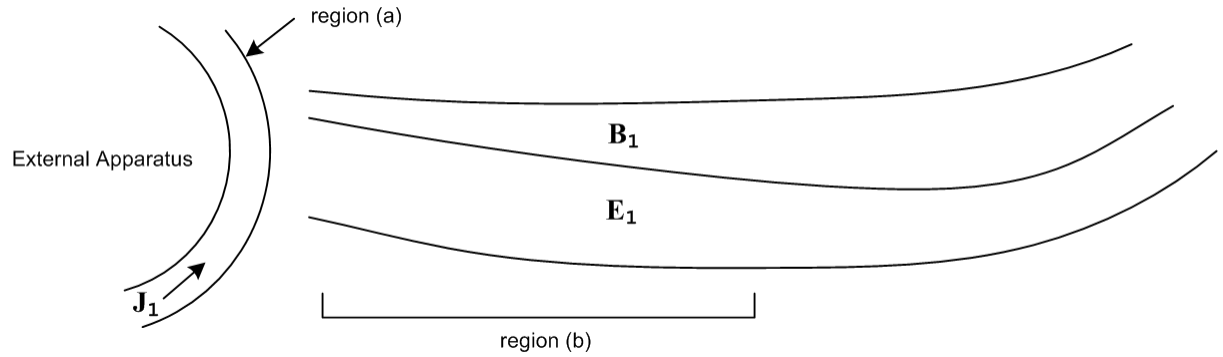
Try Again PhL 4.15.14

[I wrote this before Cape Cod, I tried to make it fly in temp3.doc, but I could not get happy with it and decided to stick with my existing Section P.1 which opens Appendix P.]

Consider this figure, where the free hand lines generically represent E and B field lines,



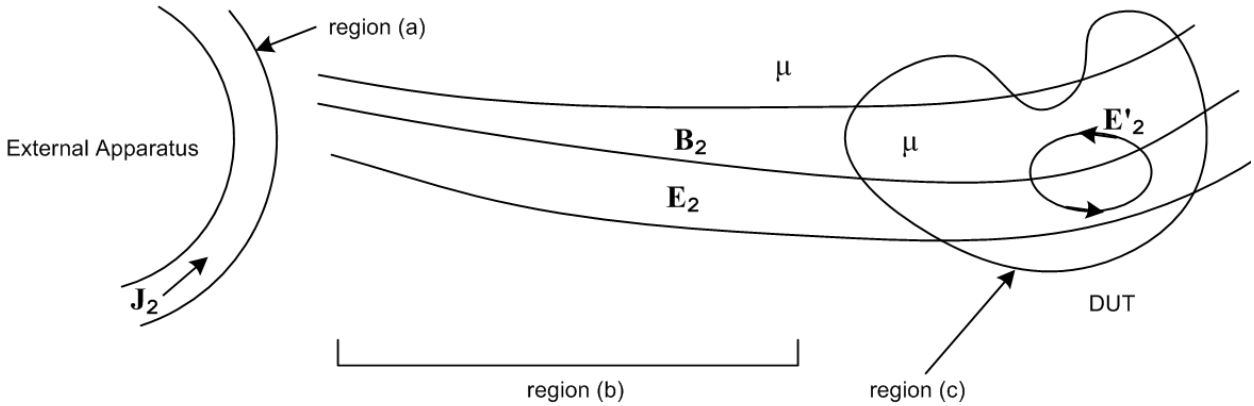
An external apparatus has current density $\mathbf{J}_1$ flowing in some wires and creates magnetic field $\mathbf{B}_1$ and there is some associated electric field $\mathbf{E}_1$ created as well. The Maxwell curl equations for the above figure are:

curl $\mathbf{E}_1 = -j\omega\mathbf{B}_1$
curl $\mathbf{B}_1 = \mu\mathbf{J}_1$ // region (a)

curl $\mathbf{E}_1 = -j\omega\mathbf{B}_1$
curl $\mathbf{B}_1 = j\omega\mu\varepsilon\mathbf{E}_1$ // region (b)

In region (a) we ignore the displacement current inside the wires. In region (b) $\mathbf{J}_1 = 0$ and there is then some displacement current $j\omega\varepsilon\mathbf{E}_1$.

