this by turning the voltage knob all the way down and then all the way up. You should see the light turn off when the voltage is high enough to supply the desired current.

To this point, V_y has been equated with V_{Hall} . In practice this does not hold true because of asymmetries in the probe element and voltage measurement contacts. Thus, even for $B_z = 0$, when a current I_x flows a voltage V_y will be measured due to the voltage drop along the length of the sample or differences in the resistivity of various parts of the sensing element.

Check this out with the sample outside the magnet: Turn the current up and note that you see a change in V_y even though there is a near-zero magnetic field (relative to the strong one between the magnet poles).

One way to eliminate the effect of this offset voltage is to measure the V_y for one direction of B_z , remove the sample from the magnet, flip the sample over 180° , put the sample back in the magnet and measure V_y again. The difference in the two readings will be twice V_{Hall} .

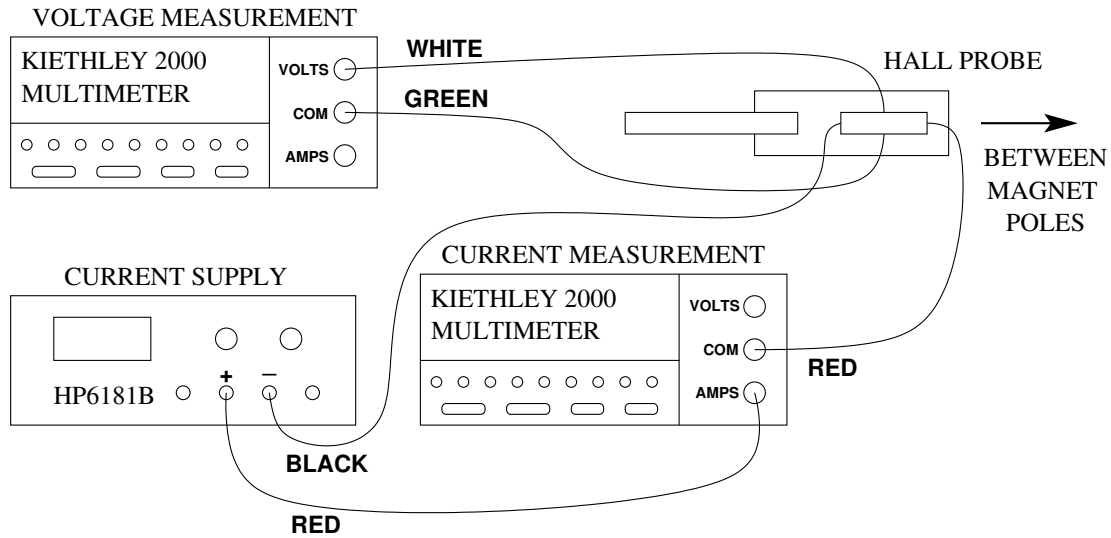


Figure 2: Schematic of Hall Effect experiment. The blade of the probe should be slipped between the magnet pole pieces so that the sensing element is centered.

When using this technique it is important that the current through the probe remain *exactly the same* for the two orientations of the probe. Your measurements should follow this procedure:

1. Set the current to the desired level.
2. Wait until the voltage reading settles to within the desired precision (3–4 significant figures is usually adequate).
3. Record the voltage, making sure to note the orientation of the probe.
4. Pull the probe stand back, loosen the clamp, rotate the probe by 180° and tighten the clamp.
5. slide the probe stand back into position.
6. Check that the current has not changed significantly (i.e., less than 1 mA).
7. After the voltage reading settles, record the voltage.
8. Go back to step 1 with a new current setting.

A big piece of useful advice: When taking data do *not* attempt to figure out all of the angles and orientations you will need to determine the sign of the charge carriers as you go. It is much more reliable to record the data by means of unambiguous visual markers, such as “green wire up”, “green wire down”, and then to draw a clear diagram of the probe layout with all wires labeled according to color, along with the probe’s orientation to the magnet. Don’t forget to pay attention to the *sign* of the voltage that you measure as well as the connections to all of the electronics. With this information you can reconstruct the relationships among the current direction, voltage sign, and magnetic field direction when you can concentrate on the data analysis.

Because you need to flip the probe orientation at each setting of the current, it is helpful to use the wood pieces on the table and the metal locking rings on the stand and probe handle to setup the probe position. Then it becomes a simple matter to move the stand and flip the probe.

In doing this experiment, you will measure I_x , V_y (for two probe orientations) and B_z for the three different probes. You should take readings of V_y for 6 to 8 different values of I_x for each sample.

For the two metal probes the Hall voltage is very small—in the microvolt range—so you will need to set the voltmeter on its most sensitive scale. Use the triangular RANGE buttons for this. It is also helpful to use the “filtering” capabilities of the meters to average out noise fluctuations in the measurements. Turn this on by pressing the FILTER button (note the “FLT” indicator on the panel below the reading), and then choosing to average many readings together (a “running average” with 50 readings works well).

