

Theory and Numerical Modelling of Lorentz Force Eddy Current Testing

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To my loving wife Nevena and daughter Viktoria ...

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Zusammenfassung

Diese Arbeit beschreibt die theoretische und numerische Untersuchung des neuen zerstörungsfreien Materialprüfverfahrens namens Lorentzkraft-Wirbelstromprüfung (LET). LET basiert auf der Messung der Lorentzkraft (LK), die auf ein Magnetsystem wirkt, welches sich in der Nähe eines leitfähigen Probekörpers bewegt. Um die zugrundeliegenden physikalischen Prinzipien vorzustellen, wird zuerst ein kanonisches Modell untersucht, welches "kriechender Magnet" genannt wird. Durch die vereinfachte Konfiguration, ist sowohl eine tiefgründige analytische Beschreibung als auch eine effektive numerische Simulation möglich.

Um realistische LET-Konfigurationen zu untersuchen, wird eine neue numerische Methodik basierend auf der Finiten Elemente Methode (FEM) entwickelt. Der sogenannte Logischer-Ausdruck-Ansatz (LEA) erlaubt eine schnelle und genaue Modellierung transienter Wirbelstromprobleme mit beweglichen Teilen in einem statischen numerischen Netz. Die Ergebnisse werden mittels der bekannten "Sliding-Mesh"-Methode (SMT) verifiziert und mittels Experiment validiert. Die Studie zeigt die Genauigkeit des LEA für alle Werte der magnetischen Reynoldszahl bei Verringerung der Gesamtsimulationszeit um mehr als das 10-fache.

Die vorgeschlagene numerische Methodik wird daraufhin angewendet, um die Anwendbarkeit und das Einsatzpotential von LET zu ermitteln. Die Simulationen werden für zwei charakteristische LET-Konfigurationen durchgeführt, (i) einen defektfreien und (ii) einen Probekörper mit definierten Defekten. Im Falle eines defektfreien Probekörpers wird beobachtet, dass LET für die kontaktlose Messung der elektrischen Leitfähigkeit des Probekörpers genutzt werden kann. Das "Lorentzkraft-Sigmometrie" genannte Verfahren ist robust gegenüber Änderungen des Lift-Off-Abstandes und der Magnetstärke. Die Studie des Probekörpers mit Defekt resultiert in Richtlinien zur effektiven Kraftmessung. Der Einfluss bestimmter Magnetformen und -größen auf die Detektion und Rekonstruktion von Defekten wird ebenfalls untersucht.

Die vorliegende Arbeit unterstreicht die Notwendigkeit einer differentiellen LK-Messung. Dafür wird ein neuer und günstiger differentieller LK-Sensor entwickelt, welcher auf etablierter Spannungsmessung beruht. Die spezifische Modifikation der ursprünglichen LET-Konfiguration wird "Differentielle Lorentzkraft-Wirbelstromprüfung" genannt.

Abstract

This thesis aims at the theoretical and numerical investigation of the novel non-destructive testing technique called Lorentz force eddy current testing (LET). LET is based on measurements of the Lorentz force acting on a magnet system moving in a close vicinity of an electrically conducting specimen. To provide insights into the fundamental principles of LET a canonical model, referred to as the creeping magnet problem, is firstly considered. Due to its simplified configuration, this problem is amenable to rigorous analytic treatment and effective numerical simulation.

To investigate realistic LET configurations, a novel numerical methodology based on the finite element method (FEM) is developed. The so-called logical expression approach (LEA) allows fast and accurate modelling of transient eddy current problems with moving parts on a fixed computational grid. The results are verified and validated using the known sliding mesh technique (SMT) and experiments, respectively. The study shows the accuracy of LEA for any value of the magnetic Reynolds number and also demonstrates the reduction of the total simulation time by more than 10 times.

The proposed numerical methodology is later applied to test the feasibility and estimate the testing capabilities of LET. The simulations are performed for two characteristic LET testing configurations, namely (i) a conductor without defects, and (ii) a conductor with pre-defined material defects. In case of non-defective conductor, it is demonstrated that LET can be applied for contactless measurement of electrical conductivity of test specimens. The technique, termed Lorentz force sigmometry, is resistant to changes in lift-off distance and magnet strength. The study involving defective conductors provides guidelines regarding more effective force measurements. The influence of the magnet shape and size on defects detection and reconstruction is analysed as well.

The thesis at hand underlines the need for differential Lorentz force measurements. Therefore, a new and low-cost differential Lorentz force sensor, based on simple and well established voltage measurements, is designed. This specific modification of the initial LET configuration is termed differential Lorentz force eddy current testing.

Preface

The work outlined in this dissertation was carried out over a period from January 2010 to December 2012, in the department of Advanced Electromagnetics at the Ilmenau University of Technology, Germany. The project was financed by the Deutsche Forschungsgemeinschaft (DFG) within the framework of the Research Training Group (RTG) "Lorentz Force Velocimetry and Eddy Current Testing" (Graduiertenkolleg "Lorentz-Kraft").

Expert supervision of the work was performed by Dr. Hartmut Brauer, Dr. Marek Ziolkowski and Prof. Hannes Töpfer. All numerical results have been validated by experiments performed by Dipl.-Ing. Robert P. Uhlig. A detailed description of the used experimental setup is contained in the thesis of Mr. Robert P. Uhlig.

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Chapter 1

Introduction

The fundamental problem of motion of a solid electrical conductor in the presence of a magnetic field is highly seminal in field of theoretical physics. In fact it was this scientific thought experiment that had led Albert Einstein to develop the special theory of relativity in his famous 1905 *Annus Mirabilis* paper entitled *Zur Elektrodynamik bewegter Körper*. However, the actual foundations of Einstein's work were laid almost 70 years earlier in 1831 by Michael Faraday through his work on electromagnetic induction [5]. Faraday discovered that when an electrically conducting body crosses the static magnetic field lines, or when it is exposed to time changing magnetic field an electric current will flow. As noted by Faraday himself, this was only possible if there is a closed electrically conducting path through which the current can circulate (Fig. 1.1). Nowadays, these currents are widely known as *eddy currents*.

However, the real physical nature of eddy currents was discovered much later in by French physicist Jean Bernard Léon Foucault in 1855. In his famous experiment Foucault discovered that the force required for the rotation of a copper disc becomes greater when being placed between the poles of a permanent magnet (Fig. 1.2). In the same time, he also noticed that the disc is heated by the currents induced inside the metal [6]. Due to his discovery eddy currents are sometimes referred to as *Foucault currents*.

The tendency of eddy currents to oppose its source has been described mathematically by Heinrich Lenz in 1834, whereas the heat produced by the current flow has been described by James P. Joule in 1840 [7]. The braking effect of eddy currents was fully understood much later by Hendrik Lorentz in 1892 [8]. Due to his contribution, the electromagnetic forces caused by the current flow or forces which act on the point charges moving across the magnetic field lines are known as *Lorentz forces*.

Eddy currents, apart for being fundamentally interesting, also find versatile fields of applications. In fact their intrinsic effects, either desirable or undesirable, form

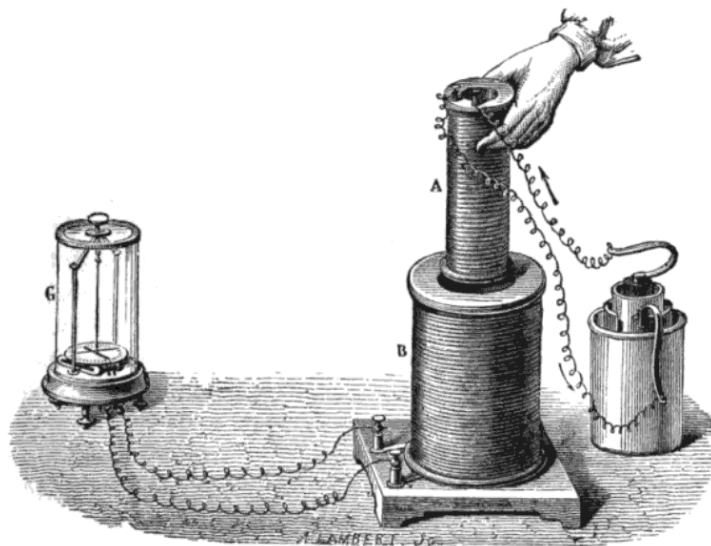


Figure 1.1: Illustration of Faraday's 1831 experiment demonstrating the principle of induction. The induced current is detected by the galvanometer. Courtesy of [1].

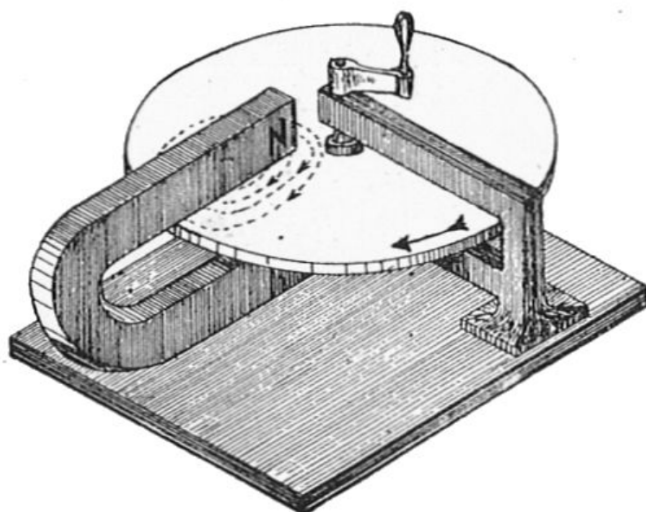


Figure 1.2: Illustration of Foucault's 1855 experiment with rotating copper disk demonstrating the effects of eddy currents. Courtesy of [2].

the operational basis of many electromechanical devices. A particularly interesting application of eddy currents lies in the field of non-destructive testing and evaluation (NDT&E). The so-called eddy current testing (ECT) involves the use of a time changing magnetic field produced by an alternating current (AC) coils to induce eddy currents inside the material under test. If the material contains a defect or any other anomaly which make the spatial distribution of the electrical conductivity non-uniform, the path of the eddy currents will be perturbed. In return, the resulting impedance of the induction coil which generates the AC magnetic field will be temporarily modified as well. Hence, by measuring the change in the impedance, the defect or anomaly can be detected.

This working principle of ECT was first demonstrated by David E. Hughes 1879 for comparing and material sorting on the basis of differences in electromagnetic parameters such as conductivity and permeability. Hughes not only showed that the principle of electromagnetic induction can be used as a basis for non-destructive evaluation of different material properties but also demonstrated the utility of differential measurements for increasing sensitivity. However, it took another 60 years for ECT to be used in industrial applications when in 1937 the German scientist Friedrich Förster applied this technique for metal sorting (Fig. 1.3) [3]. Förster used the basic principles of ECT to excite the test (P) and reference (V) specimens, whereas the so-called pickup coils were connected with a galvanometer (J) forming the secondary differential loop. In case of any imbalance caused by test specimen, e.g. difference in the electrical conductivity or due to presence of a defect, the needle of the galvanometer deflects the light to the photocell (L) which activates the sorting mechanism (S). During his work Förster also developed precise theories for many basic eddy current tests and pioneered the impedance plane display in the 1940 [9].

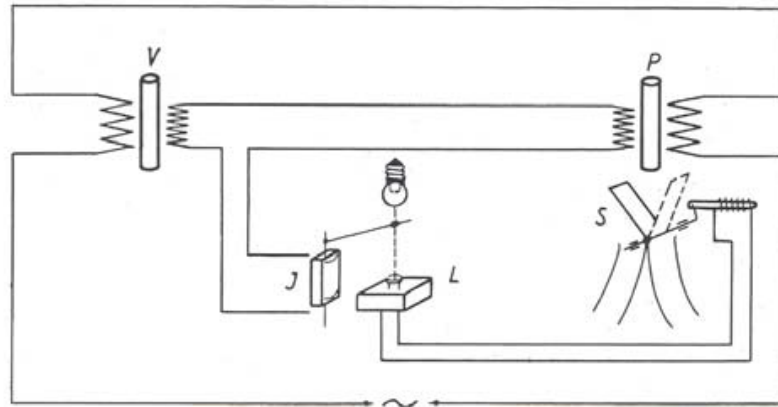


Figure 1.3: Schematic of Förster's earliest type eddy current testing device. (V) is the reference specimen, (P) test specimen, (J) galvanometer, (L) photocell and (S) sorting mechanism. The illustration has been retrieved from [3, 4].

1.1 Motivation

Since the work of Förster, there have been many advances in the field of ECT, which has led to its improved performance and new fields of application. Today ECT represents a particularly accurate and powerful NDT&E technique which is widely used for contactless defect detection as well as for measurement of the thickness, electrical conductivity and magnetic permeability of metallic materials [10, 11]. However, it is well known that ECT suffers from a fundamental limitation of low electromagnetic field penetration depth due to the skin effect [12, 13]. According to the definition, which assumes the AC magnetic fields and an electrically conducting half-space, the skin depth represents the depth at which the applied field is attenuated to $1/e \sim 37\%$ of its maximum value. It is a function of the applied testing frequency, electrical conductivity and magnetic permeability of the specimen under test [13]. The applicability of the traditional version of ECT is therefore limited to the analysis of the immediate vicinity of the surface of a material, usually of the order of a millimeter. Attempts to overcome this fundamental limitation using low frequency differential coils, giant magneto-resistance sensors (GMR) and superconducting quantum interference devices (SQUID) still have not found a widespread application in the field of NDT&E. Nevertheless, the detection of defects up to 38 mm in a multilayer aluminium stack conductors with excitation frequency of 35 Hz and testing velocity of 6 mm/s has been reported [14], which is a remarkable result. More details about the current state of the art in the field of ECT including other electromagnetic non-destructive testing (NDT) techniques can be found in [4, 15]. Another important drawback of ECT is a strong dependency in the so-called lift-off distance of the sensor and the surface conditions of the specimen [13].

All these issues are followed by a strong demand to develop new or investigate the possibility of applying hybrid NDT&E techniques which can overcome some of the limitations of already available techniques. In order to avoid or to reduce the intrinsic limitation of ECT in terms of the skin effect, an interesting method referred to as motion induced remote-field eddy current testing (MIRFEC) has been investigated in [16]. In principle, MIRFEC is a direct current (DC) version of ECT, where currents are induced due to relative motion between the testing specimen and the applied source of the static magnetic field, i.e. an electromagnet or a permanent magnet. The perturbation of induced current is measured by a pickup coil placed relatively far away from the source. Using this approach a high speed material inspection (up to 5 m/s) and detection of surface material defects has been reported in [17].

Based on MIRFEC principles, authors in [18, 19] presented the so-called drag force method for determination of hysteresis losses, fatigue and residual stresses in ferro-

magnetic electrically conducting materials. The extension of the drag force method for detection of defects by measuring the resulting Lorentz force perturbations has been first reported in [20], where the technique has been termed as Lorentz force eddy current testing (LET).

LET is based on the measurement of the Lorentz force acting on the magnet system moving relative to the conductor under test. In principle, LET differs from traditional eddy current testing in two respects, namely (i) in how the eddy currents are generated and (ii) in how their perturbation is detected. First, LET is based on generating eddy currents by setting a DC magnet system, usually a permanent magnet, into relative motion with respect to the material to be investigated. Under similar testing conditions, in terms of testing speed, this approach has the great virtue that the eddy currents penetrate the material to a much greater depth than with traditional ECT. Second, the perturbation of the eddy currents is detected through its influence upon the Lorentz force acting both inside the material and on the permanent magnet. If the permanent magnet is swept across a defect, the Lorentz force acting upon it will experience a short breakdown whose detection is the key to successful implementation of LET.

An advantage of the fact that the measurement is reduced to a force measurement is that LET can be conveniently applied during production processes since it is robust and relatively straightforward to implement. Based on all these considerations, it is therefore expected that LET can be used to detect defects lying deep within the material under test at relatively high speeds. The main aim of this work is therefore to provide fundamental insights and numerically evaluate the feasibility of the novel testing technique.

1.2 Aims and Objectives

In this work, the above mentioned working principle of the novel NDT technique using Lorentz forces is extended to provide its thorough and comprehensive theoretical foundation. Furthermore, the information about the feasibility and testing capabilities of LET is numerically evaluated using the well-established finite element method (FEM). Due to relative displacement between the conducting parts an efficient FEM-based methodology is developed allowing fast and accurate modelling of general 2D/3D LET systems. To explain the basic working principles and to improve the understanding of the theory underlying general LET systems a simplified model that is amenable to rigorous analytic treatment is presented and solved. To reach the aims of the thesis, the following objectives are defined:

- (i) Provide a detailed theoretical foundation for the underlying physics of general LET systems.
- (ii) Define and solve simplified but enlightening version of LET systems referred to as the creeping magnet problem.
- (iii) Develop a FEM-based methodology for performing 3D transient electromagnetic field problems with moving parts.
- (iv) Perform 3D numerical simulations of a given LET benchmark problem to understand the influence of varying testing parameters on the resulting Lorentz force perturbations. The parameters considered are:
 - (a) defect depth and shape,
 - (b) lift-off distance
 - (c) shape and size of the magnet system
 - (d) magnetisation of the magnet
 - (e) electrical conductivity of the test conductor,
 - (f) testing speed.

1.3 Thesis Overview

To accomplish the aims and objectives of the project, the entire thesis has been divided into several chapters. In chapter 2 the basic working principles of LET and formulation of the theory describing general LET systems is presented. In chapter 3 the current state of the art for 2D/3D numerical modelling of eddy current problems with moving parts using FEM is presented. This is preceded by a brief introduction to FEM including its nodal and edge formulations. In chapter 4 a simplified LET model referred to as *the creeping magnet* is defined and solved, both analytically and numerically. Chapter 5 describes the new FEM-based numerical methodology developed for fast and accurate modelling of general 2D/3D LET systems including parts in motion. Additionally, several simplified numerical approaches that enable fast LET analysis are presented. This also includes the determination of limits of their applicability. In chapter 6 the developed numerical methodology is applied to the given LET benchmark problem in order to study the feasibility and testing capabilities of LET. Finally in chapter 7 some concluding remarks of the research and an outlook for future work in LET is presented.

Chapter 2

Theory of Lorentz Force Eddy Current Testing

The main aim of this chapter is to explain the basic working principle and to formulate the theory describing the novel electromagnetic non-destructive testing technique (sections 2.1 and 2.2). This contactless testing technique represents a modification of the conventional eddy current testing (ECT) and it is based on the measurement of the Lorentz force acting on the magnet system moving relative to the conductor under test. Since the exploited physical effects are eddy currents and Lorentz forces the technique is termed as, Lorentz force eddy current testing (LET) [20].

2.1 Basic Principles

The problem of an electrically conducting object moving in a static magnetic field is well known and it is often used as a simple experiment to introduce the Faraday's law of induction [6, 21–23]. Let $\mathbf{B}_p$ be a primary magnetic field, i.e. the static magnetic field which exists in space free of conductors. According to Faraday's law, when an electrically conducting object moves across static magnetic field an electromotive force (EMF) is induced inside the material. The induced EMF, also known as motional EMF, according to Ohm's law further produces eddy currents $\mathbf{j}$ which follow circular paths in planes normal to the field lines (Fig. 2.1). The direction of the induced current is determined by Lenz's law which states that the current will circulate in such a way that the induced or secondary magnetic field $\mathbf{B}_s$ is opposite in polarity to the applied primary magnetic field. This yields a total magnetic field $\mathbf{B}$ which is described as a sum of the two fields, $\mathbf{B} = \mathbf{B}_p + \mathbf{B}_s$. Typically, in Lorentz force eddy current (LET) applications a permanent magnet is used as a source of $\mathbf{B}_p$. Nevertheless, the same fundamental principle applies if the magnetic field is created by a direct current (DC)

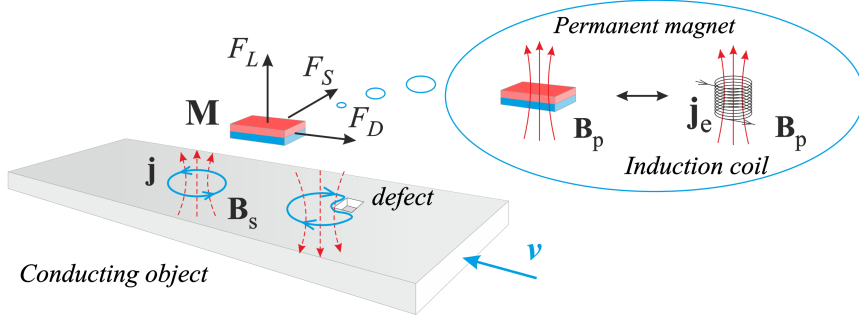


Figure 2.1: Principle sketch of the Lorentz force eddy current testing.

induction coil or by a complex magnet system which can be a combination of both (Fig. 2.1).

Due to their interaction with the magnetic field eddy currents further produce Lorentz force which acts inside the material and brakes the conductor in motion. The total value of the braking force $\mathbf{F}_B$ which can be derived from the Lorentz force law [8] describing the force on a moving charged particle is given by

$$\mathbf{F}_B = \iiint_{\Omega_C} \mathbf{j} \times \mathbf{B} d\Omega, \quad (2.1)$$

where Ω_C is the volume of the conductor. This effect is well known and it is widely used in applications such as electromagnetic braking. However, it is less widely appreciated that by Newton's third axiom *actio = reactio* the force of equal intensity but opposite direction acts on the source of the applied static magnetic field, i.e a permanent magnet or a DC induction coil. In case of a magnet this force is also referred to as the Kelvin force [24]

$$\mathbf{F} = \iiint_{\Omega_M} (\mathbf{M} \cdot \nabla) \mathbf{B} d\Omega = - \iiint_{\Omega_C} \mathbf{j} \times \mathbf{B} d\Omega, \quad (2.2)$$

where $\mathbf{M}$ is the magnetization density and Ω_M is the volume of the used magnet. However, throughout this work more common term Lorentz force has been used. Depending on the type of motion involved, Lorentz force in general comprises of all three components which are here referred to as Lorentz force drag (F_D), lift (F_L), and side (F_S) component (Fig. 2.1). This effect has been recently exploited in applications such as magnetohydrodynamics (MHD) for contactless measurement of liquid metal flows in metallurgy and it is known as Lorentz force velocimetry (LFV) [25, 26].

If the conductor contains a defect the flow of induced eddy currents will be temporary perturbed (Fig. 2.1). According to Ampère's law, the perturbation of the induced current will cause a change in the magnetic field $\mathbf{B}$ as well. In LET the perturbation of eddy currents is detected through its influence upon the Lorentz force acting both

inside the material and on the permanent magnet. For example, if the magnet is swept across a defect, the Lorentz force acting upon it will experience a short breakdown whose detection is the key to successful implementation of LET. If the material is free of defects the Lorentz force will remain unchanged.

The principle of using motion to induce eddy currents, can be used to overcome or to reduce one of the major disadvantages of the traditional ECT which is limited by the strong frequency dependent skin effect [10, 27–29], and thus to surface and near surface inspections. Therefore, it is expected that LET can be used to detect defects lying deeper inside the conducting materials compared to the traditional ECT, when the inspection is performed at similar testing conditions. Some of the preliminary results demonstrating this ability can already be found in [15, 30, 31]. However, the main aim in this work is not to provide direct comparison between LET and traditional ECT. The main intention here is to present LET as a complementary NDT method that can, together with ECT or some other testing techniques, provide better inspection results.

For practical realisation of LET it is necessary to provide the relative motion between the magnet and the conductor under test. In order to formulate the theory describing the LET, a general assumption is made that the magnet is in motion relative to a stationary conductor.

2.2 Governing Equations

Lorentz force eddy current testing belongs to a special entity of dynamic electromagnetic field phenomena in which the effect of materials in relative motion occurs. This effect is well known and it represents the basic operating principle over a wide range of electromagnetic devices [22, 23, 32]. They mainly include different types of electromechanical systems in applications such as electrical machines, magnetic levitation, eddy current brakes, magnetohydrodynamics, inductive heating, NDT&E, etc.

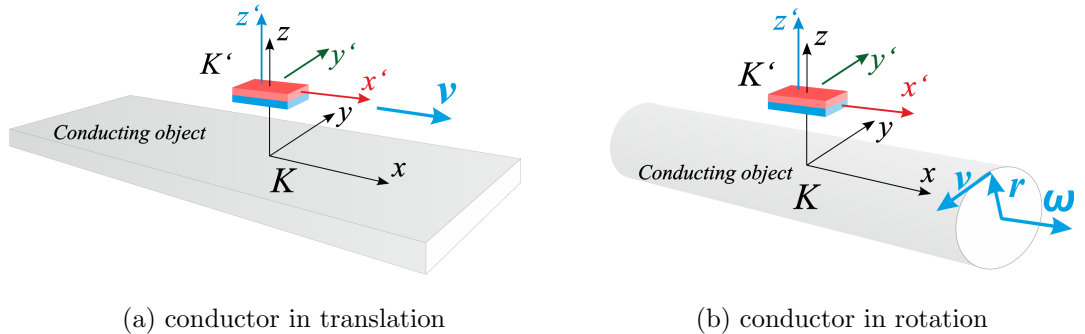


Figure 2.2: Typical motion involved in LET applications.