We now bring in a Device Under Test (DUT) to obtain a new picture:



The Maxwell curl equations are now:

$$\text{curl } \mathbf{E}_2 = -j\omega \mathbf{B}_2$$
$$\text{curl } \mathbf{B}_2 = \mu \mathbf{J}_2 \qquad \text{// region (a)}$$

$$\text{curl } \mathbf{E}_2 = -j\omega \mathbf{B}_2$$
$$\text{curl } \mathbf{B}_2 = j\omega\mu\varepsilon \mathbf{E}_2 \qquad \text{// region (b)}$$

$$\text{curl } \mathbf{E}'_2 = -j\omega \mathbf{B}_2$$
$$\text{curl } \mathbf{B}_2 = \mu \mathbf{J}_2' \qquad \text{// region (c)}$$

In regions (a) and (c) we ignore the displacement currents (the DUT is a good conductor). The quantity $\mathbf{J}'_2$ is the eddy current in the DUT, and $\mathbf{E}'_2$ is its associated electric field.

For arbitrarily large ω, the solution of the problem of the fields inside the DUT is complicated and one must solve a vector Helmholtz equation. Setting $\mathbf{J}'_2 = \sigma\, \mathbf{E}'_2$ we can write for region (c)

$$\text{curl curl } \mathbf{E}'_2 = -j\omega \text{ curl } \mathbf{B}_2 = -\, j\omega\mu(\sigma \mathbf{E}'_2)$$

But we know that

$$\text{curl curl } \mathbf{E}'_2 = \text{grad div } \mathbf{E}'_2 - \nabla^2 \mathbf{E}'_2 = -\nabla^2 \mathbf{E}'_2 \qquad \text{since div } \mathbf{E}'_2 = 0 \text{ inside the DUT } (\rho=0).$$

Thus we must solve

$$-\nabla^2 \mathbf{E}'_2 = -\, j\omega\mu\sigma \mathbf{E}'_2$$

or

$$(\nabla^2 + \beta^2)\mathbf{E}'_2 = 0 \qquad \text{where } \beta^2 = -\, j\omega\mu\sigma$$

and this is the equation encountered in *****.

If ω is small, we can take a different approach to solving the region (c) problem. We first define $\mathbf{B}_{eddy}$ as the difference between $\mathbf{B}_1$ of the first drawing and $\mathbf{B}_2$ of the second drawing, and we rename $\mathbf{J}'_2$,

$$\mathbf{B}_{eddy} \equiv \mathbf{B}_2 - \mathbf{B}_1$$

$$\mathbf{J}_{eddy} \equiv \mathbf{J}'_2$$

The region (c) curl equations are then

$$\text{curl } \mathbf{J}_{eddy} = -j\omega\sigma(\mathbf{B}_1 + \mathbf{B}_{eddy})$$

$$\text{curl } (\mathbf{B}_1 + \mathbf{B}_{eddy}) = \mu \mathbf{J}_{eddy} \qquad \text{// region (c)}$$

If ω is small, the first equation implies that $\mathbf{J}_{\mathbf{eddy}}$ is small. If this is the case, we expect that $\mathbf{B}_{\mathbf{eddy}}$ will also be small. In this case, we expect that the eddy currents don't alter $\mathbf{J}_1$ very much when the DUT is added to Fig 1 to get Fig 2, so we then set $\mathbf{J}_2 = \mathbf{J}_1$ in the second drawing. If $\mathbf{B}_{\mathbf{eddy}}$ is small, we then have roughly $\mathbf{B}_2 = \mathbf{B}_1 + \mathbf{B}_{\mathbf{eddy}} \approx \mathbf{B}_1$. The above equations are then

$$\text{curl } \mathbf{J}_{\mathbf{eddy}} \approx -j\omega\sigma\mathbf{B}_1$$

$$\text{curl } \mathbf{B}_1 \approx \mu\mathbf{J}_{\mathbf{eddy}} \qquad \text{// region (c)}$$

It is the first of these two equations that is our main interest. The second equation says that $\mathbf{B}_1$ has a small curl inside the DUT due to the eddy current there. One should not interpret this second equation as saying that the small eddy current somehow *generates* the large field $\mathbf{B}_1$. After all, in region (b) curl $\mathbf{B}_1 = 0$ and we have a large $\mathbf{B}_1$ there.

CONTINUE HERE AFTER CAPE COD!

Maybe compare the above threading to earlier attempts. Each attempt seems to fail after I let it cool off for a day, so we need to get this thing stabilized. Then it can be installed at the start of Appendix P.