For the InAs probe, the Hall voltage is much greater. This is typical of semiconductor probes, and is the reason for the use of semiconductors in commercial Hall-effect devices. But the temperature sensitivity is much greater too. It will take a bit longer for the InAs voltage to settle, since the probe element is heated by the applied current.

The magnetic field B_z remains fixed during the experiment. It can be varied by adjusting the shunt rings around the pole pieces: turning the rings toward the pole faces reduces the field strength. They are normally set at the farthest point back in order to maximize the field. Curves of field strength versus pole-face type, gap and ring setting are available in the lab. The field is measured with the Sypris (Bell) gauss meter. This meter is itself a Hall effect device, and the transducer is located at the tip of the blade. Before measuring the magnetic field, the gauss meter should be zeroed by placing the end of the probe inside the zero-field chamber and adjusting the reading to zero by pushing the zeroing button on the Sypris (and waiting until it is done). Explore the magnetic field variation over the region between the two pole faces. Use both orientations of the probe to measure the field strength. Calculate a mean value and standard deviation from your measurements. Explain in your report your method for finding the mean field and its uncertainty.

3 Analysis

From your measurements on the three samples calculate $2V_H$ by subtracting the voltage measurements for the two different probe orientations. *Be careful about the sign of the voltage!* Then plot $2V_H$ versus I_x , and fit the line with a computer to obtain the slope and its uncertainty. how?

Use the result, along with the probe data given in Table 1 to determine the magnitude and sign of the Hall coefficient for each of the samples, along with the experimental error for these quantities.

From the stated values of the dimensions and resistances of the InAs sample, it is possible to determine the sample conductivity σ :

$$\sigma = \frac{\ell}{Rwt} , \quad (17)$$

where R is the resistance of the sample. This formula assumes that the resistivity of the material is uniform over the sample and that the dimensions of the sample accurately reflect its geometry.

From the conductivity σ and the Hall coefficient R_{Hall} it is possible to compute the “Hall mobility” μ_H of the charge carriers, defined as

$$\mu_H \equiv \frac{|v_x|}{|E_x|} . \quad (18)$$

Use your values of R_{Hall} and σ to compute μ_H (and the error) for the InAs sample. Then look up mobilities for InAs in texts or handbooks and compare your value with theirs.

Some questions to research, ponder and discuss in your report:

1. Find some values for mobility in tables that are listed in texts, such as Kittel's or other references, such as the Landolt-Börnstein series of books available in the library. Compare your results with the tabulated values. How close are your results to the ones cited for InAs? Can you explain any differences?
2. In the case of the aluminum sample, you will find in such texts as Kittel's "Introduction to Solid State Physics" that the assumed charge carriers are "1-hole per atom". Why should we assume that the charge carrier in aluminum is a hole? This is not predicted by the simple "Drude" theory of electrons in metals.
3. Your results for aluminum are likely to be different from the predicted or tabulated values, which are usually quoted as being measured at low temperature and high magnetic field. See what you can find out about the Hall coefficient for a range of temperatures and field strengths, and discuss how your measurement fits in with these.

References

- [1] Hall, E. H., "On a new action of the magnet on electric currents", American Journal of Mathematics, **2**, No. 3, pages 287–292 (1879). The original paper by Hall which describes the effect. An interesting historical read.
- [2] Preston, D. W. and E. R. Dietz, The Art of Experimental Physics, John Wiley & Sons (1991), pages 303–315. Excellent discussion of the Hall effect in semiconductors.
- [3] Melissinos, A. C., Experiments in Modern Physics, Academic Press (1966), pages 85–87.
- [4] Kittel, C., *Introduction to Solid State Physics, 6th edition*, John Wiley & Sons (1986), pages 147–151. Standard undergraduate text. Contains useful tables, but does not give a detailed account of the Hall effect for metals.
- [5] Ashcroft, Neil W., and N. David Mermin, *Solid State Physics*, Saunders College, Philadelphia (1976). Chapter 1 gives the elementary theory of the Hall Effect. Chapter 3 discusses the failures of the free-electron model. Chapters 12 and 15 treat the Hall effect in the high field low temperature limit for metals.
- [6] Hurd, Colin M., The Hall Effect in Metals and Alloys, Plenum Press, New York (1972). A thorough overview of the Hall effect in metals. Predates the quantum Hall effect.
- [7] Silsbee, Robert H., and Jörg Dräger, Simulations for Solid State Physics, Cambridge University Press (1997). The book and computer simulations illustrate many important concepts in solid state physics. The simulations are available on a computer in the lab.
- [8] Ziman, J. M., Principles of the Theory of Solids, Cambridge University Press (1969). An excellent, readable and clear text covering much of the same material as Ashcroft and Mermin.

Prepared by D. B. Pengra, J. Stoltenberg, R. Van Dyck and O. Vilches
hall_effect_10-07.tex -- Updated 4 October 2007