The operating frequency and velocities involved in these devices are quite modest compared to the speed of light c , thus a quasi-static electromagnetic field theory is well suitable for their efficient analysis [23].

Relative motion involved in LET applications is typically in the form of rigid translation or rotation, e.g. along one of the coordinate axis (Fig. 2.2). From the general theory of electrodynamics in moving media it is well known that the analysis of such a system can be performed by considering different frames of reference, which are either related to the moving (K') or to the fixed part (K) of the assembly [21–23, 33–35]. If $\mathbf{v}$ is the translational velocity of the magnet described in fixed reference frame K, then in the reference frame denoted by K' the magnet is stationary and the conductor is moving in opposite direction with velocity $-\mathbf{v}$ (Fig. 2.2a). Similarly, if ω is the angular velocity of the magnet in K, then in the reference frame K' the magnet is stationary and the conductor is rotating in opposite direction with angular velocity $-\omega$ (Fig. 2.2b). Although, it might seem that these two different LET configurations (conductor in translation and conductor in rotation) require separate analysis, the relation between the linear and angular velocity $\mathbf{v} = \omega \times \mathbf{r}$ allows general LET investigation in terms of a three-component velocity vector $\mathbf{v}$ (Fig. 2.2b). Here, $\mathbf{r} = x \cdot \mathbf{e}_x + y \cdot \mathbf{e}_y + z \cdot \mathbf{e}_z$ is the position vector of a moving material point.

By introducing different frames of reference the electromagnetic field analysis requires transformation equations which relate the field quantities from one reference frame to another [22]. As already mentioned, the emphasis here is to a quasi-static electromagnetic field phenomena in the domain of Newtonian mechanics ($v \ll c$). Thus, spatial and time coordinates between K and K' frames are related by Galilean transformations [22, 23]

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t \text{ and } t' = t \quad (2.3)$$

where $\mathbf{r}$ and $\mathbf{r}'$, are position vectors, and t and t' are time coordinates in K and K', respectively. The relationship between generally valid Lorentz transformations and its low-velocity approximation ($c \rightarrow \infty$) given by Galilean transformations has been discussed in much more detail in [21–23, 33–35]. However, it is important to mention that the treatment of relative motion, either in its low-velocity range or in the relativistic range is based on the postulate that the laws of physics, and by that the Maxwell's equations, are form-invariant in any reference frame. These transformations are general and they can be applied to moving media of finite extent and of arbitrary shape [22, 23, 32, 36].

Based on the observations given above, the theory of LET has been described in two different, albeit physically equivalent frames of reference, namely the *fixed frame*

of reference and the *moving frame of reference*. In the fixed frame of reference the coordinate system (K) is referred to a stationary conductor and in the moving frame of reference the coordinate system (K') is referred to a moving permanent magnet (Fig. 2.2). The working principle of LET has been introduced by considering several fundamental laws of electromagnetism. The generalization of these laws as well as their mathematical description is given by the set of Maxwell's equations which are supplemented by the constitutive relations. Even though there exist many excellent text books on the theory of classical electromagnetism, for the completeness of this work a short overview of the Maxwell's equations in their quasi-static approximation is presented.

Maxwell's Equations

Disregarding the term accounting for displacement currents, introduced by Maxwell in the modified Ampère's law ($\partial \mathbf{D} / \partial t = \mathbf{0}$), the quasi-static approximation of Maxwell's equations is as follows

$$\text{Faraday's law: } \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.4)$$

$$\text{Ampère's law: } \nabla \times \mathbf{H} = \mathbf{j}_e + \mathbf{j} \quad (2.5)$$

$$\text{Gauss's law: } \nabla \cdot \mathbf{B} = 0 \quad (2.6)$$

where $\mathbf{E}$ is the electric field strength, $\mathbf{H}$ is the magnetic field strength, $\mathbf{B}$ is the magnetic flux density, and $\mathbf{j}_e$ is the external current density, e.g. of an induction coil (Fig. 2.1). The induced eddy current density $\mathbf{j}$ inside the moving conductor has to fulfil the continuity equation which is implied by the conservation of electric charge

$$\nabla \cdot \mathbf{j} = 0 \quad (2.7)$$

The system of partial differential equations given by (2.4)-(2.7) is not complete. Moreover, only two of the given equations are independent, i.e. the Faraday's law and the Ampère's law [37]. In order to obtain a closed-form solution, Maxwell's equations are supplemented by the constitutive relations. These relations describe the material in the presence of an electromagnetic field by relating the corresponding material properties and the electromagnetic field quantities

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \quad (2.8)$$

$$\mathbf{j} = \sigma \mathbf{E} \quad (2.9)$$

where μ_0 and σ denote for the magnetic permeability of vacuum and the electrical conductivity of the material, respectively. The equation (2.9) is widely known as Ohm's law.

The magnetization density $\mathbf{M}$ introduced in (2.8) describes how the material is magnetized when placed in an external magnetic field $\mathbf{H}$. It can be interpreted as the volume density of the induced magnetic dipole moments inside the material. Permanent magnets, e.g. have a non-zero magnetization $\mathbf{M}$ when there is no magnetic field present. The amount of magnetization left behind is also called the magnetic remanence which is described as $\mathbf{B}_r = \mu_0 \mathbf{M}$. This means that the permanent magnets can be efficiently modelled by a material having a specified distribution of magnetization density $\mathbf{M}$ or an equivalent amount of magnetic remanence $\mathbf{B}_r$. The extent to which a material is magnetized in general depends on the strength of the applied magnetic field, i.e. $\mathbf{M} = \chi_m \mathbf{H}$, where χ_m is the magnetic susceptibility. As a result the constitutive relation (2.8) can be written as:

$$\mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H} = \mu_0\mu_r\mathbf{H} \quad (2.10)$$

where μ_r represents the relative magnetic permeability of the material. Based on this non-dimensional quantity materials are classified as diamagnetic, paramagnetic and ferromagnetic materials [37]. Due to their weak influence on the resulting magnetic field distribution ($\mu_r \approx 1$) diamagnetic and paramagnetic materials are also referred to as non-magnetic, whereas ferromagnetic materials due to their high influence on the magnetic field distribution ($\mu_r \gg 1$) are often referred to as magnetic materials.

The material properties are usually non-linear, which means that they depend on the corresponding field values and temperature T , i.e. $\mu_r = \mu_r(\mathbf{H}, T)$ and $\sigma = \sigma(\mathbf{E}, T)$. For isotropic materials they are scalar quantities, whereas for anisotropic materials they represent second-order tensor quantities [37]. It should be mentioned that the electric and magnetic properties of materials are very important factors in NDT&E as they characterize its mechanical and physical state, e.g. its composition and structure, presence of defects, temperature, etc. However, they also restrict the type of the NDT&E method which can be applied. Without any loss of generality, in order to introduce LET and prove its basic principles all material properties are assumed to vary linearly with the applied magnetic field and are temperature independent. Furthermore, the conducting objects used for the analysis are considered to be non-magnetic as well ($\mu_r = 1$).

Maxwell's equations as expressed in the form given by (2.4)-(2.6) are first formulated by J. C. Maxwell for stationary media. Nevertheless, according to the relativity postulate stated earlier in the text, these equations are form-invariant when described

in the moving reference frame K' , which is related to the moving magnet

$$\nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'} \quad (2.11)$$

$$\nabla' \times \mathbf{H}' = \mathbf{j}'_e + \mathbf{j}' \quad (2.12)$$

$$\nabla' \cdot \mathbf{B}' = 0 \quad (2.13)$$

The continuity equation (2.7) and the constitutive relations (2.8) and (2.9) are also form-invariant when described in K'

$$\nabla' \cdot \mathbf{j}' = 0 \quad (2.14)$$

$$\mathbf{B}' = \mu_0(\mathbf{H}' + \mathbf{M}') \quad (2.15)$$

$$\mathbf{j}' = \sigma \mathbf{E}' \quad (2.16)$$

This is one of the main contributions of Minkowski who first formulated the theory of electromagnetic fields in the presence of moving polarizable and magnetizable media [22, 34, 35, 38]. Due to his contribution, transformations of electromagnetic field quantities between different reference frames is often referred to as Minkowski transformation.

In (2.11) to (2.16) the velocity of the moving media is not shown explicitly. It is contained implicitly in the transformation equations relating the space and time derivatives between K and K' reference frames. The transformation equations, which can be derived from the well-known Galilean transformations (2.3) are as follows [39]

$$\nabla' = \nabla \quad (2.17)$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \quad (2.18)$$

Applying (2.17) and (2.18) to Maxwell's equations described in K' (2.11)-(2.13), the equivalent system of equations is obtained

$$\nabla \times (\mathbf{E}' - \mathbf{v} \times \mathbf{B}') = -\frac{\partial \mathbf{B}'}{\partial t'} \quad (2.19)$$

$$\nabla \times \mathbf{H}' = \mathbf{j}'_e + \mathbf{j}' \quad (2.20)$$

$$\nabla \cdot \mathbf{B}' = 0 \quad (2.21)$$

The Ohm's law for moving conductors is as follows:

$$\mathbf{j}' = \sigma \mathbf{E}' = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (2.22)$$

2.2.1 Potential Functions

In order to write Maxwell's equations in simpler form usually potential functions are used. As it will be shown in the following LET analysis, they additionally provide simplifications in calculation of fields, better understanding of underlying physical phenomena and in most cases they considerably decrease overall numerical computations. To describe general and time-dependent electromagnetic fields two different vector potentials are commonly used namely, the magnetic vector potential $\mathbf{A}$ or current vector potential $\mathbf{T}$. They are defined based on divergence-free property of the magnetic field, $\nabla \cdot \mathbf{B} = 0$, and divergence-free property of the induced current density, $\nabla \cdot \mathbf{j} = 0$, which results in

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (2.23)$$

$$\mathbf{j} = \nabla \times \mathbf{T}. \quad (2.24)$$

From the vector calculus, it is known that these equations are satisfied up to a divergence of $\mathbf{A}$ and $\mathbf{T}$, respectively. To ensure the uniqueness of the solution, their divergence has to be additionally specified, which is known as gauging [40].

Due to the similarities of (2.23) and (2.24), it is sometimes a delicate problem to decide which potential formulation should be used for the analysis. The choice strongly depends on the particular type of the problem to be solved, i.e. on the used materials properties, electromagnetic field sources (voltage or current sources), and on the applied gauging condition. A comprehensive study involving different potential formulations and the possibility of combining them in a single field computation is given in [40–43]. It should be emphasised, that the use of current vector potential $\mathbf{T}$ inside the conducting regions requires always additional magnetic scalar potential ψ to be applied, whereas the magnetic vector potential $\mathbf{A}$, can be applied with or without an electric scalar potential V [42]. Apart from being the most widely used formulation, the possibility to disregard the additional electric scalar potential V , makes the magnetic vector potential $\mathbf{A}$ preferable potential function to be used in future LET analyses.

By using (2.23) together with Ampère's law, the remaining field quantity can be determined as

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V. \quad (2.25)$$

In the following sections the magnetic field diffusion equation which governs the physics underlying LET is derived. As dependent variables, the magnetic vector potential $\mathbf{A}$ augmented by electric scalar potential V is used. The resulting $\mathbf{A}$ - V formu-

lation is generally valid and it does not result from any model simplification, neither it requires some specific numerical techniques for accurate analysis [40, 42]. Furthermore, depending on the definition of the frame of reference, two equivalent types of diffusion equation are presented.

2.2.2 Fixed Frame of Reference

In the fixed frame of reference the global coordinate system K is associated with the stationary conductor, and the permanent magnet is moving with velocity $\mathbf{v}$ (Fig. 2.3). The set of Maxwell's equations which mathematically describes the LET problem in this reference frame is given by (2.4)-(2.6). According to the definition of the magnetic vector potential (2.23), the Gauss's law (2.6) is implicitly satisfied, whereas the Faraday's law (2.4) and Ampère's law (2.5) can be written as follows

$$\nabla \times \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla V \right), \quad (2.26)$$

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A} - \mathbf{M} \right) = \mathbf{j}_e + \mathbf{j}. \quad (2.27)$$

The above set of equations includes the constitutive relation (2.8) and the definition of the electric scalar potential (2.25). Using the Ohm's law (2.9) and the simple substitution in (2.26) and (2.27) the following transient magnetic field diffusion equation can be derived

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A} - \mathbf{M} \right) = -\sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) + \mathbf{j}_e. \quad (2.28)$$

This equation, together with the current continuity condition (2.7), represents the governing equation of LET systems, when described in the fixed frame of reference. It includes the external source term in form of current density vector $\mathbf{j}_e$, which can be used to describe additional DC or AC induction coils. The motion of the magnet is implicit in the time derivative of $\mathbf{A}$, which has certain advantages when being applied to FEM discretization schemes (see Section 3.1)[28].

The previous analysis has shown that, using the definition of electromagnetic po-

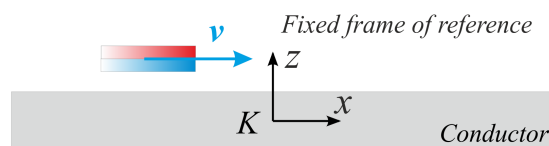


Figure 2.3: Definition of the fixed frame of reference (K).

tentials, the general LET system can be described only by $\mathbf{A}$ and V , instead of the vector field quantities $\mathbf{E}$ and $\mathbf{B}$. Thus, the number of equations to be solved is reduced from 6 to 4 for 3D LET investigations. In the 2D case only one component of the magnetic vector potential, i.e. $A_y \cdot \mathbf{e}_y$ together with the electric scalar potential V needs to be applied. This reduces the number of equations to be solved from 4 to 2.

2.2.3 Moving Frame of Reference

In the moving frame of reference the global coordinate system K' is associated with the moving permanent magnet. This makes the magnet stationary, whereas the conductor is moving with velocity $-\mathbf{v}$, i.e. opposite to the x -axis direction (Fig. 2.4). The set of Maxwell's equations which describes the LET problem in K' is given by (2.19)-(2.21). By following the same procedure as in the fixed frame of reference and using the definition of electromagnetic potentials (2.23) and (2.25), Faraday's and Ampère's law can be written as follows

$$\nabla \times (\mathbf{E} - \mathbf{v} \times \nabla \times \mathbf{A}) = \nabla \times \left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla V \right), \quad (2.29)$$

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A} - \mathbf{M} \right) = \mathbf{j}_e + \mathbf{j}. \quad (2.30)$$

The superscript " ' " has been omitted deliberately for simplicity. Additionally, to derive (2.29) and (2.30) the form-invariant constitutive relation (2.15) is applied as well. Using the Ohm's law for moving conductors (2.16) and simple substitution in (2.29) and (2.30), the following transient magnetic field diffusion-convection equation can be derived

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A} - \mathbf{M} \right) = -\sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V - \mathbf{v} \times \nabla \times \mathbf{A} \right) + \mathbf{j}_e. \quad (2.31)$$

Together with the current continuity condition (2.14), this equation represents the governing equation of LET systems, when described in the moving frame of reference, K' . By direct comparison with the equivalent diffusion equation derived in K , it can be seen that (2.31) includes an additional velocity term $(\mathbf{v} \times \nabla \times \mathbf{A})$. It describes

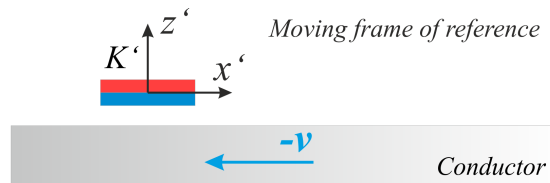


Figure 2.4: Definition of the moving frame of reference (K').

the convection of the magnetic field lines due to motion, which is in (2.28) implicit in the time derivative of $\mathbf{A}$. The form of (2.31) is fairly complicated when compared to (2.28). However, computationally this can have certain advantages. When the field configuration is time independent, i.e. for problems involving uniformly moving conductors with a constant cross-section normal to the direction of motion, only one stationary analysis can be performed to obtain an accurate steady-state solution [37]. In case of the diffusion equation (2.28) a magnetic diffusion transient response has to be calculated in order to obtain the same solution. This particular possibility makes (2.31) very useful, and therefore it is often applied in analytical studies of simplified eddy current problems with conductors in motion.

2.2.4 Magnetic Reynolds Number

As already stated in the basic working principle of LET, eddy currents induced inside the moving conductor create their own secondary magnetic field $\mathbf{B}_s$ (Fig. 2.1). It is reasonable to expect that this field will perturb the primary magnetic field $\mathbf{B}_p$ of the magnet. It can be shown that the extent of field perturbation depends on several parameters associated with the conductor in motion, namely its velocity, electrical conductivity, relative permeability and its size [32]. Nevertheless, the number of dependent variables can be reduced to only one if the so-called magnetic Reynolds number R_m is used. It represents a non-dimensional quantity, which can be interpreted as a ratio of the induced and applied magnetic field ($R_m \sim |\mathbf{B}_s|/|\mathbf{B}_p|$) [44]. An equivalent definition of R_m can be obtained by normalizing the transient diffusion-convection equation (2.31) which results in [32]

$$\tilde{\nabla} \times (\tilde{\nabla} \times \tilde{\mathbf{A}}) = -R_m \left(\frac{\partial \tilde{\mathbf{A}}}{\partial \tilde{t}} + \tilde{\nabla}(\tilde{\mathbf{v}} \cdot \tilde{\mathbf{A}}) - \tilde{\mathbf{v}} \times \tilde{\nabla} \times \tilde{\mathbf{A}} \right), \quad (2.32)$$

where " $\sim$ " indicates the non-dimensional quantities and R_m represents the magnetic Reynolds number defined as

$$R_m = \mu_0 \sigma |\mathbf{v}| L \quad (2.33)$$

A detailed derivation of (2.32) in terms of the magnetic vector potential is presented in Appendix A [32]. However, a particular attention in R_m definition (2.33) should be given to the length-scale parameter, denoted by L . There is no general rule in choosing L . It depends strongly on the particular problem at hand and it should be carefully defined (see section 5.1).

Using the obtained non-dimensional form of the diffusion-convection equation (2.32),

the magnetic field perturbation can be analysed by only one parameter R_m . If $R_m \ll 1$ the convective and the transient terms on the right-hand side of (2.32) can be neglected. The diffusion-convection equation is then considerably simplified and it describes only the stationary diffusion process. In case of $R_m \gg 1$ the right-hand side in (2.32) dominates over the diffusion term on the left-hand side. In this case the resulting magnetic field will be strongly altered due to conductor in motion (see section 6.1.1).

2.3 Optimal $\mathbf{A}^* - \psi$ Formulation

The use of the magnetic vector potential $\mathbf{A}$ and electric scalar potential V in the whole computational domain, denoted by Ω , represents the general LET solution (Fig. 2.5). In the fixed and the moving frame of reference this solution is well-described by (2.28) and (2.31), respectively. Nevertheless, by considering separately non-conducting Ω_M and Ω_A , and conducting domains Ω_C the same solution can be obtained in more economic way [40, 45].

2.3.1 Modelling of Non-conducting Domains

In non-conducting domains (Ω_A and Ω_M), where there are no currents present, it is possible to use only magnetic scalar potential ψ to represent general vector magnetic fields. Based on the resulting curl-free property of the magnetic field ($\nabla \times \mathbf{H} = \mathbf{0}$), ψ is described as

$$\mathbf{H} = -\nabla\psi. \quad (2.34)$$

The obvious advantage of using scalar potentials to represent the electromagnetic field quantities is that only one scalar function is sufficient for their analysis. Using (2.34) the governing equations for both, fixed and moving frame of reference (K and K') are

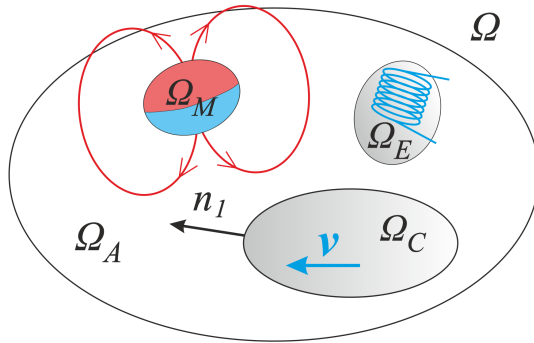


Figure 2.5: Truncation of the infinite air domain. Ω_M is the magnet domain, Ω_A is the air domain and Ω_C is the conductor domain.

simplified to Laplace type equation

$$\nabla \cdot (\mu_0 \nabla \psi + \mathbf{M}) = 0. \quad (2.35)$$

The magnetization vector $\mathbf{M}$ only applies to the domain of a permanent magnet. Due to this representation, the number of equations to be solved is reduced from three to one in both Ω_A and Ω_M .

2.3.2 Modelling of Conducting Domains

In the conducting regions, i.e. inside a moving conductor Ω_C and in general inside the external current coil domain Ω_E , the magnetic vector potential $\mathbf{A}$ along with electric scalar potential V has to be applied. This represents the so-called $\mathbf{A}$ - V formulation [42]. However, for transient eddy current analyses it is possible to modify the magnetic vector potential $\mathbf{A}$ as

$$\mathbf{A}^* = \mathbf{A} + \int_0^t \nabla V dt. \quad (2.36)$$

In this case, the electric scalar potential V is excluded from (2.28) or (2.31), which results in

$$\sigma \frac{\partial \mathbf{A}^*}{\partial t} = -\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A}^* \right) + \mathbf{j}_e, \quad (2.37)$$

for the fixed frame of reference K , and

$$\sigma \left(\frac{\partial \mathbf{A}^*}{\partial t} - \mathbf{v} \times \nabla \times \mathbf{A}^* \right) = -\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A}^* \right) + \mathbf{j}_e, \quad (2.38)$$

for the moving frame of reference K' . Due to this representation, the number of equations to be solved in Ω_C is reduced from 4 to 3.

In general, the modified vector potential $\mathbf{A}^*$ can be used only in conducting domains with homogeneous electrical conductivity. If an abrupt change of electrical conductivity occurs, e.g. due to defects, it may also result in a discontinuity of the normal component of $\mathbf{A}^*$ at the interface. This is because $\mathbf{J} = -\sigma \partial \mathbf{A}^* / \partial t$. If this is satisfied, the obtained set of equations (2.35)-(2.37) and (2.35)-(2.38) represents the so-called optimal $\mathbf{A}^*$ - ψ formulation with respect to the fixed or the moving reference frame.

2.3.3 Coupling of Potentials

The presented optimal $\mathbf{A}^* - \psi$ formulation involves the definition of different potential functions to obtain particular LET solutions. To provide the continuity of the normal and tangential components of the magnetic flux density $\mathbf{B}$, these potentials

have to be coupled on the appropriate interface. The continuity conditions are given as follows [42]

$$\nabla \times \mathbf{A}^* \cdot \mathbf{n}_1 - \mu_0 \nabla \psi \cdot \mathbf{n}_1 = 0, \quad (2.39)$$

$$\left(\frac{1}{\mu_0} \nabla \times \mathbf{A}^* \right) \times \mathbf{n}_1 - \nabla \psi \times \mathbf{n}_1 = \mathbf{0}, \quad (2.40)$$

where the unit normal vector, directed from the conducting domain Ω_C to surrounding air domain Ω_A , is denoted by $\mathbf{n}_1$ (Fig. 2.5).

Chapter 3

State of the Art in motion modelling using FEM

The intrinsic phenomena associated with electric and magnetic fields affect almost all aspects of our everyday life. This is followed by continuous development and design of more sophisticated electro-mechanical devices which result in better functionality, higher efficiency and increased safety. Depending on the particular application, this task strongly relies on the accurate modelling of the electromagnetic fields within the device, i.e. it requires solution of the governing equations described by Maxwell under well-defined conditions. Whenever it is possible, the analytical solution of the resulting system of partial differential equations (PDE) is most wanted [28, 37, 46–49]. Apart from providing exact and fast solutions, the obtained closed-form analytical expression also helps in better understanding of the underlying physical phenomena associated with the problem under investigation. Unfortunately, these solutions are not always available and they can be obtained only for some simplified device and field configurations. Thus, in NDT&E applications the development and optimization of various testing techniques is performed using the numerical methods [28].

Due to its ability to handle complex geometries, anisotropic and inhomogeneous material properties, widely used numerical method in NDT&E applications is the finite element method (FEM) [48]. One of the main aims of this work is to present new FEM-based methodology for fast and accurate modelling of non-destructive eddy current problems with moving parts. For reasons of completeness in this section only a brief introduction to FEM is presented. A detailed description of the FEM can be found, e.g. in [28, 50]. This is followed by literature review on the existing methodology for numerical modelling of general eddy current problems with conductors in relative motion usually encountered in NDT&E applications.

3.1 Brief Introduction to the Finite Element Method

The finite element method (FEM) is a numerical technique which gives approximate solutions to PDE which are commonly used in engineering applications to describe the physical behaviour of a system [28, 50]. Unlike some other numerical techniques, e.g. finite difference method, the FEM is used to approximate the solution rather than directly the partial derivatives appearing in the governing equations. Its fundamental idea is to divide the region under investigation into small well-described elementary domains called the finite elements. In 2D investigations they represent simple geometrical forms such as triangles and quadrilaterals, whereas in 3D these domains are typically in form of tetrahedra and hexahedra. The assembly of all elements is called the finite element mesh [50].

Within each finite element the unknown solution, i.e. the unknown scalar or vector potentials (see section 2) are usually approximated with low-order polynomial functions, which are in FEM applications widely known as shape functions. They can be defined as scalar or vector quantities. The scalar shape functions are associated with nodes of a finite element and they can be used to approximate both, scalar and vector potentials. The vector shape functions are associated with the corresponding edges of an element and they can only be used to approximate vector potentials. Depending on the particular choice, these elements are also referred to as nodal or edge finite elements.

3.1.1 Nodal Finite Elements

In the optimal $\mathbf{A}^* - \psi$ formulation, the non-conducting regions of the LET problems are described using the magnetic scalar potential (2.35). In the finite element analysis (FEA) the scalar quantities are approximated using nodal finite elements [51–53]. Assuming the discretization of the computational domain Ω as shown in Fig. 3.1 the unknown scalar potential can be estimated as follows

$$\psi(x, y, z) \approx \psi^a(x, y, z) = \sum_{k=1}^N N_k(x, y, z) \psi_k^a, \quad (3.1)$$

where $N_i(x, y, z)$ is the resulting scalar shape function for particular node k and N is the total number of nodes forming the entire finite element mesh. The actual value of N is determined by the order of the used polynomial approximation of the unknown potential within the element. This approximations is typically of first or second order

a system of algebraic equations for N degrees of freedom (DoF). The resulting matrix of the system, also referred to as the stiffness matrix, is symmetric and sparse [48, 50, 55]. To simplify the calculation of integrals in (3.5), instead of using the global coordinates x, y and z , the shape functions are usually defined in a local (normalised) coordinate systems [48, 50].

Using the nodal elements, vector quantities are discretized simply by considering each of its components separately. Applying this procedure at the material interfaces, which are also the interfaces between different elements of the mesh, makes the approximated vector potential normally and tangentially continuous across that interface. When solving for modified magnetic vector potential $\mathbf{A}^*$ (see section 2.3) the FEM analysis would be restricted to isotropic and homogeneous materials [42]. To overcome this limitation, edge finite elements are widely applied.

3.1.2 Edge Finite Elements

In case when anisotropic and inhomogeneous conducting materials are described only by a single magnetic vector potential, the applied FEM formulation must account for possible discontinuity of its normal component. This requirement can be easily fulfilled using the edge finite element formulation [42]. Otherwise, instead of using only modified vector potential $\mathbf{A}^*$ to describe linear conductors in general LET systems, the electric scalar potential V has to be additionally applied (see section 2.2). In this case V accounts for discontinuity of induced eddy currents [56, 57].

Geometrically, the edge finite elements are identical to their nodal counterparts (see Fig. 3.2). Instead of using only nodes, edges of a finite element can also be used to approximate the unknown vector potential $\mathbf{A}^*$ as

$$\mathbf{A}^*(x, y, z) \approx \mathbf{A}_a^*(x, y, z) = \sum_{k=1}^N \mathbf{N}_k(x, y, z) \mathbf{A}_{tk}^*, \quad (3.6)$$

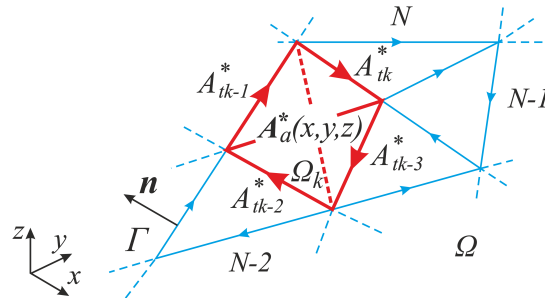


Figure 3.2: Sketch of a finite element mesh obtained by using 1st order edge triangular elements.

where $\mathbf{A}_{tk}^*$ is the line integral of $\mathbf{A}^*$ along edge k and $\mathbf{N}_k$ is the vector shape function at edge k (see Fig. 3.2). This makes only the tangential component of the unknown vector potential continuous across the element interfaces, allowing a discontinuity of its normal component [42, 56, 57].

By following a similar procedure as in the nodal FEM formulation, i.e. using the Galerkin's weighted residual procedure, the weak form of (2.37) and (2.38) can be obtained as

$$\begin{aligned} \sum_{k=1}^N \left[\iiint_{\Omega} \sigma \mathbf{N}_k \cdot \frac{\partial \mathbf{A}_a^*}{\partial t} d\Omega + \oint_{\Gamma} \left(\frac{1}{\mu_0} \nabla \times \mathbf{A}_a^* \times \mathbf{N}_k \right) \cdot \mathbf{n} d\Gamma + \right. \\ \left. + \iiint_{\Omega} \frac{1}{\mu_0} (\nabla \times \mathbf{A}_a^*) \cdot (\nabla \times \mathbf{N}_k) d\Omega \right] = - \sum_{k=1}^N \iiint_{\Omega} \mathbf{N}_k \cdot \mathbf{j}_e d\Omega, \end{aligned} \quad (3.7)$$

in the fixed reference frame K and

$$\begin{aligned} \sum_{k=1}^N \left[\iiint_{\Omega} \sigma \mathbf{N}_k \cdot \left(\frac{\partial \mathbf{A}_a^*}{\partial t} - \mathbf{v} \times \nabla \times \mathbf{A}_a^* \right) d\Omega + \oint_{\Gamma} \left(\frac{1}{\mu_0} \nabla \times \mathbf{A}_a^* \times \mathbf{N}_k \right) \cdot \mathbf{n} d\Gamma + \right. \\ \left. + \iiint_{\Omega} \frac{1}{\mu_0} (\nabla \times \mathbf{A}_a^*) \cdot (\nabla \times \mathbf{N}_k) d\Omega \right] = - \sum_{k=1}^N \iiint_{\Omega} \mathbf{N}_k \cdot \mathbf{j}_e d\Omega, \end{aligned} \quad (3.8)$$

in the moving reference frame K' . To obtain (3.7) and (3.8) vector identity $\mathbf{b} \cdot \nabla \times \mathbf{a} = \nabla \cdot (\mathbf{a} \times \mathbf{b}) + \mathbf{a} \cdot \nabla \times \mathbf{b}$ together with the divergence theorem has been applied [42]. For simplicity, the source terms associated with external current density have been placed on the right-hand side of (3.7) and (3.8). As in the case of nodal elements, to simplify the calculation of integrals in (3.7) or (3.8), instead of using the global coordinates x, y and z , the shape functions are usually defined in a local (normalised) coordinate system [48, 50].

Finally, to solve the LET problem using the optimal $\mathbf{A}^* - \psi$ formulation, nodal FEM formulation (3.5) has to be coupled with the corresponding edge FEM formulations (3.7) or (3.8) depending on the chosen frame of reference (K or K'). In case of linear conducting materials the coupling can be obtained directly on the outer surface of the conductor by satisfying the continuity conditions (2.40). It should be mentioned that the resulting stiffness matrix in the fixed frame of reference K is symmetric, whereas in the moving reference frame K' due to the additional velocity term the stiffness matrix is non-symmetric [48, 50, 55].

3.2 Modelling Motion Using the Finite Element Method

The Lorentz force eddy current testing (LET) belongs to a special class of electromagnetic field phenomena in which the effect of materials in relative motion occurs (see section 2). In fact, this effect represents the basic operating principle over a wide range of electro-mechanical devices in application areas such as electrical machines, magnetic levitation systems, inductive heating, eddy current brakes, non-destructive testing and evaluation (NDT&E), etc. Due to its tremendous industrial relevance the application of FEM to this particular type of field problems, also known as the moving eddy current problems [58], has undergone an extensive research over the past decades. As a result many different techniques for 2D/3D motion modelling using FEM have been developed, mostly referring to rotational electrical machines [58–61]. However, either due to their high computational requirements or difficulties in their implementation only a few techniques are nowadays commonly used in general-purpose commercial FEM codes. The main aim here is to give an overview of these techniques, whereas a more comprehensive study due to a large number of publications would be far beyond the scope of this work. Additional information concerning the particular technique can be found in the extensive reference list presented.

In principle, independent on the actual type of motion (translation or rotation) all existing techniques for simulation of general moving eddy current problems can be classified into (i) fixed grid methods and (ii) moving or time changing grid methods

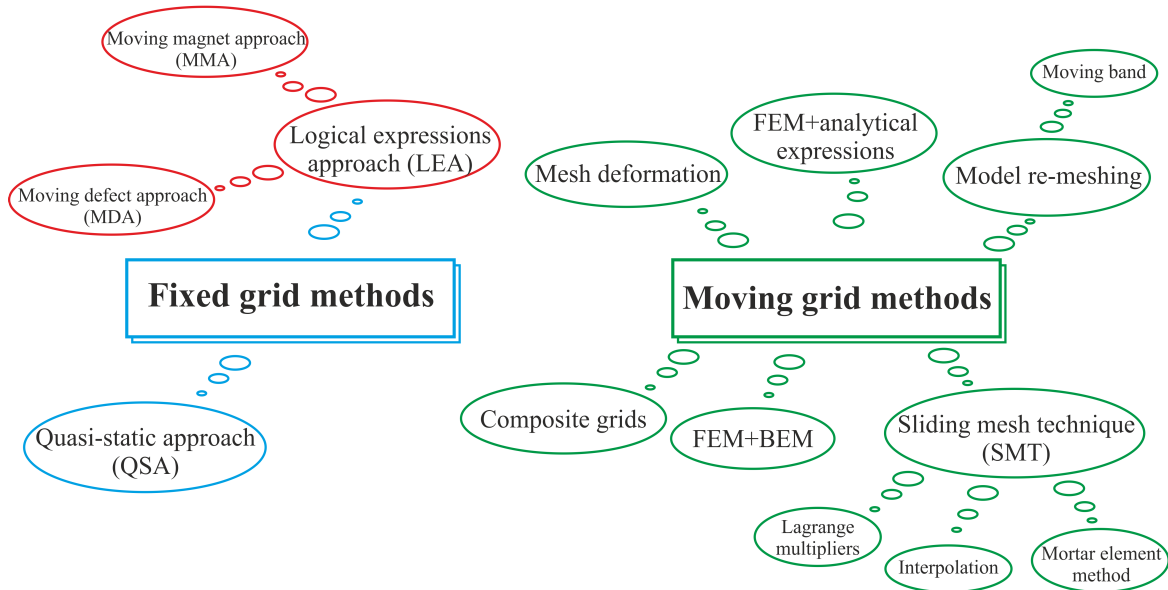


Figure 3.3: FEM-based methods for modelling eddy current problems involving motion.

(see Fig. 3.3) [59].

The fixed grid methods are usually applied to 2D/3D static or time-harmonic eddy current problems involving uniformly moving conducting parts having invariant cross-section in the motion direction [27, 58, 59, 62–66]. The field problem is formulated in the moving frame of reference (see section 2.2) where the additional velocity term ($\mathbf{v} \times \nabla \times \mathbf{A}$) is used to describe the induced eddy currents within the moving conducting part. This approach, also referred to as the quasi-static approach (QSA) [66], is very efficient in terms of computational time since only one stationary analysis needs to be performed to obtain an accurate steady state solution. For some simple device configurations several authors combine FEM with analytical solutions as well [65, 67]. Considerably reduction of the simulation time and increased accuracy has been reported. However, apart from simple geometries the analysis is restricted to stationary and time-harmonic problems. The convection of the magnetic field lines introduced in the fixed grid methods can cause spurious numerical oscillations when the resulting Péclet number ($P_e = \mu\sigma v\Delta x$) is large [68–70]. This can be avoided by performing the mesh refinement within the conductor region or by technique known as the upwinding [71, 72]. The logical expression approach (LEA) presented in this work, extends the applicability of the fixed grid methods to 2D/3D transient moving eddy current investigations (see Fig. 3.3). Furthermore, the moving conductors can have variable cross-section in the motion direction as well. Depending on the type of the object in motion LEA can be described in the fixed or in the moving frame of reference.

The moving grid methods are more general and they can be applied to simulate wide variety of electromechanical devices involving linear or rotational movement. In principle, from the model topology point of view all available techniques are quite similar. The main idea is to decompose the whole computational domain (the finite element mesh) into two parts associating them with the moving or with the fixed part of the assembly [58, 59, 73]. Within each part the governing equations are solved in their own frame of references, whereas the relative displacement and the field coupling is provided on the introduced interface [61, 74]. Depending on the actual interface, which can have constant or variable lift-off distance, to achieve the coupling many different techniques have been applied, each of them having certain advantages and disadvantages.

One of the earliest techniques to model relative displacement of 2D induction machines is based on mesh deformation [75]. During the movement this technique can produce elements with large aspect ratios leading to the loss of accuracy [61, 76]. Thus, its application is usually restricted for modelling 2D electromagnetic devices requiring small relative displacements [77–79].

In order to provide arbitrary large displacements techniques based on the continuous re-meshing of the computational domain have been reported [80]. When re-meshing is performed only in the small air-gap region of the device, these techniques are referred to as the step-by-step finite elements or the moving band techniques [74, 80–84]. Due to time changing grids the obtained solution can be numerically noisy. Additionally, for 3D complex geometries the re-meshing procedure can also be very time-demanding. This is why these techniques are mostly applied to 2D eddy current problems. In order to avoid tedious re-meshing of the model several authors combine the advantages of the FEM and analytical expressions which provide the field coupling without meshing the air-gap of the machine. The resulting technique is often referred to as the air-gap element or macro element method [76, 85–88]. Whenever they can be obtained, the analytical expressions are simply coupled with the FEM formulation providing high accuracy compared to all other methods available [58]. However, as reported in [88] the coupling procedure can reduce the sparsity of the resulting stiffness matrix increasing the computational costs. Furthermore, due to difficulties in obtaining the analytical expressions, the analysis is usually restricted to 2D field problems [89]. On the same note, to avoid the re-meshing of the computational domain, hybrid methods based on the coupling of boundary element method (BEM) and FEM have been introduced. The BEM provides all the advantages to model linear unbounded air regions in which the conducting objects are free to move [70, 90–93]. The conducting regions, which in general can be non-linear and inhomogeneous, are effectively modelled by means of FEM. Similarly as before, coupled BEM-FEM technique can produce partially dense matrices. To reduce the computational costs several authors propose parallelization techniques based on a domain decomposition to BEM and FEM part [94, 95].

One of the widely applied moving grid methods to model relative displacements in general eddy current problems is the so-called sliding mesh technique (SMT) [96]. Similarly to all other methods, SMT also referred to as the moving mesh method [58] or slip surface method [97], requires two independent meshes to be defined. To provide the relative displacement, the meshes are simply slid relative to each other eliminating any need to alter their structure. The governing equations are solved independently in the fixed reference frame of each moving part, thereby avoiding the convection (velocity) terms. Depending on the mesh distribution along the introduced interface, which can be conforming or non-conforming, the field continuity can be preserved using several coupling techniques. In case of conforming meshes, the unknown potentials on each side of the sliding interface are made equal in the same way as a Dirichlet boundary conditions are imposed [58, 97–101]. However, the displacement is strictly controlled by the size of the finite elements in the motion direction and the time-

step size of the transient solver. To overcome this limitation non-conforming meshes along with Lagrange multipliers have been introduced [62]. The Lagrange multiplier approach introduces an additional set of variables on the sliding interface which ensures the continuity of the field in a weak sense [102–105]. Unfortunately, the existence of additional variables considerably deteriorates the conditioning of the stiffness matrix [105–107]. Furthermore, the matrix contains zeros on the main diagonal, which means that some standard iterative solvers, such as incomplete Cholesky conjugate gradient (ICCG) or conjugate gradient (CG), either have slow convergence or fail completely [79, 105, 107]. All these issues led to development of other coupling techniques for non-conformal meshes, such as interpolation method [80, 105] and the mortar element method [96, 106, 108–110]. Using the interpolation methods the field continuity is only globally conserved across the interface. Additionally the coupling increases the bandwidth of the resulting matrix system increasing the computational costs for the same number of unknowns [105, 111, 112]. The mortar element method results in a positive definite stiffness matrix, which makes it well suited for 3D moving eddy current problems. However, up to now this method has been only applied to solve 2D moving eddy current problems. Additional issues such as complex implementation and increase of non-zero elements in the resulting matrix system have also been reported as well [58, 107].

In order to solve electromagnetic field problems with parts in arbitrary motion (variable air-gap sizes) the so-called overlapping or composite grid methods have been developed [58, 113–116]. The method applies an iterative algorithm to couple two sets of overlapped grids. The grid of the moving part is discretized with a fine grid, whereas the fixed part is discretized with coarser grids. Due to longer simulation time and increased number of unknowns [115] the method is usually applied to 2D field problems [58]. However, some 3D implementations using nodal finite elements have been reported [115, 117].

Chapter 4

Canonical Model: The Creeping Magnet

In order to better understand the underlying physics of LET it is desirable to formulate and study highly simplified models which can be treated analytically. The definition of such a model is the main aim of the following analysis. Thus, the motion of a free-falling permanent magnet in an electrically conducting pipe, which contains an idealized defect, has been analysed (Fig. 4.1). This problem represents a slight modification of a popular educational experiment used to introduce the Faraday's law of induction, and it is a highly simplified yet enlightening version of the general LET system. The working principle is quite similar to the previously described LET setup (Fig. 2.1). Due to the motion of a magnet, eddy currents are induced inside the pipe walls. These currents produce their own secondary magnetic field $\mathbf{B}_s$ which is causing a braking Lorentz force on the magnet that is decelerating its free fall to a constant and relatively low velocity (usually in the order of several centimetres per second). The effect can be recognized by the significantly longer falling time of the magnet compared

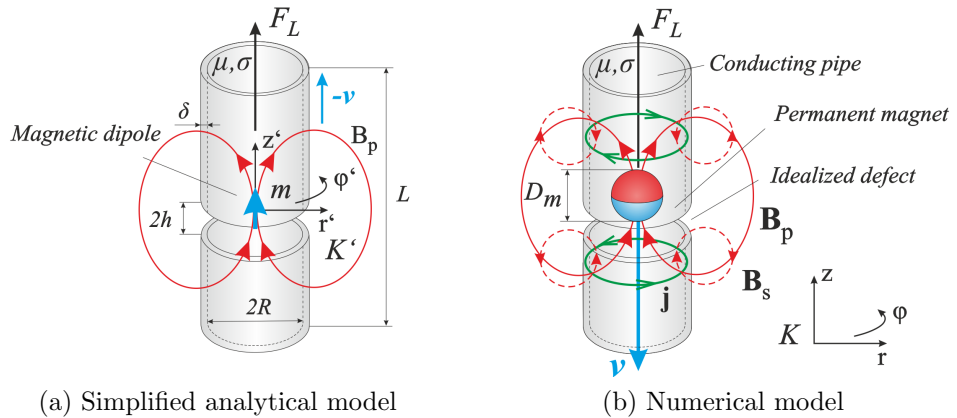


Figure 4.1: Sketch of the creeping magnet problem.

with its free fall. This is the reason why the experiment is sometimes referred to as the creeping magnet. If a highly simplified defect is introduced inside the pipe walls, e.g. an axisymmetric cut, the induced eddy currents will be weakened. The braking force will decrease and the magnet will temporarily increase its falling velocity, resulting in a shorter falling time. As it will be demonstrated below, this problem is amenable to rigorous analytic treatment and efficient numerical simulation, which provides a number of unexpected phenomena useful for further development of LET. The investigation at hand is a combination of analytical theory, numerical simulation and experimental validation.

4.1 Analytical Solution

The analytical theory consists of the derivation and solution of a non-linear ordinary differential equation for the time-dependent position $z(t)$ of the falling magnetic dipole, representing the permanent magnet (Fig. 4.1a). The conducting pipe is considered to be infinitely long with thin walls compared with its radius ($\delta \ll R$). Under this assumption the decay of the magnetic field within the pipe walls can be neglected. Due to the fact that the creeping magnet problem results in low a magnetic Reynolds number, which is estimated as $R_m = \mu_0 \sigma v \delta \sim 10^{-4}$ (see section 2.2.4), the secondary magnetic field $\mathbf{B}_s$ associated with induced eddy currents has been neglected as well.

Differential Equation of the Creeping Magnet Motion

The creeping magnet problem can be analysed in a very elegant way using the cylindrical coordinate system K' attached to the moving dipole. In this reference frame the dipole is fixed and the pipe is moving with velocity $-v(t)$ in the direction of z' -axis (Fig. 4.1a). The primary magnetic field $\mathbf{B}_p$ of a magnetic dipole is well-described and it is given by [118]

$$\mathbf{B}_p(\mathbf{r}') = \frac{\mu_0}{4\pi} \cdot \left\{ 3 \cdot \frac{(\mathbf{m} \cdot \mathbf{r}') \mathbf{r}'}{r'^5} - \frac{\mathbf{m}}{r'^3} \right\}, \quad (4.1)$$

where $\mathbf{r}'$ is the position vector and $\mathbf{m}$ is the magnetic dipole moment. Assuming the dipole orientation along the z' -axis ($\mathbf{m} = m \cdot \mathbf{e}_{z'}$) the axial symmetry of the problem allows considerable simplification of the analysis. The resulting magnetic field has only two components and it is described as

$$\mathbf{B}_p(\mathbf{r}') = m \frac{\mu_0}{4\pi} \cdot \left\{ 3 \cdot \frac{r' z'}{(r'^2 + z'^2)^{5/2}} \cdot \mathbf{e}_{r'} + \frac{2z'^2 - r'^2}{(r'^2 + z'^2)^{5/2}} \cdot \mathbf{e}_{z'} \right\}. \quad (4.2)$$

Using Ohm's law for moving conductors (2.22), eddy currents induced inside the pipe walls follow circumferential paths. The density of the induced current is given by

$$\mathbf{j} = j_{\varphi'} \mathbf{e}_{\varphi'} = -\frac{3\mu_0 m \sigma v(t)}{4\pi} \cdot \frac{r' z'}{(r'^2 + z'^2)^{5/2}}. \quad (4.3)$$

Due to axial symmetry, the resulting Lorentz force (2.2) acts only in the direction of the z' -axis and brakes the free fall of the magnet. Using (4.3) it is described as

$$F_{z'} = \frac{9\mu_0^2 m^2 \sigma v(t)}{16\pi^2} \cdot \iiint_{\Omega_p} \frac{r'^2 z'^2}{(r'^2 + z'^2)^5} d\Omega, \quad (4.4)$$

where Ω_p is the volume of the pipe. This equation can be further simplified by assuming that the pipe has thin walls ($\delta \ll R$), i.e. assuming that the magnetic field has a constant value, $\mathbf{B}_p(r' = R)$ within the pipe walls. In this case the volume integral is simplified to only a line integral over the pipe length ($L \rightarrow \infty$) and it is given by

$$F_{z'} = \frac{9\mu_0^2 m^2 \sigma v(t)}{8\pi} R^3 \delta \cdot \left[\int_{-\infty}^{\infty} \frac{z'^2}{(R + z'^2)^5} dz' - \int_{z'-h}^{z'+h} \frac{z'^2}{(R + z'^2)^5} dz' \right]. \quad (4.5)$$

The first integral in (4.5) can be solved analytically. Together with pre-factor it gives a constant braking force $F_{z'}^0$ acting on the dipole for a pipe free of defects

$$F_{z'}^0 = \frac{45\mu_0^2 m^2 \sigma \delta v}{1024R^4}. \quad (4.6)$$

When the braking force is equal to the weight ($\rho V_m g$) of the magnet (4.6) can be used to estimate its equilibrium velocity v_e

$$v_e = \frac{1024R^4 \rho V_m g}{45\mu_0^2 m^2 \sigma \delta}. \quad (4.7)$$

Here, ρ is the density of the magnet, V_m is its volume, and g represents the acceleration of gravity. The second integral describes the perturbation of the Lorentz force due to the presence of an axisymmetric defect. It has to be calculated for each relative position z' between the dipole and the defect. The total Lorentz force which brakes the free fall of the magnet is then described as

$$F_{z'} = F_{z'}^0 - \frac{9\mu_0^2 m^2 \sigma v(t)}{8\pi} R^3 \delta \cdot \int_{z'-h}^{z'+h} \frac{z'^2}{(R^2 + z'^2)^5} dz'. \quad (4.8)$$

This is a very important result since it shows that the relative Lorentz force perturbation, $F_{z'}/F_{z'}^0$, depends only on the geometry of the defect. Whereas, the force perturbation, $\Delta F_{z'} = F_{z'} - F_{z'}^0$, depends linearly on the conductivity of the pipe, its velocity and thickness (R_m), and quadratically on the strength of the applied magnetic field and magnetic permeability of the material. This effect can be used to verify numerical results of real 3D LET systems as well (see section 6.2).

The differential equation of motion, which gives the instantaneous position of the dipole $z(t)$ within the pipe, can be obtained from Newton's second law of motion in the form

$$M \ddot{z}(t) = F_{z'} + Mg, \quad (4.9)$$

where $\ddot{z} = d^2z/dt^2$ is the acceleration of the magnet. From (4.8) it can be observed that the braking Lorentz force $F_{z'}$ depends on several parameters such as: electrical conductivity, thickness and radius of the pipe, weight of the magnet and size of the defect. To reduce the number of variables (4.9) can be normalized on the basis of R and v_e/g for length and time, respectively. As a consequence, the time derivatives are calculated as $d\tau = g/v_e dt$, where τ represents the non-dimensional time variable. The resulting non-dimensional form of differential equation of motion (4.9) is given by

$$\ddot{\xi} + \left[1 - \frac{128}{5\pi} \int_{\xi-\beta}^{\xi+\beta} \frac{\xi'^2}{(1+\xi'^2)^5} d\xi' \right] \dot{\xi} = \alpha, \quad (4.10)$$

where $\xi = z/R$ is the non-dimensional position coordinate, $\dot{\xi} = d\xi/d\tau$ is the non-dimensional velocity and $\ddot{\xi} = d^2\xi/d\tau^2$ is the non-dimensional acceleration of the falling magnet. Using (4.10) the analysis of the creeping magnet problem can be performed by considering only two non-dimensional parameters, namely the forcing parameter $\alpha = v_e^2/(gR)$ and the defect size $\beta = h/R$. The derivation of this equation constitutes a key result of the analytical model. To highlight the mathematical structure of the obtained equation it can be re-written as

$$\ddot{\xi} + f(\xi, \beta) \cdot \dot{\xi} = \alpha. \quad (4.11)$$

The function $f(\xi, \beta)$ represents a position-dependent electromagnetic friction coefficient which differs from its unperturbed value ($f = 1$) only in the region of the defect ($\xi = 0$). All geometrical data and material properties are contained in α . This parameter can be interpreted as a forcing parameter since it appears on the right-hand side of (4.11). If there is no defect, this equation reduces to $\ddot{\xi} + \dot{\xi} = \alpha$ whose solution given by $\xi = \alpha\tau$ describes steady as well electromagnetically damped motion of the

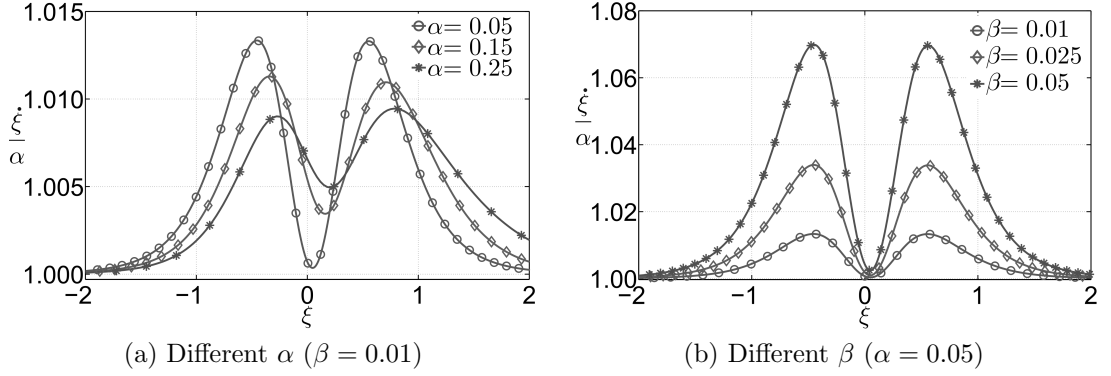


Figure 4.2: Finite difference solution of the creeping magnet problem.

magnet with constant downward velocity α . It is important to notice that α represents the ratio between the squared unperturbed velocity v_e^2 and the squared velocity gR which a freely falling body would attain after having traversed a height R in the absence of electromagnetic damping. Hence, small values of α correspond to low velocity and strong electromagnetic damping, whereas higher values of α indicate a higher velocity and weak damping, accordingly. The geometry of the defect is described by the parameter β , where $\beta = 0$ represents the pipe without defect.

The solution of the (4.11) can not be obtained analytically in a closed form. Nevertheless, for small values of β it is possible to further simplify (4.10) which is then amenable for completely analytical treatment. Due to the fact that this simplification does not contribute to any better understanding of LET, it will not be presented here. More details can be found in [119] where this analysis is referred to as the narrow defect approximation. The solution of the differential equation of motion (4.10) can be obtained very efficiently by implementing the finite difference method (FDM), e.g. using the general purpose code Matlab[®]. It is illustrative to provide the solution of (4.10) in order to highlight the role of non-dimensional parameters α and β and test their influence on the resulting velocity and Lorentz force perturbations. The results are presented in the form of $\dot{\xi} = f(\xi)$ showing the velocity of the falling magnet as a function of its temporal position. This representation is more convenient for the creeping magnet problem, than the seemingly more natural form used in LET investigations $F_z = f(\xi)$. However, the same analysis in terms of magnet velocity applies for the braking Lorentz force as well.

Fig. 4.2a shows the solution of (4.10) for different values of α when the size of defect is fixed to $\beta = 0.01$, whereas Fig. 4.2b shows the influence of β for fixed $\alpha = 0.05$. One apparently counter-intuitive feature is common to all solutions shown in Fig. 4.2. Based on qualitative reasoning it could be expected that the presence of the defect would lead to a temporary rise in velocity and that the curve $\dot{\xi} = f(\xi)$ would therefore have a

bell-shape with a single maximum velocity close to the location of the defect, i.e. close to $\xi = 0$. By contrast, all curves shown in Fig. 4.2 have two maxima rather than one. This indicates that the falling magnet experiences two phases of acceleration when passing the defect. The reason for this behaviour can be readily understood by invoking Fig. 4.4c in which the qualitative structure of the eddy currents is shown. It can be seen that the eddy currents induced by the moving magnet consist of two structures with opposite orientation. This leads to the fact that the Lorentz force has two minima rather than just one and that $\dot{\xi} = f(\xi)$ has two maxima as long as the defect is not bigger than the size of the falling magnet ($2h < D_m$). The shape of $\dot{\xi} = f(\xi)$ is symmetric for small values of the forcing parameter. When α increases, the velocity distribution becomes asymmetric due to the increasing influence of inertia.

Larger defects cause an asymmetry in velocity distribution as well (Fig. 4.2a). Here, the acceleration phase is dominating when the magnet is experiencing a decrease of the braking Lorentz force due to the presence of a defect. More important to notice is a very strong influence of the defect size β on the velocity, i.e. Lorentz force perturbation. This is a promising result, which can be used to reconstruct the shape of the defect by solving an inverse LET problem.

The resulting trajectory of the dipole is shown in Fig. 4.3. When comparing it with the unperturbed trajectory represented by the solid line two features become apparent. In a given time the magnetic dipole travels a distance that is by $\Delta\xi$ longer than in the unperturbed case. Observed, from a different perspective, the dipole arrives by $\Delta\tau$ earlier at any position that is sufficiently far downstream of the defect. Here $\Delta\tau$ is referred to as the falling time difference and $\Delta\xi$ as the trajectory expansion with respect to defects free pipe.

Since $\Delta\tau$ can be measured, it is a particularly important quantity. It can be shown analytically [119] that $\Delta\tau$ increases linearly with the defect width 2β , i.e. $\Delta\xi$ and $\Delta\tau$

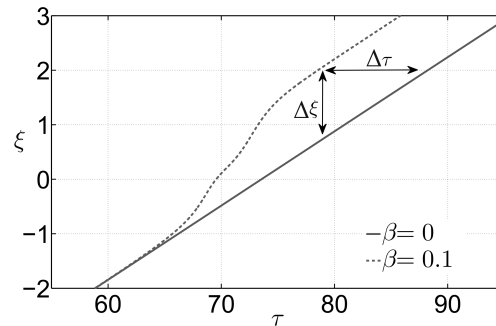


Figure 4.3: Path of the dipole for the pipe with and without a defect
 $\Delta\tau$ - time shift, $\Delta\xi$ - trajectory expansion ($\alpha = 0.05$).

obey rigorous relations given by

$$\Delta\xi = 2\beta \text{ and } \Delta\tau = -\frac{2\beta}{\alpha} \quad (4.12)$$

In principle, if there is only one defect within the pipe its size can be determined using (4.12) simply by time measurements. In order to localise and identify the structure of more than one defect, a time resolved force signal acting on the magnet (or the pipe mounting) needs to be measured. This actually is the basic idea of LET.

4.2 Numerical Solution

The analytical model developed so far is suitable for fast determination of the falling time and the characteristic Lorentz force profiles. It was derived using several simplifications which lead to lower accuracy than a full 3D electromagnetic field computation would provide. The numerical simulation allows to overcome three limitations inherent in the analytical theory and to take into account (i) the finite size and shape of the falling magnet, (ii) the finite thickness of the pipe and (iii) the finite value of the magnetic Reynolds number.

The analysis has been divided into two parts. The first part comprises the consideration of a non-defective pipe. As a result the falling time of the magnet through the pipe is determined. The calculated falling time is used for the verification and validation of the numerical simulations. In the second part an ideal defect is introduced into the pipe walls. The main aim of this analysis is to estimate the falling time difference $\Delta\tau$ with respect to the non-defective pipe. This can help to prove the linear relation between $\Delta\tau$ and β (4.12).

Pipe Without Defects

The geometry used for the numerical simulation of the creeping magnet problem is shown in Fig. 4.1b. The magnetic dipole is replaced with a permanent magnet of a spherical shape, and the conducting pipe is modelled as a hollow cylinder with finite wall thickness δ . If the magnet moves parallel to the z -axis, the induced eddy currents flow in the azimuthal direction (Fig. 4.1b). Thus, a 2D axisymmetric model can be applied (Fig. 4.4a).

As in the presented analytical model the moving reference frame K' associated with the falling magnet has been applied. In this reference frame the magnet is fixed, and the pipe is moving with velocity $-v$, in the z' -axis direction. The governing equation is formulated as the diffusion-convection equation (2.31), which includes an additional

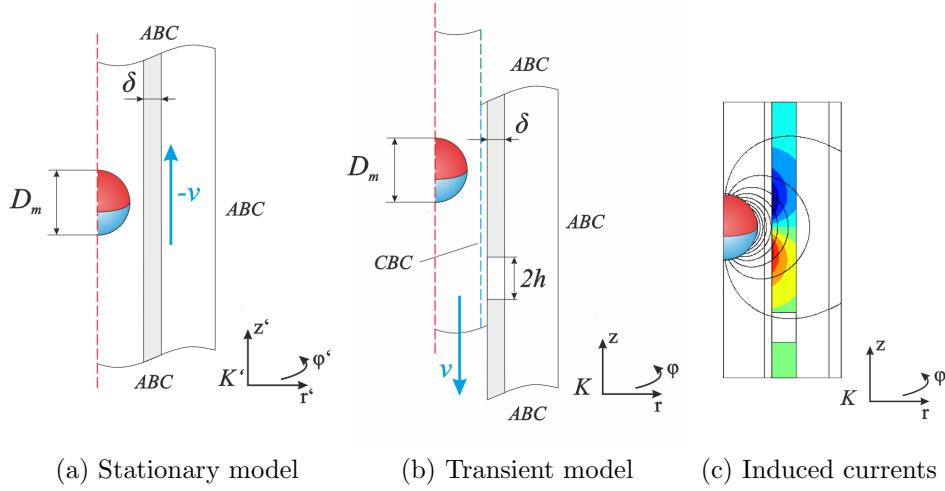


Figure 4.4: Numerical solution of the creeping magnet problem.

velocity term. Since there is no defect within the pipe and its length is assumed to be infinite, a stationary form of (2.31) can be applied ($\partial \mathbf{A} / \partial t = \mathbf{0}$). In order to account for the infinite long pipe, the asymptotic boundary conditions (ABC) [120] have been applied on the outer model boundaries (Fig. 4.4a). This, considerably reduces the computational expenses due to smaller size of the resulting air domain.

The computational algorithm is performed in two steps. Initially, for the zero braking force ($F_{z'} = 0$) the velocity of the magnet is calculated using (4.9) and FDM implemented in Matlab[®]. In the second step, the resulting braking force $F_{z'}$ is determined using FEM and Comsol Multiphysics[®]. In this step, since there is no defect in the pipe, the stationary form of (2.31) is used to calculate the induced currents and magnetic field distribution, whereas the braking force is determined by (2.2). This process is repeated until the equilibrium velocity of the magnet v_e is reached. As a final result, the falling time of the magnet is calculated using

$$T = T_e + \frac{(L - L_e)}{v_e} \quad (4.13)$$

where T_e is the time needed to reach the equilibrium state, and L_e is the travelled distance during the time T_e .

In order to validate both, the proposed numerical procedure and the analytical prediction of the equilibrium velocity (4.7) an experiment has been performed [15, 119]. Hence, two different spherical permanent magnets (NdFeB) have been dropped through a non-defective copper pipe and its falling time has been measured with a stop watch. All necessary geometrical and material properties are given in Table 4.1. The obtained results are summarized in Table 4.2. The relative errors are calculated using $e_{A,N} = 100 \cdot$

Table 4.1: Geometrical and material properties

Parameter	Value
Pipe radius R	8 mm
Pipe length L	1 m
Wall thickness δ	1 mm
Conductivity σ	44.5 MS/m
Magnet diameter D_m	15 mm
Magnet density ρ	7588.85 Kg/m ³
Magnetization density M	$0.9054 \cdot 10^6$ A/m

Table 4.2: Experimental validation [15]

D_m	t_{EXP}	t_A	t_N	α_{EXP}	α_A	α_N	e_A	e_N
15 mm	11.966 ± 0.092 s	14.690 s	11.663 s	0.095	0.063	0.099	22.7%	2.5%
10 mm	3.426 ± 0.092 s	4.353 s	3.497 s	1.73	1.014	1.67	27.1%	1.5%

$(t_{EXP} - t_{A,N})/t_{EXP}$, where A and N denote for the analytical and numerical estimation of the falling time, respectively. As expected, the analytical model overestimates the falling time for both used magnets. This is because the decay of the magnetic field within the pipe walls has been neglected which resulted in higher braking Lorentz force. The numerical solution properly takes into account the field decay, thus it agrees better with experiments.

Pipe Containing an Idealized Defect

The main aim of the following analysis is to verify and to validate used numerical procedure additionally proving the linear dependency between the falling time difference $\Delta\tau$ and the defect size 2β (4.12). Furthermore, various effects of the model simplifications assumed in the analytical model are discussed and quantified as well.

The entire dynamic behaviour of the velocity and the Lorentz force acting on the creeping magnet has been analysed using an efficient 2D-axisymmetric transient FEM model. The cylindrical coordinate system K is fixed to the pipe and the magnet is

Table 4.3: Experimental validation for a pipe with an ideal defect [15]

2β	t_{EXP}	e_A	e_N
$R_{PM} = 7.5mm$			
2.66	10.23 ± 0.08 s	22.45%	0.19%
5.33	10.20 ± 0.09 s	22.89%	0.14%
8	10.15 ± 0.11 s	23.45%	0.57%
10.66	10.00 ± 0.11 s	25.30%	2.06%
16	10.03 ± 0.07 s	24.95%	1.74%
$R_{PM} = 5mm$			
2.66	2.81 ± 0.06 s	32.73%	1.62%
5.33	2.79 ± 0.06 s	33.96%	0.73%
8	2.82 ± 0.06 s	32.42%	1.89%
10.66	2.80 ± 0.06 s	33.57%	1.08%
16	2.74 ± 0.06 s	36.15%	0.76%

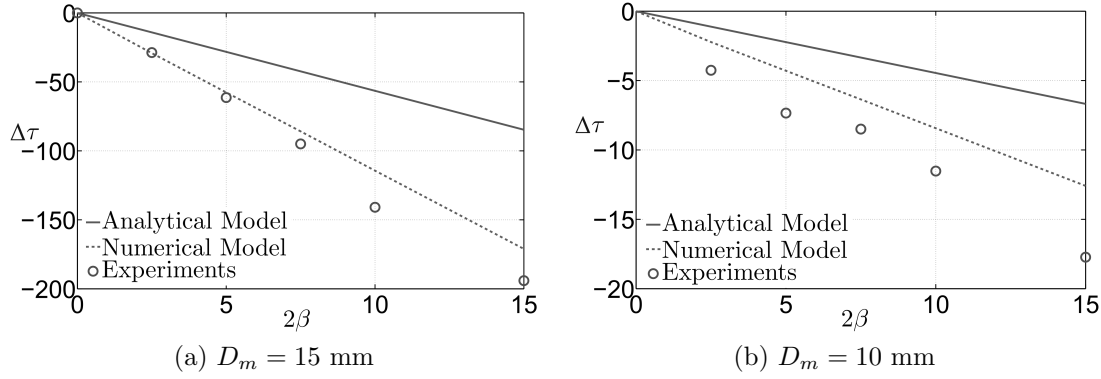


Figure 4.5: Validation of the analytical and numerical models

moving with velocity v opposite to the z -axis (Fig. 4.4b). The governing equation is given by (2.28), where $\mathbf{j}_e = 0$, since there are no external current excitations present. In order to provide relative motion between the magnet and the pipe the sliding mesh technique (SMT) is used (see chapter 3). Here, SMT is adapted for modelling linear displacement according to the given creeping magnet problem. The computational domain has been divided into two independent parts, namely the stationary and the moving part (Fig. 4.4b). The moving part comprises the magnet and the part of the surrounding air region, whereas the stationary part comprises the pipe with an idealised defect and the rest of the air region. The coupling interface has been placed in air between the magnet and the pipe. To assure the continuity of the magnetic field across the interface, the continuity boundary conditions (2.39) and (2.40) along with the Lagrange multipliers have been used. Similarly, as in the case of a non-defective pipe, the asymptotic boundary condition (ABC) are applied on the outer boundaries of the stationary part. Initially, the magnet was placed far enough from the defect so that the equilibrium velocity v_e can be reached before the defect region. After passing the defect the equilibrium velocity has been reached again so that the falling time of the magnet can be calculated according to (4.13).

In order to validate the proposed models, both the analytical and the numerical one, experiments have been performed additionally [15, 119]. For given defect sizes the falling time has been measured with a stop watch. The obtained results are summarized in Table 4.3 (Fig. 4.5). All results show linear dependency between the falling time difference $\Delta\tau$ and the defect size 2β . The slope of the graph obtained analytically differs from the ones obtained numerically and experimentally due to the difference in the forcing parameter α , which depends on the resulting equilibrium velocity (see Table 4.2). The numerical results agree better with experiments for both magnet sizes (Table 4.3). The estimation of the discrepancy is given in Table 4.3.

It should be mentioned that the refinement of the experiment to a more precise

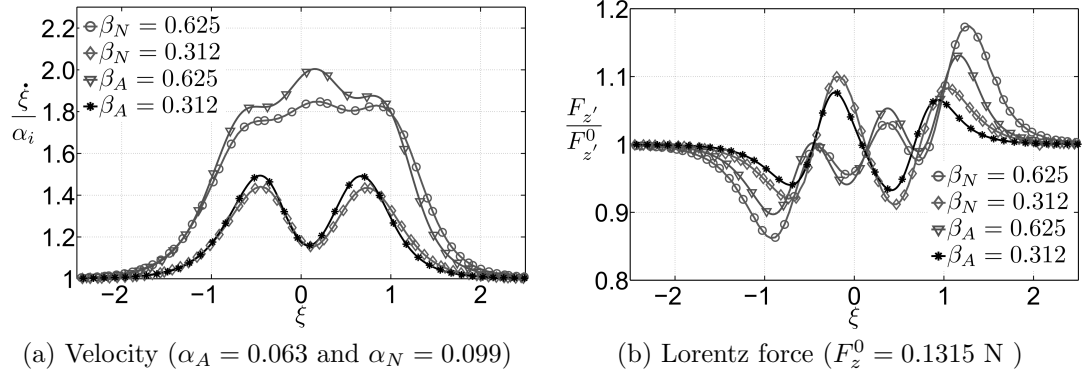


Figure 4.6: Comparison of the analytical (A) and numerical (N) models in terms of velocity and Lorentz force perturbation ($D_m = 15$ mm, $\delta = 1$ mm)

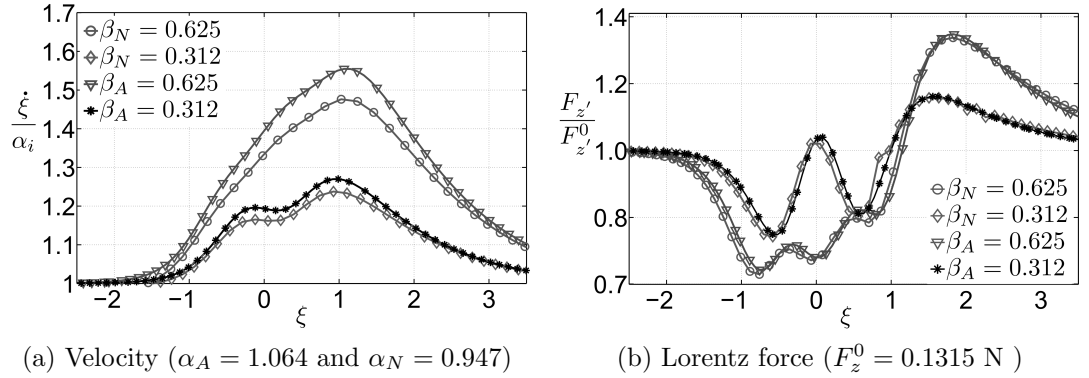


Figure 4.7: Comparison of the analytical (A) and numerical (N) models in terms of velocity and Lorentz force perturbation ($D_m = 15$ mm, $\delta = 0.25$ mm)

time measurement system is necessary, since uncertainties of time measurements using a stop-watch are quite high compared to the falling time changes caused by defects. Additionally, when passing by a defect and entering the pipe region again, the magnet hits the edge of the pipe which affects its falling time [15, 119]. Therefore, a better guidance for the magnet in the defect region needs to be provided as well. On the other hand, even though a simplified experiment has been applied, a linear dependency can still be observed which proves the predictions given by (4.12).

It is illustrative to show the effect of the defect size 2β on the characteristic velocity profiles and the resulting Lorentz force (Fig. 4.6). The results have been presented in form of $\dot{\xi}/\alpha_i = f(\xi)$ and $F_z/F_z^0 = f(\xi)$, where index $i \in \{A, N\}$ denotes for the analytically and numerically obtained forcing parameter. In spite of considerable simplification assumed in the analytical model, the obtained results qualitatively agree well. On the other hand the absolute values, in terms of the forcing parameter α_i do not match. The relative errors, compared to the experimental results are presented in Table 4.3. In order to better understand this error, and find out which of the assumed simplifications mostly influence the obtained results, the analysis has been performed

by reducing the thickness of the pipe to $\delta = 0.25mm$. The obtained results are shown in Fig. 4.7. A considerable improvement of results can be observed. Moreover, the value of the forcing parameter obtained analytically is closer to the one obtained numerically ($\alpha_A = 1.064$ and $\alpha_N = 0.947$). These results are expected, since other assumed simplifications in terms of the low R_m approximation and magnetic dipole representation of the spherical magnet do not influence the accuracy of results.

Although the presented creeping magnet model has no direct practical application, it helps in elucidating the basic laws of LET. Using (4.8) it has been shown that the relative Lorentz force perturbation caused by an idealized defect depends only on its geometry. This is an important feature of LET which can be used in order to localise and identify different types of defects. Furthermore, the linear dependency between the falling time difference $\Delta\tau$ and the defect size 2β was proved. This effect can be used in order to characterise the defect size as well.

Chapter 5

Numerical Modelling of Lorentz Force Eddy Current Testing

The main aim of this chapter is to introduce new FEM-based methodology, which can be used to analyse and develop future LET systems. The particular emphasis is placed on the reduction of the overall computational requirements and the total simulation time, while maintaining the accuracy of the solution. Additional goals include development of simplified numerical models which enable fast 2D/3D LET analysis in conjunction with the validation of assumed simplifications (sections 5.3 and 5.4). For comparison and validation of different approaches a benchmark problem which represents a typical LET configuration has been considered (section 5.1).

5.1 Benchmark Problem Definition

In this section a typical LET benchmark problem representing a generic conductor with pre-defined artificial defects, moving across the static magnetic field is described (Fig. 5.1). The conductor under test is considered to be non-magnetic with the conductivity denoted by σ and permeability equal to the permeability of vacuum $\mu = \mu_0$. It has a rectangular cross-section determined by its width W_c and height H_c , whereas its length is denoted by L_c . A cylindrical permanent magnet described by magnetization $\mathbf{M}$ is used as a source of the static (primary) magnetic field. The diameter and the height of the magnet are denoted by D_m and H_m , respectively. The magnet is placed centrally above the conductor under test ($\delta y = 0$) at a lift-off distance δz . For the analysis, three different types of artificial defects, namely long ("|"), wide ("—") and cross ("+") have been considered. Defects are placed centrally within the conductor at depth d below its surface. They are characterised by their width w , height h and length l . The conductivity of the defect is denoted by σ_d .

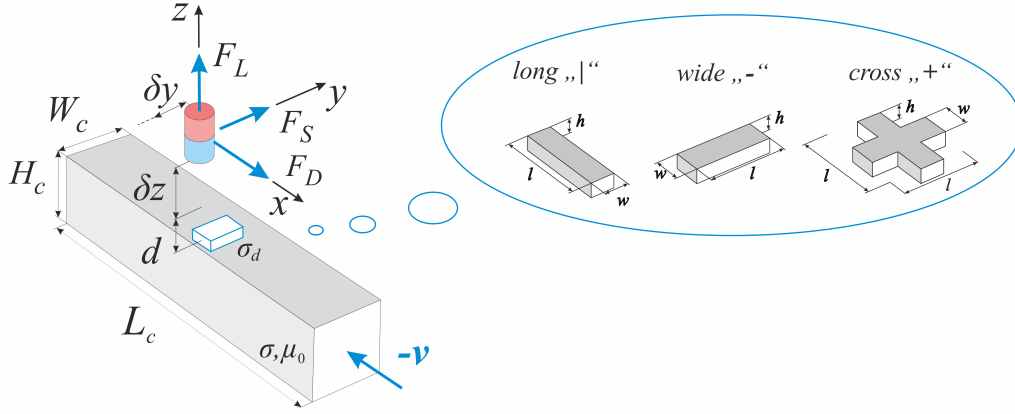


Figure 5.1: Definition of the LET benchmark problem. The conductor contains three types of defects: long ("|"), wide ("—") and cross ("+").

In order to reduce the number of dependent variables, the magnetic Reynolds number (R_m) has been used for the analysis (2.33). It involves the characteristic length-scale parameter L whose definition depends on the particular problem at hand and characterises the conductor in motion (see section 2.2.4). For the given benchmark problem (Fig. 5.1), there are several possibilities to define L . In fluid dynamics, in order to measure the efficiency of different channel flows, all channel configurations are usually approximated with an equivalent circular pipe having the equivalent diameter D_e defined as

$$D_e = \frac{2A}{P} = \frac{2W_c \cdot H_c}{W_c + H_c}, \quad (5.1)$$

where A is the cross-section area of the conductor, and P is its perimeter [32]. Following the same principle and choosing the radius of an equivalent cylinder ($D_e/2$) as the characteristic length-scale for the given LET benchmark problem, the value of R_m has been defined as follows

$$R_m = \mu_0 \sigma |\mathbf{v}| \frac{W_c \cdot H_c}{W_c + H_c}. \quad (5.2)$$

5.2 Logical Expressions Approach

In this section the logical expressions approach (LEA) that allows fast computations of 2D/3D eddy current problems including parts in relative motion is presented [121]. Using the proposed methodology, the spatial coordinates of moving parts, either conducting or non-conducting, are modelled on a fixed computational grid using logical expressions (LE). By applying the principles of Boolean algebra directly in finite element analysis (FEA) the shape of moving parts is determined on the fly by calculating

Table 5.1: Logical expressions for basic geometric primitives

Shape	Logical expression (LE)
Box (B)	$[X_{start} + vt \leq x \leq X_{start} + L_B + vt] ? 1 : 0$
Cylinder (C)	$[(x + X_{start} + vt)^2 + y^2 \leq R_C^2] ? 1 : 0$
Sphere (S)	$[(x + X_{start} + vt)^2 + y^2 + z^2 \leq R_S^2] ? 1 : 0$

Note: x, y, z – Cartesian coordinate system, X_{start} – starting position of the moving part, v – prescribed velocity of the displacement, t – actual simulation time.

constraints given by LE and filtering the finite elements in those domains where LE are introduced. Fig. 5.2 shows three basic geometrical primitives, i.e. box, cylinder, and sphere which are modelled using LE.

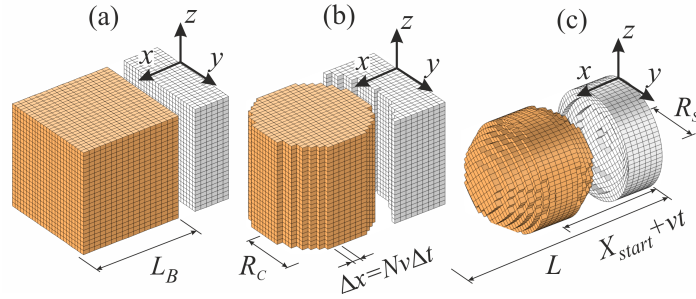


Figure 5.2: Basic geometric primitives defined by logical expressions: (a) box (b) cylinder (c) sphere.

Independent to the type of reference frame used in LET analysis, modelling of moving parts using LE requires the existence of a homogeneous zone in which these expressions are applied. This zone is referred to as the moving domain and it is determined by the shape of the moving part and its relative displacement L (Fig. 5.2). In order to introduce the motion and determine the shape of moving parts, the constraints given by LE are defined as time dependent. This step represents the basic idea of the logical expression approach [121]. Table 5.1 presents some sample logical expressions for basic primitive shapes moving with velocity v along the model x -axis (Fig. 5.2). Logical constructions "[condition]?1 : 0" in Table 5.1 should be interpreted as: if the *condition* in square brackets is valid then take 1. Otherwise if the condition is false take 0.

The implementation of LEA in the existing finite element codes can be performed easily by multiplying material properties assigned to the moving domain with the appropriate LE. This modification does not require any major changes in the FEM code and it can be applied in any commercially available FEM software. To assign material properties to the moving domain not occupied by the moving part, a term multiplying these properties with the negation of the pre-defined logical expression, i.e. by $1 - LE$ should be added as well.

As a summary, any implementation of LEA requires the following modelling steps:

- (i) creation of the moving domain
- (ii) definition of the corresponding LE
- (iii) modification of material properties of the moving domain by combining the properties of the part under motion and the remaining region with appropriate logical expressions. This, the corresponding material properties have to be multiplied by LE_i or $1 - LE_i$, respectively.

During the discretization of the moving domain, the element size in the moving direction Δx has to satisfy the following constraint:

$$N\Delta x = v\Delta t \quad (5.3)$$

where Δt represents the size of the time step used during the solution process, and N is any integer number greater than zero. Then the outer surface of the moving part is forced to stay unchanged during its motion. Otherwise, its volume can fluctuate in subsequent time steps introducing additional errors that are observed as non-physical Lorentz force oscillations.

One of the obvious disadvantages of the proposed LEA is its limitation to the relatively simple shapes of the moving objects (Fig. 5.2). More complex shaped moving parts can be modelled by combining several moving domains and repeating the steps from (i) to (iii). Additionally, to model curved geometries using LEA the finite element mesh in the moving region should be relatively fine.

In order to test the proposed LEA, LET benchmark problem (Fig. 5.1) been considered. Depending on the definition of the frame of reference, the LET analysis has been performed using two different implementations of LEA, (i) in case of fixed frame of reference, the logical expressions are used to model the motion of the permanent magnet, (ii) in case of moving frame reference, the logical expressions are used to model the relative motion of the defect. These two specific LEA implementations are referred to as the moving magnet approach (MMA) and the moving defect approach (MDA), respectively.

5.2.1 Moving Magnet Approach

In the implementation of the moving magnet approach (MMA), the global coordinate system is associated with the conducting object (fixed frame of reference) and the logical expressions are used to describe motion of the used cylindrical permanent magnet (Fig. 5.3). The magnet is moving with constant velocity v along the model

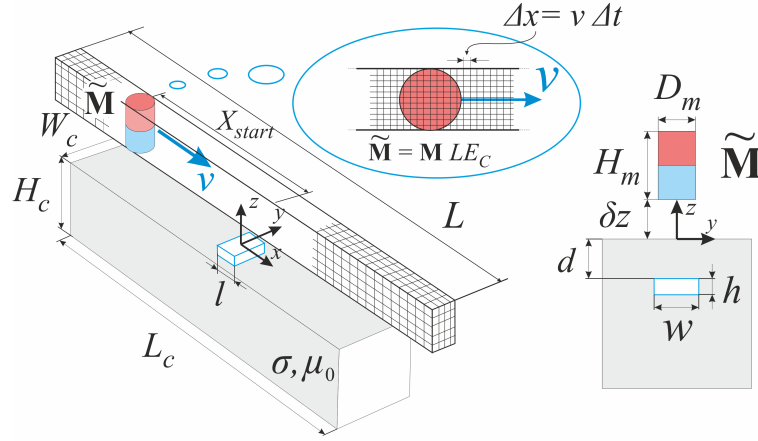


Figure 5.3: Implementation of the moving magnet approach (MMA). The moving domain is defined in the air region.

x -axis in close vicinity of the conductor containing an artificial defect below its surface. Using steps (i)-(iii) which describe the implementation of the LEA, the moving domain is defined in the surrounding air region (Fig. 5.3). The cross section of the moving domain is determined by the height H_m and the diameter D_m of the magnet, while its length depends on the starting position X_{start} and its relative displacement L (Fig. 5.3). In general, the starting position X_{start} is the distance of the moving object to the origin of the coordinate system at $t = 0$. It has to be large enough to avoid any influence of the used initial conditions on the resulting Lorentz force perturbations.

To model the motion of the magnet in the moving domain, the magnetisation $\mathbf{M}$ assigned to the moving domain has been modified as $\tilde{\mathbf{M}} = \mathbf{M} LE_C$, where LE_C is the logical expression for a cylinder (Table 5.1). When defining the particular logical expression (LE), it is important to keep the equivalent volume of the cylinder V_{LE_C} obtained by LE and the volume of the real cylinder given by $V_{cyl} = \pi(D_m/2)^2 H_m$ the same ($V_{LE_C} = V_{cyl}$). This can be achieved, e.g. by changing the diameter of the magnet D_m . In the given LET configuration, the relative velocity between the magnet and the conductor under test is constant. Thus, the size of the finite element mesh in the moving direction Δx is uniform (5.3).

Applying the optimal $\mathbf{A}^* - \psi$ formulation, the governing system of equations in MMA is given by (2.35) and (2.37). According to section 3.1 this requires both nodal and edge finite element formulations in single computational domain. The nodal FEM formulation approximating the magnetic scalar potential ψ is given by (3.5), whereas the corresponding edge formulation for the magnetic vector potential $\mathbf{A}^*$ is given by (3.7). Due to the fact that (3.7) does not involve the additional velocity term the resulting system of equations remains symmetric. Another important feature of MMA is that the modification of the magnetization vector $\mathbf{M}$ by time dependent LE contributes

as a source only to the right-hand side in the resulting FEM weak formulation (see section 3.1.2). Therefore, in MMA the resulting stiffness matrix has to be assembled only once during the entire motion of the magnet which reduces the total computational time considerably.

5.2.2 Moving Defect Approach

In the moving defect approach (MDA) the global coordinate system is assigned to the permanent magnet (moving frame of reference). In this reference frame the magnet is stationary and the conductor is moving in opposite direction with velocity $-v$ along the x -axis (Fig. 5.4). For the given LET problem, the use of LE to describe the motion of the whole conducting domain would be computationally very expensive. However, the implementation of LEA in the moving reference frame can be considerably simplified if only perturbations of Lorentz force caused by defects are required, which is in fact a typical LET problem. Thus, it is sufficient to model only the movement of the defect relatively to the magnet, instead of modelling the motion of the whole conductor (Fig. 5.4).

In MDA the moving domain is defined entirely inside the conductor where LE are used to model the motion of the particular defect [121]. The shape and position of the moving domain is defined by the cross-section of the defect $w \times h$, its relative displacement L , and depth d (Fig. 5.4). In order to simulate the defect motion, the electrical conductivity assigned to the moving domain is modified as $\tilde{\sigma} = \sigma(1 - LE_B) + \sigma_d LE_B$, where LE_B is the logical expression given in Table 5.1, and σ_d is the electrical conductivity of the defect. Due to the constant velocity, the size of the finite element mesh in the moving direction Δx is uniform (5.3).

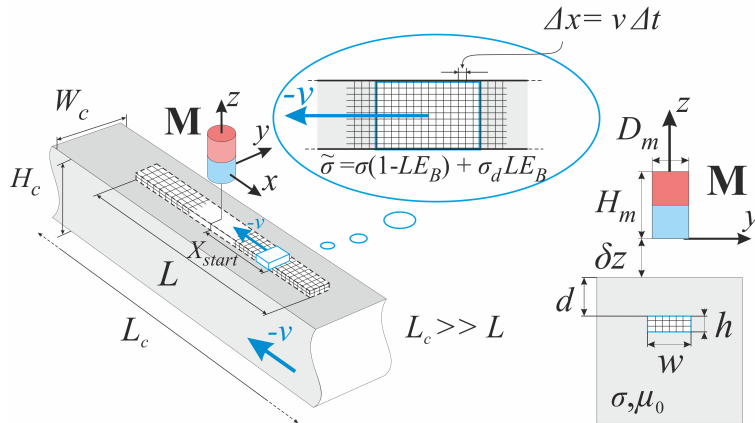


Figure 5.4: Implementation of the moving defect approach (MDA). The moving domain is defined inside the conductor.

Similarly, to the previous MMA applying the optimal $\mathbf{A}^* - \psi$ formulation in MDA results in the system of governing equations given by (2.35) and (2.38). According to section 3.1 this requires both nodal and edge finite element formulations to be applied in the computational domain as well. The nodal FEM formulation approximating for the magnetic scalar potential ψ is given by (3.5), whereas the corresponding edge formulation for the magnetic vector potential $\mathbf{A}^*$ is given by (3.8). In contrast to MMA (3.7) the formulation used in MDA (3.8) involves the additional velocity term which makes the resulting system of equations non-symmetric. Additionally, the modification of the electrical conductivity by time dependent LE, introduced by MDA, modifies the resulting stiffness matrix as well. Therefore, the stiffness matrix has to be re-assembled at every time step which increases the computational time compared to MMA.

5.3 Quasi-Static Approach

The moving magnet approach (MMA) and the moving defect approach (MDA) assume no simplifications for the given LET analysis. They offer accurate results for any relative testing velocity $\mathbf{v}$ between the magnet system and the conductor, and for any material and geometry parameters involved. Thus, they are valid for finite values of the magnetic Reynolds number (5.2), whether the conductor contains material defects or not.

Conductor Free of Defects

If LET configuration under investigation is time independent, i.e. it involves uniformly moving conductors with a constant cross-section normal to the direction of motion (conductors free of defects) the analysis can be considerably simplified [37]. In principle, only one stationary can be performed to obtain an accurate steady state solution, e.g. the Lorentz force acting on the magnet system. This assumption requires the moving frame of reference (2.31), where the additional velocity term ($\mathbf{v} \times \nabla \times \mathbf{A}$) is used as a source of the induced eddy currents inside the conductor in uniform motion

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A} - \mathbf{M} \right) = -\sigma (\nabla V - \mathbf{v} \times \nabla \times \mathbf{A}) + \mathbf{j}_e, \quad (5.4)$$

$$\nabla \cdot (\sigma \nabla V - \mathbf{v} \times \nabla \times \mathbf{A}) = 0. \quad (5.5)$$

The second equation results from the current conservation law (2.7) and it is an additional equation for the electric scalar potential V . This system of equations takes the deformation of the magnetic field lines correctly into account making it valid for any

value of the R_m .

Conductor with Defects

Although (5.4) and (5.5) provide fully correct solutions only for conductors without any material defects, their use can be still extended to NDT applications. In [66], it has been shown that for LET systems resulting in small R_m , they can be used for fast Lorentz force calculations on the moving magnet even for conductors with defects. Here, this method is referred to as a quasi-static approach (QSA). As a direct consequence of low magnetic Reynolds numbers ($R_m < 1$), the diffusion time of the magnetic field into the conducting object, estimated as $\tau \sim R_m \cdot L/v$, where L is the characteristic length-scale of the conductor (5.1), is small as well [24, 122]. This basically justifies the instantaneous field reaction ($\partial \mathbf{B}/\partial t \rightarrow \mathbf{0}$) to any perturbation of induced currents, which is assumed in QSA. Nevertheless, if this is not the case, the full transient form of (5.4) and (5.5) has to be considered, which in fact represents the governing equation of already presented moving defect approach (MDA).

In the implementation of QSA, only the change in relative position between the magnet and the defect must be provided. This is done either by moving the magnet system relative to the defect, or vice-versa [66]. In any case, this requires a time consuming re-meshing procedure of the entire model geometry for each new configuration. The re-meshing of the geometry can be avoided if the basic principle of the logical expression approach (LEA) is combined with the quasi-static formulation given by (5.4) and (5.5). In this LEA implementation the time variable used in different LE is just a parameter which needs to be changed from one stationary solution to another providing the displacement of the moving part (magnet or defect). Basically, this means that the same geometry, used for the implementation of the LEA, i.e. MMA and MDA (Fig. 5.2 and Fig. 5.3), can be used for implementation of QSA as well. The only difference introduced by QSA is in the governing equation in the conducting region, which is now in its stationary form and contains an additional scalar potential V . In regions free of eddy currents (surrounding air region and permanent magnet), the magnetic scalar potential formulation ψ is used (section 2.3).

5.4 Weak Reaction Approach

If the general assumption is made, that the secondary magnetic field $\mathbf{B}_s$ associated with induced currents $\mathbf{j}$ is low, compared to the primary magnetic field ($\mathbf{B}_s \ll \mathbf{B}_p$) the QSA formulation (5.4) and (5.5) can be even further simplified. By neglecting the reaction field ($\mathbf{B}_s = \mathbf{0}$) the curl-free property of the total magnetic field $\mathbf{B}$ is preserved

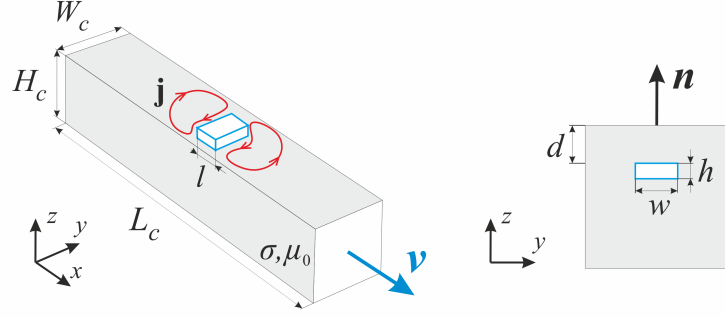


Figure 5.5: Implementation of the weak reaction approach (WRA). Only conducting region needs to be considered.

within the conductor region as well

$$\nabla \times \mathbf{B} = \nabla \times (\mathbf{B}_p + \mathbf{B}_s) = \nabla \times \mathbf{B}_p = \mathbf{0}. \quad (5.6)$$

In terms of the magnetic Reynolds number R_m (section 2.2) the constraint given by (5.6) is fulfilled when $R_m \ll 1$. In this case the electric potential V and induced eddy currents are described by the Ohm's law for moving conductors (2.22) which results in

$$\mathbf{j} = \sigma(-\nabla V + \mathbf{v} \times \mathbf{B}_p), \quad (5.7)$$

Applying the current conservation law $\nabla \cdot \mathbf{j} = 0$ the following Laplace's equation for electric scalar potential is obtained

$$\nabla^2 V = \nabla \cdot (\mathbf{v} \times \mathbf{B}_p) = \mathbf{B}_p \cdot (\nabla \times \mathbf{v}) - \mathbf{v} \cdot (\nabla \times \mathbf{B}_p) = 0. \quad (5.8)$$

The specific value of V is defined by the boundary conditions for the given problem. They are obtained by making the normal component of the induced current density on the walls of the conductor to be zero (Fig. 5.5) [67]

$$\mathbf{n} \cdot \mathbf{j} = 0, \quad (5.9)$$

If the conductor contains a defect, this condition has to be imposed on its walls as well. The Laplace's equation (5.8) is actually the governing equation of LET systems when the reaction magnetic field $\mathbf{B}_s$ of the induced eddy currents is neglected. In fact, this represents the so-called weak reaction approach (WRA). It should be mentioned that WRA is an extension of the method developed in [25, 67, 123] to moving conductors with artificial defects.

In the numerical implementation of the WRA there are two possibilities to prescribe the primary magnetic field $\mathbf{B}_p$. If the used permanent magnets have simple shape, like of a sphere or a rectangle, $\mathbf{B}_p$ can be obtained in the closed form analytically [124]. For a spherical magnet with radius $\mathbf{r}_m$, the magnetic field outside the magnet region ($\mathbf{r} > \mathbf{r}_m$) is described by magnetic point dipole formula (4.1). The magnetic moment $\mathbf{m}$ of an equivalent dipole is given by $\mathbf{m} = \mathbf{M} \cdot V_m$, where $\mathbf{M}$ is the magnetization density of used spherical magnet, and V_m its volume [118, 124]. In case of a rectangular magnet, the expression for $\mathbf{B}_p$ can be found in [124]. For complex magnet systems, the primary magnetic field distribution inside the conductor can be calculated numerically, e.g. by using the magnetic scalar potential ψ (2.35). However, this requires time taking interpolation of the field on the finite element mesh of a conductor.

As in QSA, the relative displacement between the conductor and the magnet can be performed in two ways. In case, when the magnetic field is described analytically it is most simple to move the magnet from one stationary solution to another and leave the conductor at a fixed position. When the magnetic field can only be obtained numerically, than it is most efficient to change the position of the conductor instead. In this case the field distribution $\mathbf{B}_p$ has to be calculated only once whose solution is than interpolated for each new position of the conductor. This however, results in longer simulation time compared to the previous case where $\mathbf{B}_p$ can be described analytically in the closed form.

The main benefit of WRA compared to QSA is that instead of both, the magnetic vector potential $\mathbf{A}$ and V , the solution of LET system involves only V as the unknown quantity (5.8). This is a considerable reduction of the total number of degrees of freedom (DoF), as the number of equations to be solved is reduced from 4 to 1. Furthermore, due to the absence of the reaction field $\mathbf{B}_s$ it is sufficient to model only the region of the conductor (Fig. 5.5).

5.5 Sliding Mesh Technique

The sliding mesh technique (SMT) is one of the widely applied FEM techniques for modelling relative motion in general eddy current testing problems. Similarly to all other moving grid methods, SMT (also referred to as the moving mesh method [58] or slip surface method [97]) requires the definition of two independent finite element meshes. In order to account for motion the meshes are simply slid relative to each other eliminating any need to alter their structure. However, this yields some kind of coupling procedure providing the continuity of the resulting magnetic field across the interface where the meshes overlap. In the presented literature review (see section

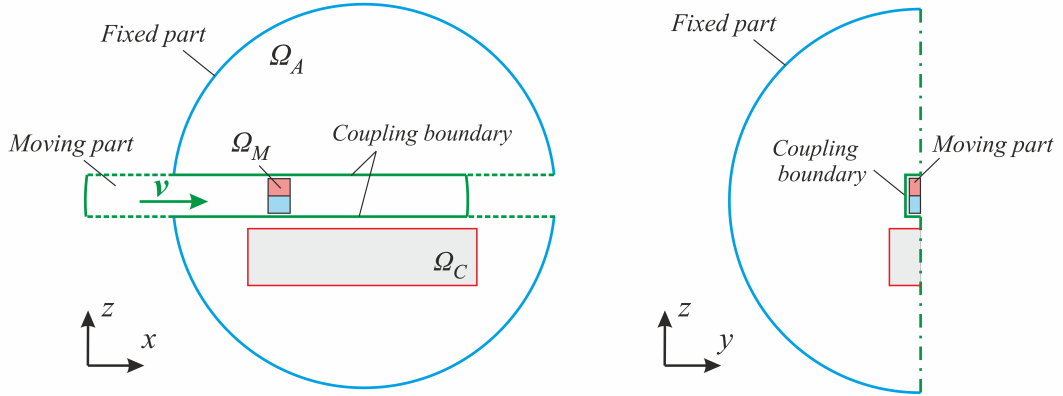
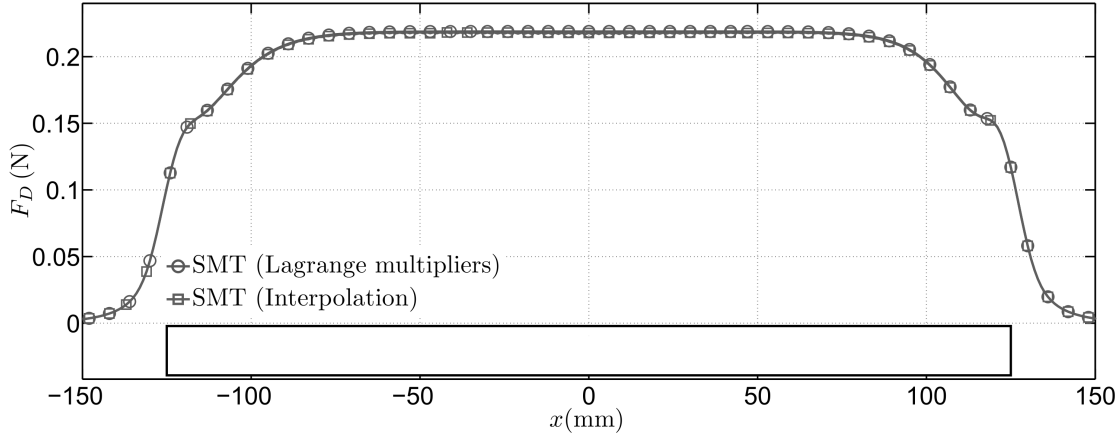


Figure 5.6: Implementation of SMT. In the assumed fixed frame of reference (K) the small air region including the magnet has been moved (Moving part). The rest of the computational domain was fixed (Fixed part).

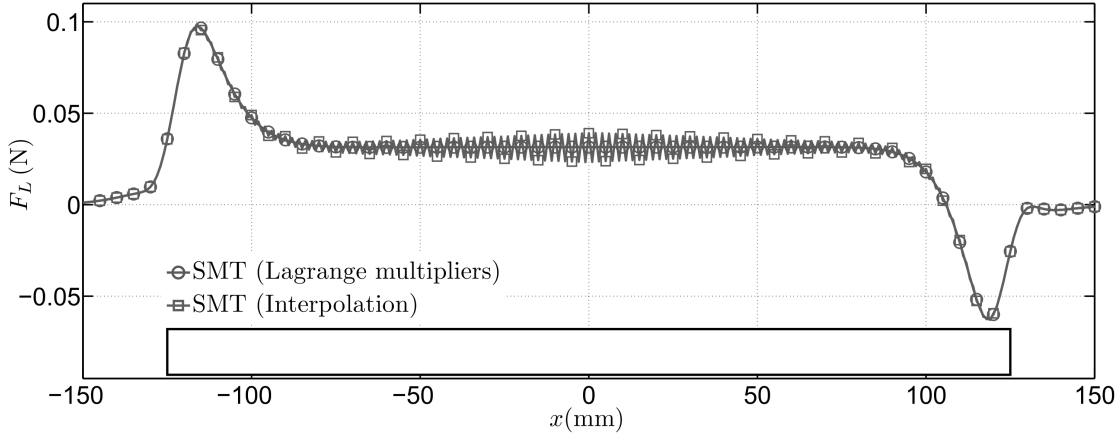
3.2) several coupling techniques have been reported. Nevertheless, only two of them are nowadays commonly applied in 3D electromagnetic field computations. They are referred to as (i) Lagrange multipliers and (ii) field interpolation techniques. The main aim here is to present one of the possible SMT implementations to solve the given LET benchmark problem (Fig. 5.1). The resulting SMT model will be used throughout the work in order to verify results and estimate the computational costs of the novel fixed grid methods presented above.

In any SMT implementation it is of main interest to minimize the coupling interface (boundary) between two independent meshes, i.e. the number of variables which need to be coupled. In order to minimize the size of the coupling surface resulting from the given 3D LET problem, only a small region around the magnet has been used as the moving part. The rest of the geometry was fixed (Fig. 5.6). Additionally, the contact boundary is placed in the air region which has been described only with the scalar potential ψ (see section 2.3). The coupling boundary actually consists of two surfaces, i.e. a source boundary which belongs to the moving part and a destination boundary which belongs to the fixed part. Even though this is not directly required, the source and destination boundaries were meshed using identical finite element meshes, so that the meshes could conform during the entire motion. This helps to reduce possible errors due to non-matching grids.

On the coupling boundary the field continuity has been obtained by applying both, Lagrange multipliers and interpolation methods. On the rest of the boundary, which is not overlapping, the magnetic field insulation boundary condition has been used (see section 5.6.2). The Lagrange multipliers method introduces an additional set of variables on the sliding interface which ensures the continuity of the field in a weak sense [102–105]. Unfortunately, the existence of additional variables considerably deteriorates



(a) Drag Lorentz force component



(b) Lift Lorentz force component

Figure 5.7: Comparison of two SMT solutions of the LET benchmark problem for conductor free of defects. The field continuity across the interface has been obtained by means of Lagrange multipliers and by the interpolation technique.

the conditioning of the stiffness matrix [105–107]. Furthermore, the matrix contains zeros on the main diagonal, due to which only direct solvers provided by Comsol Multiphysics® could be used [79]. On the other hand, the interpolation technique does not introduce additional variables [80, 105]. The scalar potentials at node points on the destination side are made to be equal to the potentials at corresponding points on the source side. If the mesh is conforming, this should actually work well as long as the dependent variable which is being constrained is continuous between elements. If the mesh nodes are not conforming, then the interpolation at node points and inter-element boundaries can lead to instabilities, i.e. to oscillations of the Lorentz force. However, the matrix system remains well-conditioned for which iterative solvers behave well.

It should be emphasised that irrespective of the approach being used to provide continuity of the field across the interface, the resulting stiffness matrix of SMT is changing at each time step of the simulation time. This is because due to mesh displacement

Table 5.2: Computational requirements and simulation time

Method	Solv. type	NoE	DoF	RAM	T_{total}
SMT (Lagr.)	Direct	258 905	967 862	~ 19.21 GB	17h : 7min
SMT (Interp)	Iterative	258 905	923 180	~ 3.1 GB	9h : 56min

the connections between different finite elements on the coupling boundary are changing. For large displacements, this can also lead to considerable increase of the matrix bandwidth resulting in higher memory consumption, as reported in [105, 111, 112].

The results obtained by both, Lagrange multipliers and interpolation methods are presented in Fig. 5.7. As a benchmark problem the LET setup presented in Fig. 5.1 for the conductor free of defects has been used. The cylindrical magnet has been moved from left end of the conductor to the right starting at $X_{start} = -160$ mm. The lift-off distance was $\delta z = 5$ mm. All other parameters are given in Table 5.3. For analysis, the generalized- α solver with $\alpha = 0$ has been used. Although the same finite element mesh has been applied, the results obtained by the interpolation technique tend to oscillate. Due to lower force values, the oscillation is more obvious in the Lorentz force lift component, with respect to corresponding drag force component (compare Fig. 5.7a and Fig. 5.7b). This is mostly because the continuity of the field across the interface can be only globally provided by the interpolation technique [105, 111, 112]. Furthermore, it is also possible that the finite element mesh on the coupling interface was not completely conforming. Important to mention is that higher values of α lead to higher oscillations of the Lorentz force as well. The Lagrange multipliers, on the other hand provide smooth force signals in both force components independent on the value of α . For that reason only SMT along with Lagrange multipliers was used to verify the results obtained by LEA.

The computational costs resulting from both SMT implementations are summarized in Table 5.2. Since SMT together with Lagrange multipliers require direct solvers, its memory consumption was considerably higher than corresponding interpolation technique for the same number of elements (NoE). Due to additional variables on the coupling boundary the resulting degrees of freedom (DoF) are slightly higher as well. Consequently the total solution time T_{total} of SMT with Lagrange multipliers was $\sim 50\%$ longer than with the interpolation technique.

5.6 Numerical Implementation

In this section the results obtained by applying the proposed numerical methodology to the LET benchmark problem shown in Fig. 5.1 are presented. All approaches have been implemented using the commercial finite element code Comsol Multiphysics®

[79]. The verification of the obtained results has been performed using the sliding mesh technique (SMT). In order to provide direct control over the time-step size required by MMA and MDA, all simulations have been performed using the generalized- α transient solver. The other possibility provided by Comsol[®], namely the backward differential formula (BDF) transient solver is based on a variable time-step size algorithm [79, 125, 126]. Thus, it could only be applied in SMT simulations. Nevertheless, the solution of LET problems using SMT and BDF as a transient solver required a relatively large number of time steps, which resulted in an extremely long simulation time. In order to provide a fair comparison the generalized- α solver has been used in the implementation of SMT as well.

The implicit and second order accurate generalized- α transient solver was first developed for second-order systems arising in structural mechanics problems. Later its application has been extended to first-order systems [127, 128]. One of its main advantages is the possibility to control the amount of high frequency numerical dissipation introduced by time discretization. This is provided through a single parameter α which can take values between 0 and 1. If α is chosen to be 0 the method damps the high frequency response in a single time step, whereas if α is chosen to be 1 no numerical dissipation is introduced [127]. For MMA and SMT approaches a relatively strong dependency of the Lorentz force lift component with particular choice of α has been observed (see Appendix B). This is because in the assumed velocity range ($R_m \sim 1$) the lift force is around two orders of magnitude smaller than the corresponding Lorentz force drag component. The discrepancy was reduced when α was approaching 1. However, for large values of α ($\alpha > 0.85$) the solutions obtained by MMA, MDA and SMT tends to be unstable. This practically limited the application of the generalized- α solver to $\alpha \leq 0.85$. Fortunately, results obtained by MMA or SMT can be corrected in a simple way as proposed in Appendix B.

If not stated otherwise, the material properties and geometrical parameters are defined as summarized in Table 5.3. To compare all approaches, the total Lorentz force acting on the cylindrical permanent magnet has been used (2.2). Due to the symmetry of the model with respect to the xz -plane, the side component F_S of the Lorentz force vanishes, and for that reason it was not used for the analysis. The drag F_D and the lift F_L components of the Lorentz force were compared quantitatively with a window normalized root mean square error (WNRMS error) δ_c , $c \in \{D, L\}$ has been used

$$\delta_c = \frac{\sqrt{\frac{1}{\Delta W} \int_{-\Delta W/2}^{\Delta W/2} (F_c - F_c^{Ref})^2 dx}}{\max F_c^{Ref} - \min F_c^{Ref}} 100\%, \quad (5.10)$$

Table 5.3: Geometrical parameters and material properties of the LET problem

Parameter	Value [mm]	Description
D_m	15	Diameter of the magnet
H_m	25	Height of magnet
L_c	250	Length of conductor
W_c	50	Width of conductor
H_c	50	Height of conductor
l	12	Length of defect
h	2	Height of defect
w	2	Width of defect
d	2	Defect depth
δz	3	Lift-off distance
L	$2X_{start}$	Length of moving domain
Δx	1	Mesh size in moving domain
Property	Value	Description
B_r	1.17T	Remanence of the magnet
σ	30.61 MS/m	Conductor conductivity
σ_d	0 MS/m	Defect conductivity

where F_c and F_c^{Ref} are Lorentz force values received by the method to be compared and by the reference method, and ΔW is a window size for which we calculate the errors.

5.6.1 Initial Conditions

In order to develop specific LET systems, it is sufficient to study the Lorentz force perturbations caused by material defects. At the initial simulation time $t = 0$, there are actually two possibilities to prescribe $\mathbf{A}_i$, i.e. either (i) the zero magnetic field ($\mathbf{A}_i = \mathbf{0}$) or (ii) the magnetic field of the magnet diffused in free-space ($\mathbf{A}_i = \mathbf{A}_{mag}$) can be used (Fig. 5.8a). In the first case a considerable amount of simulation time is wasted on unnecessary calculations related to the magnetic diffusion transient response until the steady state of the solution is reached [129]. In the second case, since at initial time ($t = 0$) there are no eddy currents induced inside the conductor ($\mathbf{j} = \mathbf{0}$), the response time of the Lorentz force needs to be calculated. In principle, the former approach is more realistic, but it still requires additional computation. In order not to affect the Lorentz force perturbations caused by defects the initial position of the magnet, denoted by X_{start} , has to be increased by the distance $\Delta X_{start} = \tau_r \cdot v$, where τ_r is the response time of the Lorentz force for the given LET system. It is reasonable to expect that the

specific value of τ_r depends on the testing velocity v geometric and material properties of the conductor under test. As an first estimation of τ_r , the magnetic field diffusion time, $\tau_r = \tau \sim R_m \cdot L/v$, can be used. Assuming the testing velocity of $v = 1$ m/s and a conductor with the square cross-section ($W_c = H_c = 50$ mm) with typical electrical conductivity of aluminium ($\sigma = 35$ MS/m), the elongation of the initial position can be calculated as $\Delta X_{start} = \tau \cdot v \sim 25$ mm. To confirm this observation the response time of the Lorentz force to the step change in the testing velocity $v(t) = v \cdot h(t)$ has been calculated numerically. Thus, the LET benchmark problem presented in Fig. 5.1 has been used, where all geometry and material parameters are given in Table 5.3. The results for two different values of testing velocities are presented, i.e. $v = 1$ m/s and $v = 10$ m/s, which corresponds to $R_m \sim 1$ and $R_m \sim 10$, respectively (Fig. 5.9). From the obtained results, it can be seen that for the fast estimation of ΔX_{start} , the magnetic diffusion time τ , can only be used for moderate velocities ($v = 1$ m/s). For higher, testing velocities ($v = 10$ m/s) this would overestimate the value of ΔX_{start} .

To avoid these unnecessary computations the quasi-static solution received from (5.4) and (5.5), for a defect-free conducting object ($\mathbf{A}_i = \mathbf{A}_{qs}$), can be used as an initial condition (Fig. 5.8b). In principle, this makes the elongation of the initial position ΔX_{start} between the magnet and the defect to zero. If the optimal $\mathbf{A}^* - \psi$ formulation is used (section 2.3), at the initial time ($t = 0$), both the magnetic scalar potential ψ and magnetic vector potential $\mathbf{A}^*$, which are resulting from quasi-static solution have to be initialized, i.e. $\mathbf{A}_i = \mathbf{A}_{qs}$ and $\psi_i = \psi_{qs}$ (Fig. 5.10).

In case when the magnet is initially placed next to the conductor under test, i.e. in the air region the initial conditions are simply obtained as $\mathbf{A}_i = \mathbf{A}_{mag}$, (Fig. 5.8a).

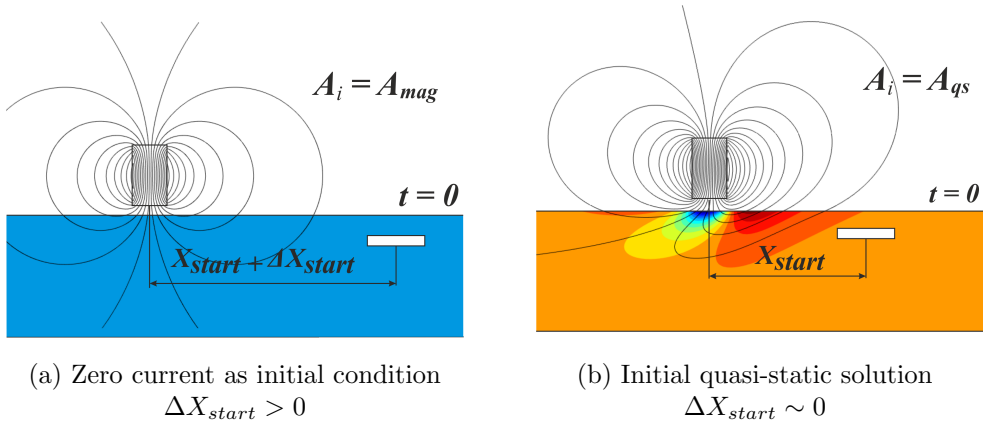
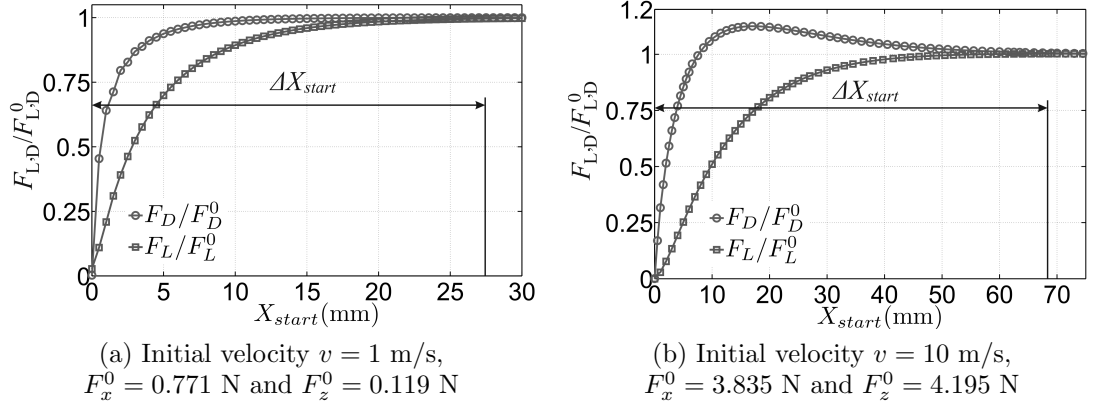
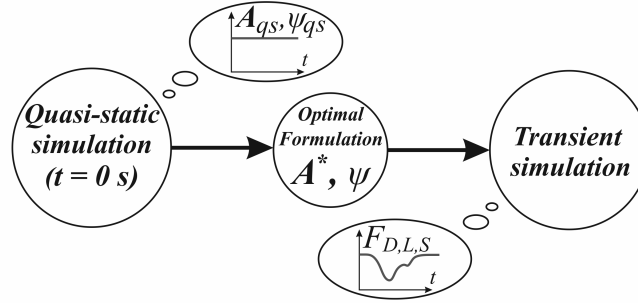


Figure 5.8: Distribution of the magnetic field $\mathbf{B}$ and induced current $\mathbf{j}$ at $t = 0$. ΔX_{start} is the elongation of the initial position of the magnet which depends on the applied initial conditions


 Figure 5.9: Elongation of the initial position ΔX_{start}

 Figure 5.10: Initial quasi-static solution $(\mathbf{A}^*, \psi)$ for the transient simulation.

5.6.2 Boundary and Interface Conditions

Boundary Conditions

Lorentz force eddy current problems are characterised as open boundary problems. The easiest way to solve these problems is to truncate the infinite computational domain by an artificial outer boundary Γ (Fig. 5.11). This approach is based on the assumption that the obtained solution is not altered by setting the potential or its normal derivative to zero (applying the Dirichlet or the Neumann boundary conditions) on the boundary which is sufficiently far away from the region of interest [130]. If the normal component of the magnetic field $\mathbf{B}$ along Γ is set to zero, the so-called magnetic field insulation boundary condition is obtained

$$\mathbf{n} \cdot \mu_0 \nabla \psi = 0, \quad (5.11)$$

where $\mathbf{n}$ is the unit normal vector on outer boundary surface Γ (Fig. 5.11). If the tangential component of the magnetic field $\mathbf{B}$ is set to zero, the following condition has to be fulfilled

$$\psi = \psi_0. \quad (5.12)$$

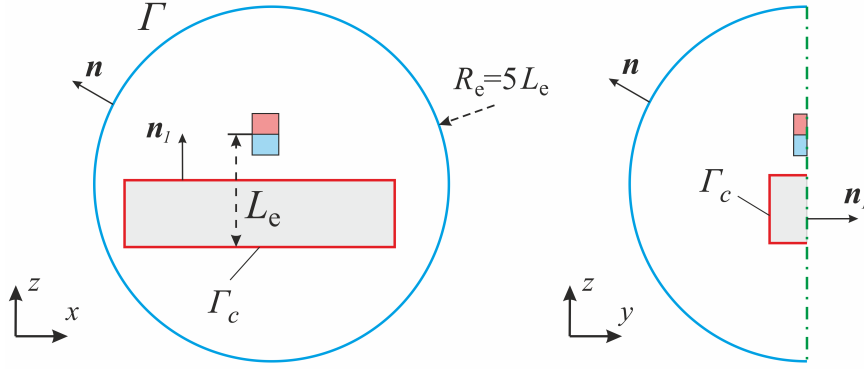


Figure 5.11: Truncation of the infinite air domain.

Here ψ_0 is some prescribed value of the magnetic scalar potential, which is usually zero, $\psi_0 = 0$.

In all considered approaches, the surrounding air domain has been modelled in form of a sphere along with magnetic insulation boundary conditions (Fig. 5.11). Based on the sensitivity study used to determine the influence of the air domain size on the computed Lorentz force, a wide range of domain configurations were investigated. Finally, for the equivalent radius of a sphere the value of $R_e = 5L_e$ has been used, where L_e is the characteristic length shown in Fig. 5.11. Further increase of R_e did not affect the obtained Lorentz forces. The fact that proposed LEA is implemented on the fixed computational grid, enables the use of other techniques dealing with open boundary problems as well [130]. Nevertheless, techniques like asymptotic boundary conditions (ABC) or infinite elements resulted in considerable increase of the simulation time compared to simple truncation technique.

To further reduce the computational requirements, the symmetry of the model with respect to the xz -plane, allows only half of the model to be considered (Fig. 5.11). On the resulting outer and symmetry boundary of the model, the magnetic insulation boundary conditions (5.11) are applied. Additionally, if inside the conducting domain the electric scalar potential V is used, the Dirichlet boundary condition $V = 0$ has to be applied as well.

Interface Conditions

For the given LET benchmark problem the surface of the conductor under test has been used as the coupling interface (Γ_c) between two potentials associated with the optimal $\mathbf{A}^* - \psi$ formulation. (Fig. 5.11). This represents the most economical solution. The conditions providing the field continuity across the interface have been described in section 2.3.3 and they are given by (2.40).

5.6.3 Meshing of the Conductor

In FEM, the overall accuracy of results is strongly influenced by the spatial discretization of the computational domain into finite elements [50]. A finer finite element mesh commonly gives better accuracy (see chapter 3). Since this would significantly increase the memory consumption, the maximum number of finite elements is usually restricted by the computational resources available. Therefore, in any numerical model it is very important to control the distribution of the finite elements, providing higher mesh density in regions where higher magnetic field gradients are present, e.g. in the close vicinity of the permanent magnet.

To reduce a possible influence of the domain discretization on the calculated values of Lorentz forces, a similar mesh distribution inside the conducting domain is used for the comparison of all considered numerical approaches. The schematic of the used finite element mesh is shown in Fig. 5.12. The mesh consists only of quadratic tetrahedral elements [48], where the particular distribution is based on a performed sensitivity analysis reducing the influence of the mesh size on the resulting value of the Lorentz force. Similar mesh distribution is used in non-conducting domains (permanent magnet and air) as well, and it is not shown here for simplicity.

The particular choice of tetrahedral elements considerably simplifies meshing of complex domains including the conductor with defects. In case of simple geometries better control over the mesh distribution can be obtained by applying the hexahedral finite elements (see chapter 6). Nevertheless, for approximately the same number of DoF the analysis has been performed to make sure that the LET solution is not influenced by the type of the finite elements applied. A comprehensive study on the influence of different finite element shapes on the accuracy of the solution can be found

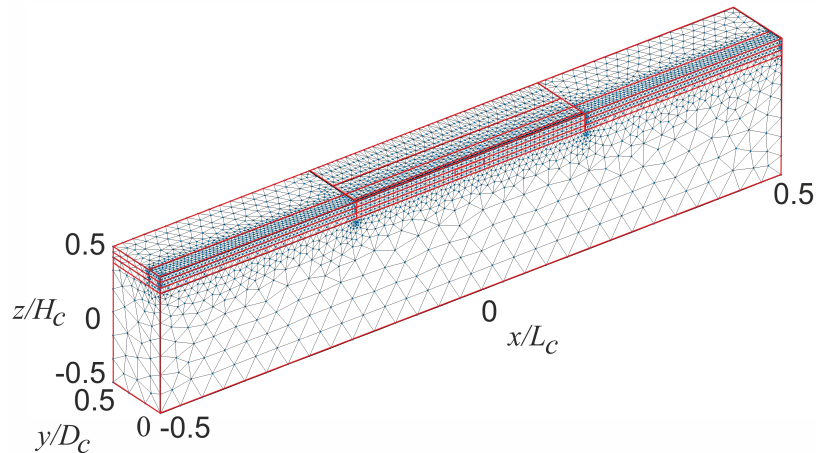


Figure 5.12: Distribution of the finite element mesh inside the conducting domain which is used in all numerical calculations for conductors with defects. The mesh consists of only tetrahedral elements.

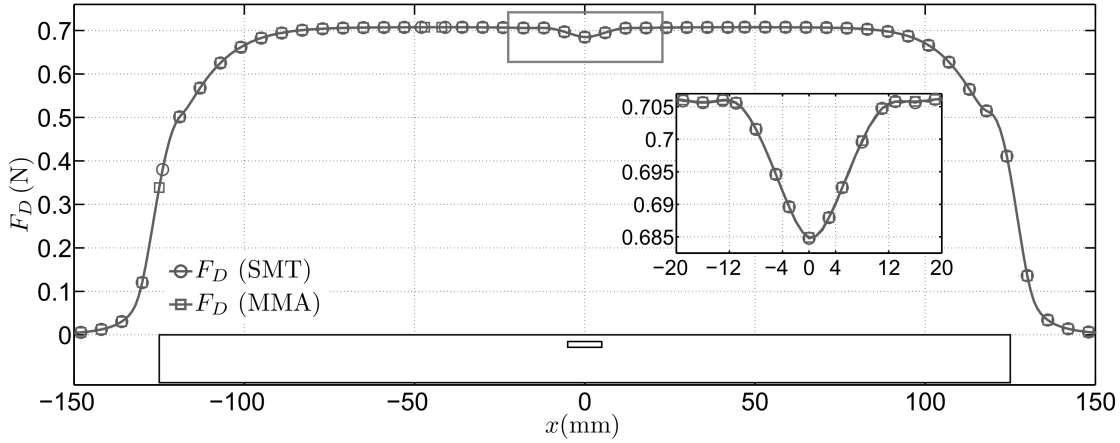
in [131].

5.7 Numerical Verification

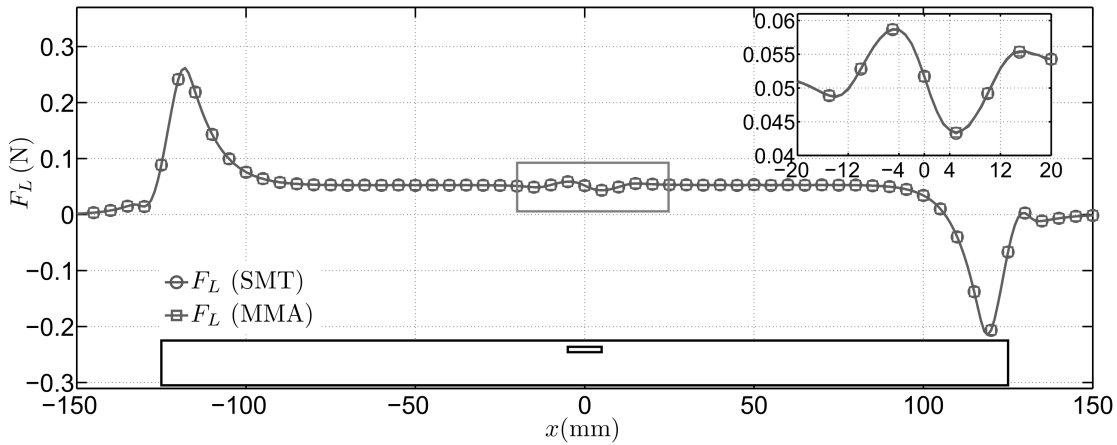
In order to test the accuracy and determine the application range to which each of the proposed numerical approaches can be applied, simulations of the LET benchmark problem (Fig. 5.1) have been performed considering various values of R_m . Finally, the computational costs and simulation time for all proposed approaches are presented and discussed.

5.7.1 Verification of the Moving Magnet Approach

First, the verification of the moving magnet approach (MMA) with reference to the sliding mesh technique (SMT) has been performed. As a benchmark problem the LET



(a) Drag Lorentz force component



(b) Lift Lorentz force component

Figure 5.13: Verification of the moving magnet approach (MMA) with the sliding mesh technique (SMT). The long (”|”) defect is located $d = 2$ mm below the surface of the conductor.

configuration shown in Fig. 5.1 has been used. All material and geometrical parameters are given in Table 5.3. The conductor has a finite length $L = 250$ mm and contains the long (”|”) defect located symmetrically in its centre at $d = 2$ mm. The cylindrical magnet has been moved from left to right at constant velocity $v = 0.5$ m/s, starting at $X_{start} = -160$ mm. During the solving process, the generalized- α transient solver with a fixed time step $\Delta t = \Delta x/v$ and $\alpha = 0.85$ has been used. Since, both MMA and SMT would require the same constant correction factor (see Appendix B) the correction of obtained results has not been performed. The simulations resulted in a total number of 320 time steps (see Table 5.3). Fig. 5.13 shows the characteristic Lorentz force profiles obtained by both, SMT and MMA. Furthermore, a small Lorentz force perturbation produced by the long (”|”) defect in the middle of the force profiles can be observed. As expected, much stronger force perturbations are caused by the front and back walls of the solid conductor. Using SMT as a reference method and a window size of $\Delta W = 250$ mm, the WNRMS errors (5.10) of the drag and lift Lorentz force components are equal to $\delta_D = 0.03\%$ and $\delta_L = 0.06\%$. The small WNRMS error values additionally confirm a very good agreement between MMA and SMT.

As already mentioned in section 5.2.1 the stiffness matrix resulting from MMA is symmetric, well-conditioned and remains unchanged during the simulation time. This means that the time taking procedure involving the assembly of the stiffness matrix needs to be performed only once, yielding a fast and stable simulation. Due to the additional variables in form of the Lagrange multipliers introduced on the coupling boundary, the structure of the stiffness matrix in SMT is not symmetric and changes at each time step because of the mesh displacement (see section 5.5). Furthermore, the resulting matrix system contains zeros on the main diagonal which renders the SMT analysis to use of direct solvers only. All these issues along with the ease of implementation makes MMA much faster than SMT for approximately the same number of DoF. For detailed comparison of the computational requirements and solution time see section 5.7.4. Since, both SMT and MMA solve a transient form of the magnetic field diffusion equation (2.37) the obtained results are valid for any finite value of the R_m .

5.7.2 Verification of the Moving Defect and Quasi-Static Approaches

In what follows the results obtained by MDA and QSA have been verified with respect to SMT. In order to test the validity of both approaches, the LET benchmark problem (Fig. 5.1) has been solved by considering two different values of R_m , i.e., $R_m = 0.1$ and $R_m = 10$, which correspond to relative velocities of $v = 0.103$ m/s and 10.3 m/s, respectively (see Table 5.3). In the implementation of MDA the long (”|”) defect has

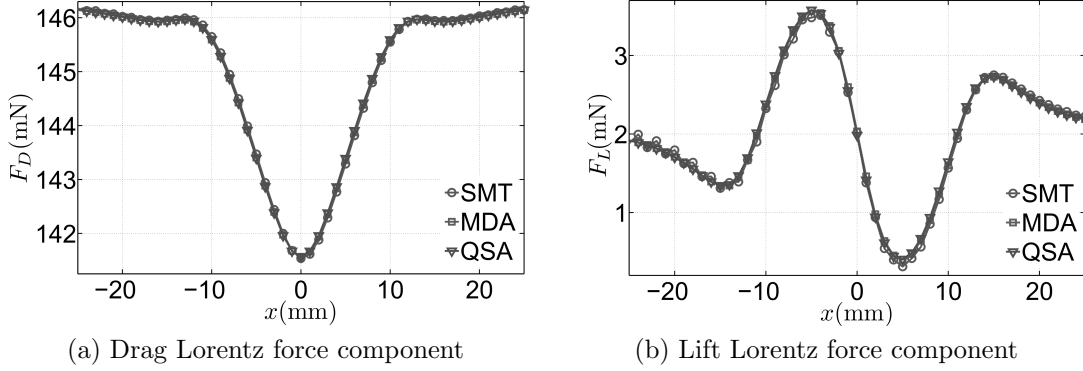


Figure 5.14: Verification of the moving defect approach (MDA) and quasi-static approach (QSA). The comparison is performed for $R_m = 0.1$ ($v = 0.103\text{m/s}$). The long (") defect is located $d = 2$ mm below the surface of the conductor.

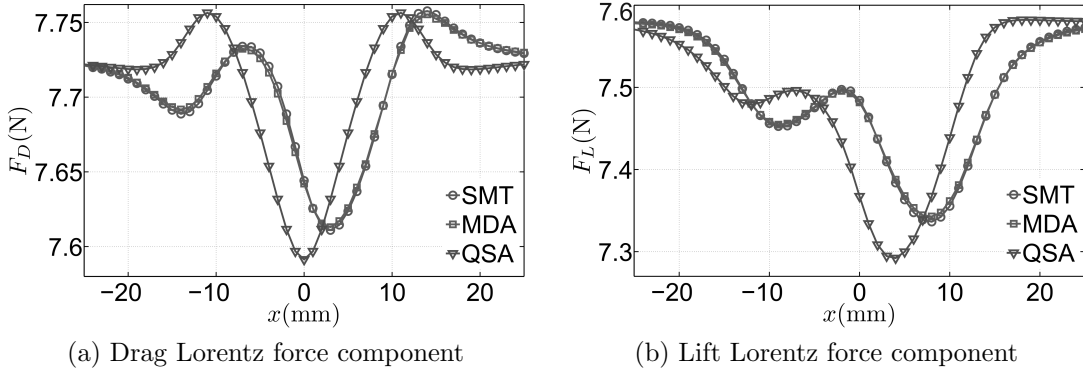


Figure 5.15: Verification of the moving defect approach (MDA) and quasi-static approach (QSA). The comparison is performed for $R_m = 10$ ($v = 10.3\text{m/s}$). The long (") defect is located $d = 2$ mm below the surface of the conductor.

been moved with a constant velocity from right end of the conductor to left end, starting at $X_{start} = 55$ mm (Fig. 5.4). The generalized- α transient solver with a fixed time step $\Delta t = \Delta x/v$ and $\alpha = 0.85$ has been used for both values of R_m , resulting in total 110 time steps. Thus, the constant correction of the transient results obtained by SMT has been performed according to (B.1) and (B.2) (see Appendix B).

As expected, all approaches, i.e., MDA, SMT, and QSA are in good agreement with each other for small values of R_m , $R_m = 0.1$ (see Fig. 5.14a). Using SMT as a reference method and the window size $\Delta W = 48$ mm, WNRMS errors (5.10) calculated for MDA and QSA are equal to $\delta_D^{MDA} = 0.92\%$, $\delta_L^{MDA} = 1.68\%$, and $\delta_D^{QSA} = 1.20\%$, $\delta_L^{QSA} = 2.03\%$.

For higher values of R_m ($R_m = 10$), the QSA produces erroneous Lorentz force perturbations due to the assumed instantaneous magnetic field reaction ($\partial \mathbf{B}/\partial t = 0$). As a result, the WNRMS errors of QSA with reference to SMT are equal to $\delta_D^{QSA} = 28.14\%$, $\delta_L^{QSA} = 27.36\%$. The MDA, on the other hand agrees well with the SMT

solution for higher values of R_m (Fig. 5.15a). The obtained WNRMS errors are equal to $\delta_D^{MDA} = 1.57\%$, $\delta_L^{MDA} = 1.23\%$.

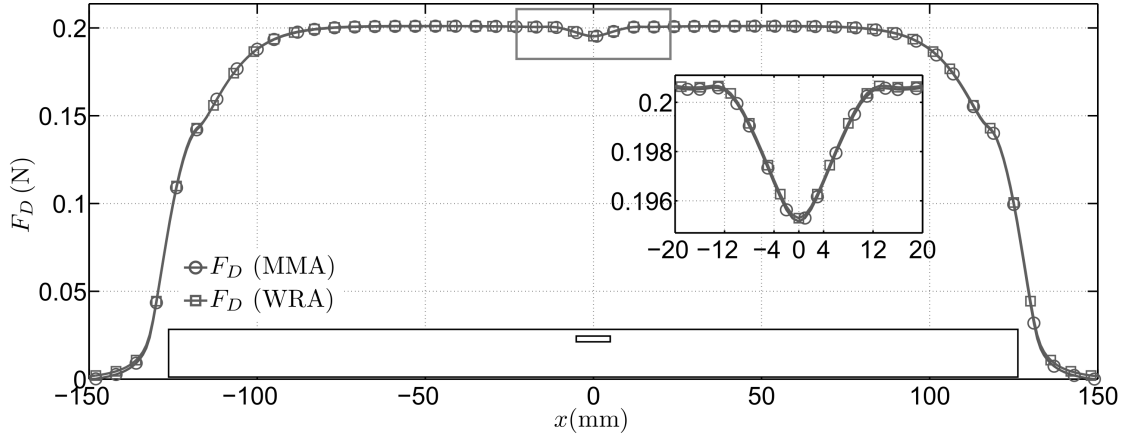
It should be mentioned again that, SMT like MMA offer accurate LET results for any value of R_m . The reason for that is that both methods are described in the fixed frame of reference in which the transient magnetic field diffusion equation without the additional velocity term applies (2.28). The MDA is described in the moving frame of reference (2.31) which is time dependent and contains the additional velocity term $(\mathbf{v} \times \nabla \times \mathbf{A})$ as well. Due to a good agreement with SMT for any value of R_m , the presented analysis additionally confirms the validity of applying (2.31) for transient eddy current problems having conducting parts in non-uniform motion and variable cross-section in the moving direction (Fig. 5.15a). The QSA assumes instantaneous magnetic field reaction ($\partial \mathbf{B} / \partial t = 0$). Strictly speaking this would restrict its applicability only to uniformly moving objects with constant-cross section in the moving direction. Nevertheless, as it has been shown above that QSA offers good results when applied for LET problems and conductors with defects under the condition $R_m < 1$ (Fig. 5.14a).

5.7.3 Verification of the Weak Reaction Approach

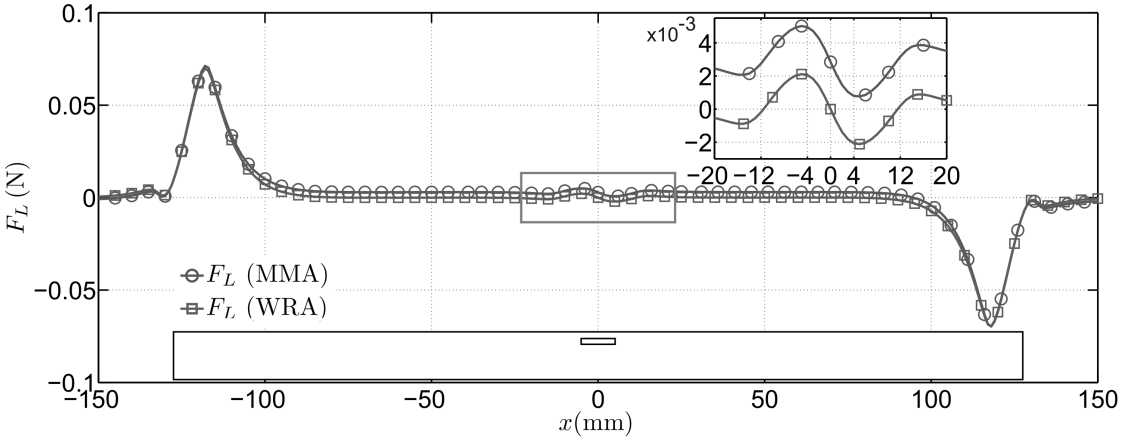
The closed-form analytical solution describing the magnetic field distribution of a cylindrical permanent magnet could not be obtained [124]. Therefore, in order to verify the so-called weak reaction approach (WRA), the rectangular shape of the magnet has been considered instead. The closed-form analytical solution for a rectangular shape of the magnet can be found in [124] and it is not presented here for simplicity. The dimensions of the used rectangular magnet are the same as for the cylindrical magnet described before, i.e. $W_m = D_m$, where W_m and D_m are its width and depth, respectively. The height of the magnet is given by H_m (see Table 5.3). Other geometry and material parameters are described by the LET benchmark problem (Fig. 5.1).

Similarly as in the verification of MMA, in WRA the magnet has been moved from the left end of the conductor to its right end, i.e. starting at $X_{start} = -160$ mm at testing velocity $v = 0.103$ m/s ($R_m = 0.1$). The obtained Lorentz force profiles are shown in Fig. 5.16. Due to a considerably shorter simulation time (section 5.7.1), instead of SMT as a reference method MMA has been applied. Furthermore, the rectangular shape of the magnet in MMA can be simply modelled by changing the corresponding logical expression (Table 5.1). For implementation of MMA, the generalized- α transient solver with a fixed time step $\Delta t = \Delta x / v$ and $\alpha = 0.85$ has been used. This yields the correction of MMA results according to (B.1) and (B.2) (see Appendix B).

From the obtained results, a very good agreement in the drag component of the



(a) Drag Lorentz force component



(b) Lift Lorentz force component

Figure 5.16: Verification of the weak reaction approach (WRA) with the moving magnet approach (MMA). The long ("|") defect is located $d = 2$ mm below the surface of the conductor.

Lorentz force can be observed (Fig. 5.16a). Using the window size of $\Delta W = 250$ mm, the WNRMS error for the drag component obtained using WRA, with reference to MMA is equal to $\delta_D^{WRA} = 0.35\%$. In case of a lift force the agreement is deteriorated (Fig. 5.16b). The WNRMS error for window size of $\Delta W = 250$ mm is equal to $\delta_L^{WRA} = 1.87\%$. The relatively small error is expected since the value of R_m used for comparison was small $R_m = 0.1$. For higher values of R_m the WNRMS would increase considerably. The main reason for such a behaviour is that WRA neglects any reaction of the induced eddy currents. Due to resulting symmetry of the current distribution within the conductor this makes the lift component of the Lorentz force acting on the magnet (its integral over the conducting domain) to zero. However, since the induced Lorentz force density within the conductor is not zero WRA actually calculates the perturbation of the lift force component correctly. The MMA on the other hand accounts for the reaction field of induced currents resulting in non zero lift force component. Due to this, the applicability of WRA is restricted to LET analysis resulting in low

values of $R_m \ll 1$.

The obvious advantage of WRA over all other methods is that only one scalar potential V is sufficient to describe LET systems (see section 5.4). This considerably reduces the computational costs and total simulation time. Moreover, only the conducting domain needs to be modelled, which significantly reduces model complexity and modelling time.

5.7.4 Computational Requirements and Simulation Time

In the preceding sections the verification of the numerical results obtained by various numerical approaches (MMA, MDA, QSA and WRA) with respect to SMT together with Lagrange multipliers has been performed. For the analysis the LET benchmark problem defined in Fig. 5.1 has been applied, i.e. the resulting Lorentz force acting on the magnet has been used as a comparison parameter. The main aim in this section is to estimate the computational costs and quantitatively compare the simulation time of each of these methods (Table 5.4).

As expected, the reference SMT method results in longest simulation time ($T_{total} = 18\text{h} : 28\text{min}$). This is mostly because SMT is restricted to direct solvers only. To obtain the solution, these solvers use a large amount of available random access memory (RAM) and take relatively long time for matrix factorization which is required in each time step due to mesh displacement (see section 5.5). This results in a relatively long calculation time, even per single time step $T_{step}^{SMT} = T_{total}/NT = 3\text{min} : 28\text{s}$, where NT is the total number of time steps (see Table 5.4). In the implementation of SMT the multifrontal massively parallel sparse direct solver (MUMPS) has been used [132]. Even for approximately the same number DoF, MMA solves the given LET problem in much shorter time ($T_{total} = 1\text{h} : 35\text{min}$). Apart the fact that iterative solvers can be used, this is mostly because the stiffness matrix resulting from MMA is symmetric, well-conditioned and remains unchanged during the whole simulation time. Thus, the simulation time per time step is considerably reduced to $T_{step}^{MMA} = 17.7\text{s}$.

Due to similarities in their implementation the computational costs resulting from MDA and QSA are very much alike. Nonetheless, since in QSA the time derivative of magnetic vector potential is set to zero ($\partial\mathbf{B}/\partial t \rightarrow \mathbf{0}$) the calculation time of QSA is

Table 5.4: Computational requirements and simulation time

Method	Solv. type	NoE	DoF	RAM	NT	T_{total}	$T_{step} = T_{total}/NT$
SMT	Direct	257 343	922 110	~ 20.1 GB	320	18h : 28min	3min : 28s
MMA	Iterative	303 613	1 066 535	~ 4.7 GB	320	1h : 35min	17.7s
MDA	Iterative	213 876	744 233	~ 2.17 GB	80	41min : 32s	32.2s
QSA	Iterative	213 876	869 056	~ 2.59 GB	80	31min : 05s	23.4s
WRA	Iterative	89 430	132 499	~ 0.73 GB	320	26min : 10s	4.9s

still shorter, with respect to MDA (see Table 5.4). Even though MDA and QSA require the same total number of finite elements (NoE), due to the additional scalar potential V used inside the conducting domain the resulting number of degrees of freedom (DoF) in QSA is larger compared to MDA. Similarly as in SMT, due to motion of the defect the stiffness matrix resulting from MDA and QSA is changing during the simulation which yields additional assembling procedure at every time step (see Sections 5.2 and 5.3). Moreover, the additional velocity term makes the stiffness matrix non-symmetric as well. All this issues result in longer simulation time per time step compared to MMA ($T_{step}^{MDA} = 32s$ and $T_{step}^{QSA} = 23s$).

The WRA compared to all other approaches results in lowest computational costs and shortest simulation time (see Table 5.4). This is expected, since WRA requires only one scalar potential V to be applied to solve 3D LET problems. The very short simulation time per single time step $T_{step} = 4.9s$ makes WRA preferable to all other approaches when the assumed simplifications apply $R_m \ll 1$.

5.8 Experimental Validation

In order to validate the obtained numerical results and to prove the feasibility of LET for the first time, the LET benchmark configuration presented in Fig. 5.1 has been experimentally realised in [15]. Two different typed of test conductors have been considered, namely (i) the solid conductor with linear surface defects and (ii) an equivalent package of metallic sheets (Fig. 5.17). Each sheet is 2 mm thick and it is made of the same material as the solid conductor. This configuration allows to consider various types of defects (long ”|” wide ”—”, and cross ”+”) and to change their position within the conductor in an easy and flexible way (Table 5.5). For completeness of this work the measuring system has been only briefly described here, whereas its detailed description can be found in [15]. In order to provide relative motion between the specimen and the

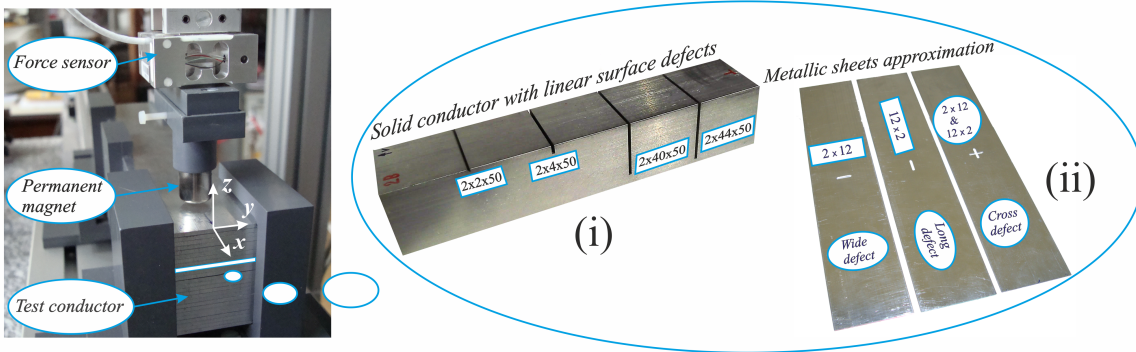


Figure 5.17: Experimental LET setup used for validation of the numerical results. (i) Solid conductor with linear surface defects (ii) Package of metallic sheets with artificial defects.

Table 5.5: Dimensions of the artificial defects

Defect type	Long "I"	Wide "—"	Cross "+"
$l \times w \times h$	$12 \times 2 \times 2$ mm	$2 \times 12 \times 2$ mm	$12 \times 2 \times 2 \cup 2 \times 12 \times 2$ mm

Table 5.6: LET measurement system - technical details [15]

Parameter	Linear drive	Positioning stage $y - z$	3D-Force sensor
Type	belt driven	step drive	strain gauge
Range	0..4m/s		$3 \times 3 \times 10$ N
Accuracy	100 μ m	30nm	$15 \times 15 \times 50$ mN
Operation	3m	45×45 mm	10kHz

Table 5.7: Parameters of applied filters [15]

Source of interferences	Filter type	Frequency [Hz]	Sampling rate [Hz]
Basement	Notch	10.9	44
Power supply	Notch	50	200
Construction	Band-stop	60 ... 90	300
Uncertain	Wavelet (Dmey)	50 ... 230	continuous

magnet, a linear belt-driven servo-drive has been used. The velocity range from 0 up to 4 m/s allows the the validation numerical simulations in the range of small and medium magnetic Reynolds numbers. The measurement frame is decoupled from the drive and carries the force sensor and the magnet system together with yz -positioning stage that provides a precise positioning in y and z directions. A rear-earth neodymium permanent magnet (NdFeB) has been mounted directly on a three component force sensor that is built in strain gauge technology providing medium accuracy and high dynamics. The main technical details of this LET measurement system can be found in Table 5.6.

It should be mentioned that the used force sensor was not an optimal choice for the given LET setup, as the accuracy in all force components is rather low (Table 5.6). This is because the overall resolution of the sensor is determined by the maximum measurement range, which is up to 3 N in the x -axis and y -axis direction, and 10 N in z -axis direction [15]. Additionally, due to the noisy laboratory environment which strongly influences measured force signals the resulting signal-to-noise ratio is rather poor. Therefore, the raw force signals cannot be used directly. The signal quality can be significantly improved by applying appropriate filter techniques whose parameters are summarized in Table 5.7.

5.8.1 Metallic Sheets Approximation

As already mentioned, for arbitrary positioning of artificial defects one of the test conductors used in experiments has been approximated by a package of metallic sheets, each 2 mm thick. From the basic principles of LET (section 2.1) it can be shown that eddy currents induced by relative motion form two-dimensional concentric loops in the planes normal to the magnetic field lines [37] (in the yx -plane of the model). In order

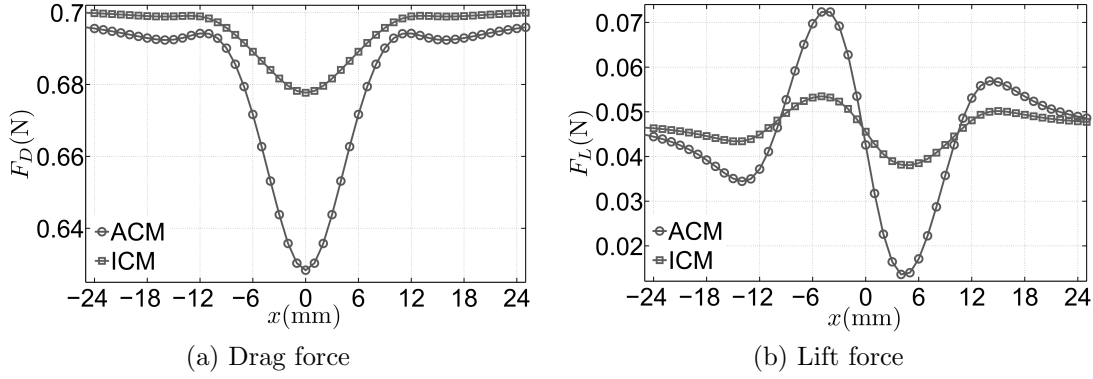


Figure 5.18: Comparison between the anisotropic conductivity model (ACM) and isotropic conductivity model (ICM). The long (”|”) defect is located at $d = 2$ mm and $v = 0.5$ m/s.

not to affect their distribution within the conductor the laminations used in experiments have been directed along the model z -axis (Fig. 5.17). If there are no defects within the conductor this approximation is completely justified since the z -component of induced eddy currents is zero, $j_z = 0$ [133, 134]. Numerically, this would require no additional considerations and the laminated structure of the conductor could be modelled simply by applying an isotropic electrical conductivity to the conductor.

$$\sigma_{ICM} = |\sigma|. \quad (5.13)$$

This will be further referred as the isotropic conductivity model (ICM).

In case of a defective conductor it is reasonable to expect that the distribution of the induced eddy currents around the defect is completely three-dimensional, i.e. $j_z \neq 0$. This actually means that the laminated structure of the test conductor used in experiments will affect the resulting perturbations of the Lorentz force. In order to test how strong this effect is a 3D numerical simulation based on MDA has been performed. The laminated structure of the conductor has been numerically handled by applying an anisotropic electrical conductivity in the form of a diagonal tensor with zero value at the position corresponding to z -axis

$$\sigma_{ACM} = \begin{vmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 0 \end{vmatrix}. \quad (5.14)$$

The so-called anisotropic conductivity model (ACM) is computationally more efficient than the modelling of each individual sheets would provide [135, 136]. Fig. 5.18 shows the perturbation of the Lorentz force caused by the long (”|”) defect which is $d = 2$ mm below the surface of the conductor. The calculation has been performed for both, ICM

and ACM. By a direct comparison, a considerable increase in Lorentz force perturbations is obtained in case of ACM (Fig. 5.18). The maxima in force perturbation are increased by factors of 3 and 4 for the drag and the lift force components, respectively. As expected, sufficiently far away from the defect both ACM and ICM give identical results, which additionally confirms the observations made before. In order to exclude any influence of possible discretization errors, identical finite element meshes are used for both ACM and ICM.

The given analysis has shown that in order to obtain correct Lorentz force perturbations for laminated conductors, an anisotropic conductivity model (ACM) has to be applied. In the following sections both, ICM and ACM have been validated with corresponding experiments (section 5.8).

5.8.2 Validation of the Moving Magnet Approach

Similarly, as in the presented verification of the MMA approach for its validation it has been assumed that the conductor has a finite length ($L = 250$ mm) and a long (”|”) defect located at $d = 2$ mm. The cylindrical magnet has been moved from left end of the conductor to its right end ($X_{start} = -160$ mm) at constant testing velocity $v = 0.5$ m/s and lift-off distance $\delta z = 1$ mm (Fig. 5.3). During the solving process, the generalized- α transient solver with a fixed time step $\Delta t = \Delta x/v$ and $\alpha = 0.85$ has been used, which resulted in total number of 320 time steps (see Table 5.3). Thus, the obtained force signals have been corrected according to (B.1) and (B.2) (see Appendix B). To approximate the laminated structure of the test conductor used in experiments, the anisotropic conductivity model (ACM) (5.14) has been applied. Fig. 5.19 shows the results obtained by the MMA and the corresponding experiments (EXP). A very good agreement can be observed in the whole range of the resulting Lorentz force perturbations. The calculated WNRMS errors (5.10), considering only perturbations caused by the defect, i.e. $\Delta W = 48$ mm are equal to $\delta_D^{MMA} = 5.6\%$ and $\delta_L^{MMA} = 4.9\%$, for the drag and the lift Lorentz force components, respectively. For the whole length of the conductor, i.e. $\Delta W = 250$ mm the calculated WNRMS errors are equal to $\delta_D^{MMA} = 2.8\%$ and $\delta_L^{MMA} = 1.5\%$. In the experimental force signals a constant drift can be observed in the Lorentz force drag component (Fig. 5.19a). It is caused by a small tilt in the mounting of the specimen leading to a constant change in the lift-off distance between the magnet and the conductor. Due to low force values which lie in the uncertainty range of the used force sensor the tilt is not so expressed in the lift force component (Fig. 5.19b). Nevertheless, the results further underline the necessity of applying the ACM to the conductor, whenever the metallic sheets approximation of the solid conductor is used.

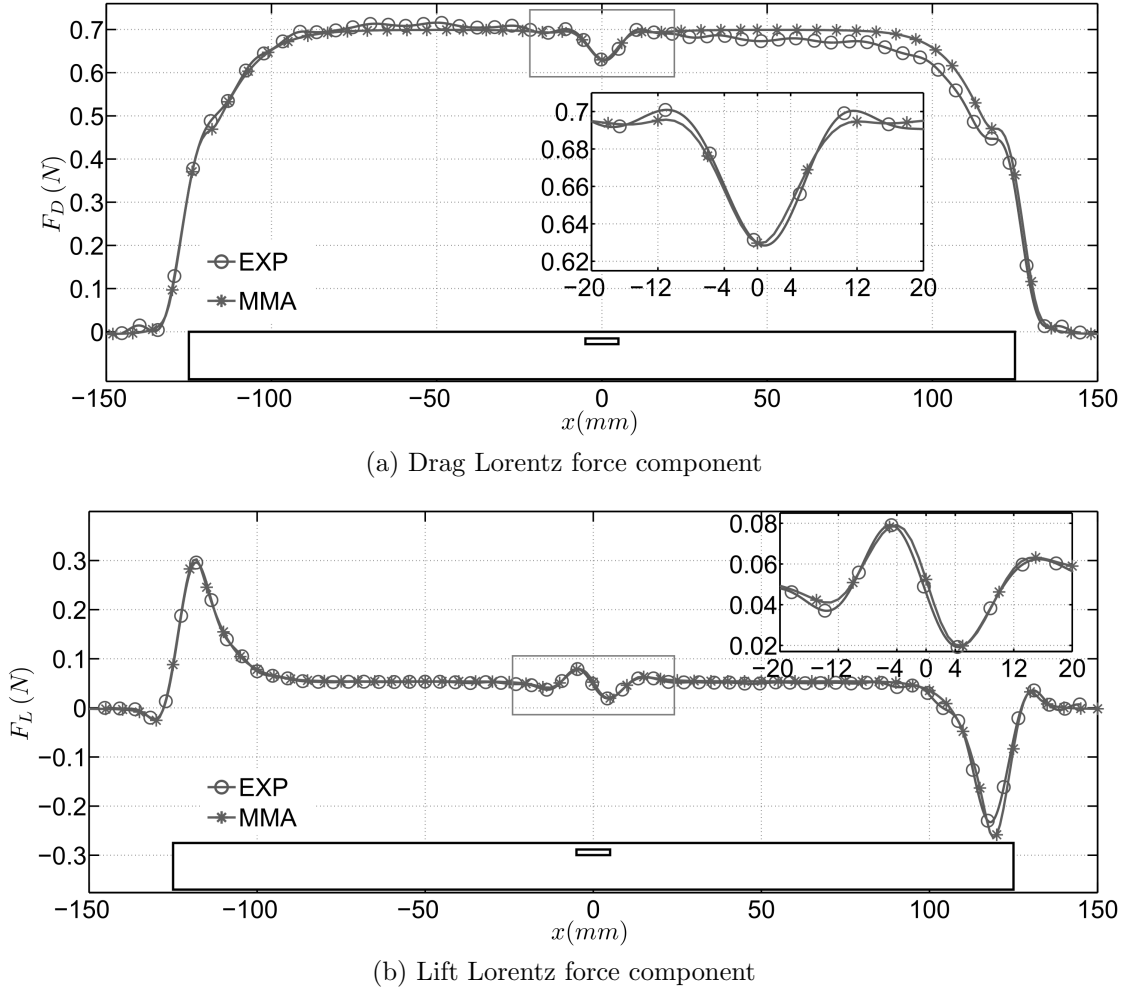


Figure 5.19: Validation of numerical results obtained by the moving magnet approach (MMA) with experiments (EXP). The long (”|”) defect is located in the second sheet from the top surface of the conductor ($d = 2$ mm) which is modelled using ASM.

5.8.3 Validation of the Moving Defect Approach

In order to validate the moving defect approach (MDA) the wide (”—”) and cross (”+”) defects have been considered additionally (Fig. 5.17). Furthermore, the same LET benchmark problem (Fig. 5.1) and material properties as for the MMA have been used (Table 5.3). In the implementation of the MDA the defects are moving from right side of the magnet to its left side starting at $X_{start} = 55$ mm (Fig. 5.4). As a transient solver the generalized- α solver with a fixed time step $\Delta t = \Delta x/v$ and $\alpha = 0.85$ has been used. Since, MDA is not depending on the particular value of damping coefficient α (see Appendix B) no correction of the obtained results was necessary. Due to shorter displacement ($L = 2 \cdot X_{start}$) compared to MMA, the simulation resulted in total number of 110 time steps (see Table 5.3). Due to the laminated structure of the test conductor the anisotropic electrical conductivity model (ACM) has been applied as well. Fig. 5.20 shows the Lorentz force perturbations only in the vicinity of defects,

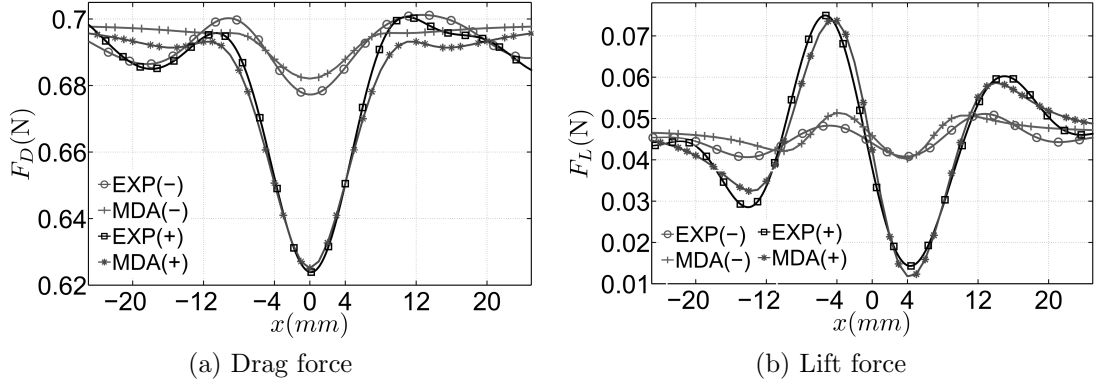
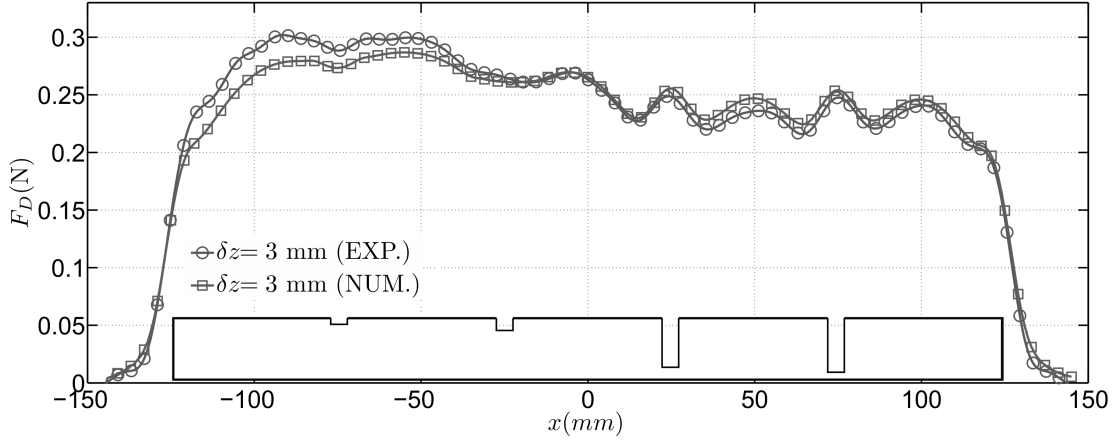


Figure 5.20: Validation of numerical results obtained by the moving defect approach (MDA) with experiments (EXP). The long ("|") defect is located in the second sheet from the top surface of the conductor ($d = 2$ mm) which is modelled using ASM.

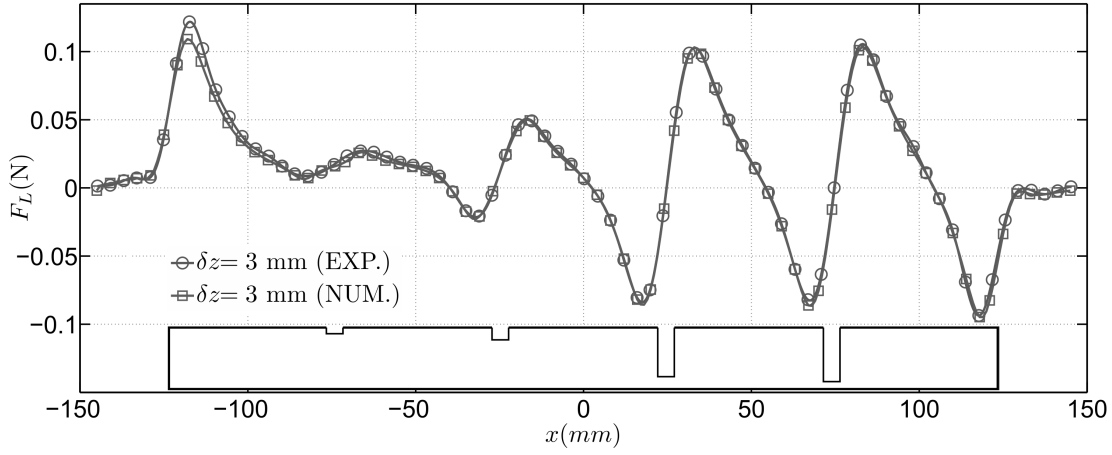
since the front and back walls of the conductor are far away from defects and do not influence the Lorentz force perturbations (Fig. 5.19). The calculated WNRMS errors of the drag and lift Lorentz force components are equal to $\delta_D^{"-"} = 16.7\%$, $\delta_L^{"-"} = 18.4\%$, and $\delta_D^{"+"} = 5.5\%$, $\delta_L^{"+"} = 5.4\%$, for wide ("—") and cross ("+") defects, respectively. The WNRMS errors have been calculated with reference to the corresponding experimental signals (EXP) and window size of $\Delta W = 48$ mm. Higher discrepancy between obtained results has been observed for the wide ("—") defect compared to the cross ("+") defect. This is due to the fact that wide ("—") defects produce relatively small force perturbations which are strongly influenced by the low signal-to-noise-ratio of the measurement signal (section 5.8). Beside validation of the numerical results, the analysis also helps to determine the influence of the shape of the defect on the resulting Lorentz force perturbations. For the given size of the defects, the strong dependency of the Lorentz force perturbations on the actual shape of defects can be observed. This is a promising result since it can be used to identify and localise defects by solving the corresponding inverse problem.

5.8.4 Validation of the Isotropic Conductivity Model

For the validation of MMA and MDA, experiments with the laminated conductor has been used (Fig. 5.17). This configuration has been numerically handled by applying the anisotropic conductivity model (ACM), which resulted in a very good agreement with measured force signals. In order to additionally validate the isotropic conductivity model (ICM), described by (5.13), MMA has been applied to the solid test conductor with linear surface defects (Fig. 5.17). The size and the material properties of the conductor are the same as used for the metallic sheets analysis (see Table 5.3). Fig. 5.21 shows the obtained Lorentz force perturbations for the whole solid conductor, i.e.



(a) Drag Lorentz force component



(b) Lift Lorentz force component

Figure 5.21: Validation of numerical results obtained for isotropic conductivity model (ICM) and moving magnet approach (MMA) with experiments (EXP). The test conductor is solid with idealised surface defects in form of linear slits.

taking its front and back walls into account. Similar to the previous ACM analysis the results agree well with experiments (EXP). The calculated WNRMS errors of the drag and lift Lorentz force components, with respect to the corresponding experimental signals (EXP) and a window size of $\Delta W = 250$ mm, are equal to $\delta_D^{ICM} = 4.7\%$ and $\delta_L^{ICM} = 6.8\%$. The same constant drift in the experimental force signals is observed again (Fig. 5.19 and Fig. 5.21). This only confirms the observation made before that the drift is caused by a small tilt in the mounting of the specimen leading to a constant change in the lift-off distance between the magnet and the conductor.

Chapter 6

Results and Discussion

In this chapter results of 3D finite element simulations of a solid electrically conducting bar moving in the vicinity of a permanent magnet are presented. In particular, parametric studies have been carried out in order to quantify effects of various parameters on the total value of the Lorentz force acting on the magnet. The simulations have been performed for two characteristic LET testing configurations, namely (i) for a conductor without defects and (ii) for a conductor with pre-defined material defects. In case of a non-defective conductor, the parametric studies have been used to quantify the effect of the lift-off distance, magnetization of the used magnets, and the magnetic Reynolds number on the resulting Lorentz force (section 6.1). The study involving defective conductors provides guidelines regarding more effective force measurements. The influence of the magnet shape and size on defects detection and reconstruction is analysed as well (section 6.2).

6.1 Non-defective Conductor

Considering a conductor free of any material defects, the actual problem under investigation is described in Fig. 5.1. If not stated otherwise all geometrical and material parameters described in Table 5.3 apply here as well. Since the length of the conductor is much larger than the diameter of the used cylindrical permanent magnet ($L_c \gg D_m$), and the conductor is free of defects, the quasi-static formulation given by (5.4) and (5.5) is well suited for the analysis. In the assumed moving frame of reference the additional velocity term ($\mathbf{v} \times \nabla \times \mathbf{A}$) describes the motion of the conductor enabling only one stationary analysis to be performed for an accurate steady state solution. In the non-conducting domains (air and magnet) the magnetic scalar potential (2.35) is used. Due to identical geometry of the problem, the same boundary and interface conditions described in section 5.6.2 also apply. In order to obtain more efficient mesh

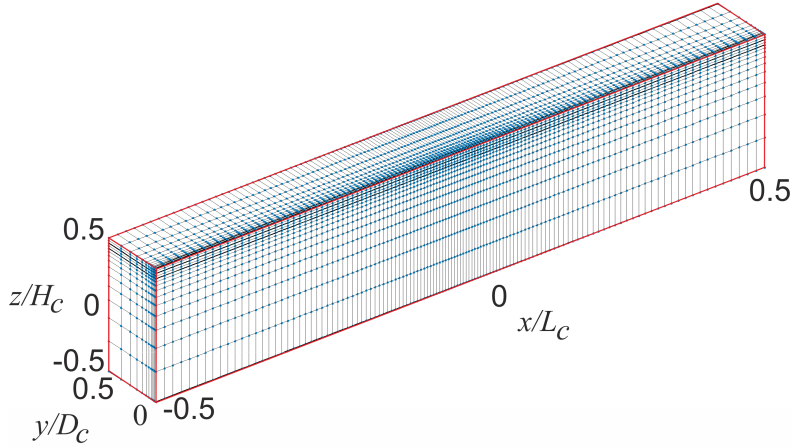


Figure 6.1: Distribution of the finite element mesh inside the non-defective conductor used in the QSA modelling. The mesh consists of 2^{nd} order hexahedral elements.

Table 6.1: Mesh convergence study ($R_m = 10$)

NoE	DoF	$F_D(N)$	$F_L(N)$
4370	255711	3.8438	4.1884
6704	301499	3.8392	4.1920
9638	361805	3.8371	4.1938
13220	440826	3.8362	4.1947
17498	537158	3.8356	4.1952
22520	649873	3.8353	4.1955

distribution inside the non-defective conductor it is more appropriate to use hexahedral instead of tetrahedral finite elements (Fig. 6.1). The size of successive elements along the edges of the conductor has been controlled by means of the geometric progression. The applied common ratio of elements was $r_x = 1/6$, $r_y = 1/5$, and $r_z = 1/7$ along the x , y and z -axis, respectively. The total number of elements (NoE = 22520) used for the analysis is based on the performed sensitivity study providing the converged Lorentz force values independent on the actual mesh distribution (Table 6.1).

Due to more complex geometries, the magnet and the surrounding air region are meshed with quadratic tetrahedral elements having finer distribution in the close vicinity to the magnet. The resulting mesh distribution has not been shown here for simplicity.

6.1.1 Magnetic Reynolds Number Study

To illustrate the validity of QSA for any value of the magnetic Reynolds number (R_m) when conductor is free of defects, a semi-analytical model described by Lee and Menendez [134] has been used. The method provides exact expressions for Lorentz force acting on a current carrying coil moving above a conducting sheet of arbitrary thickness which are calculated by means of Fourier transform approach. To approximate the

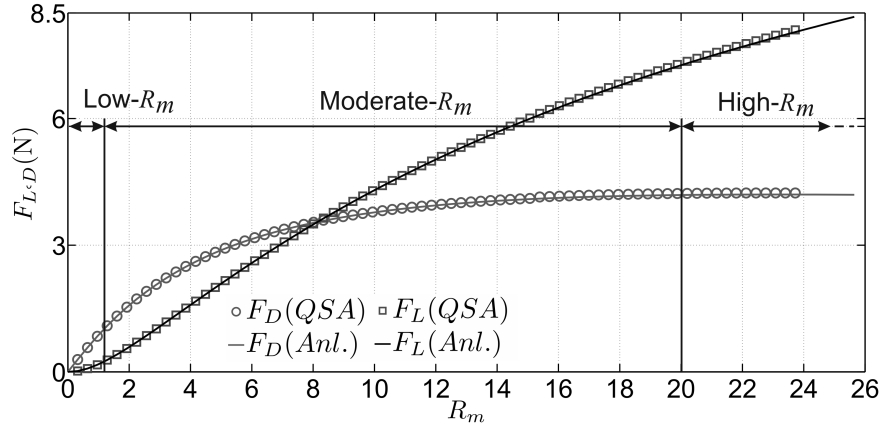
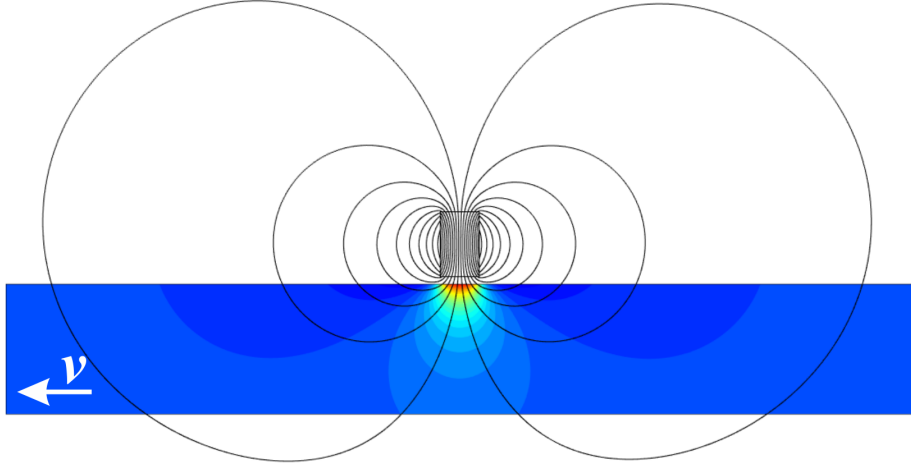


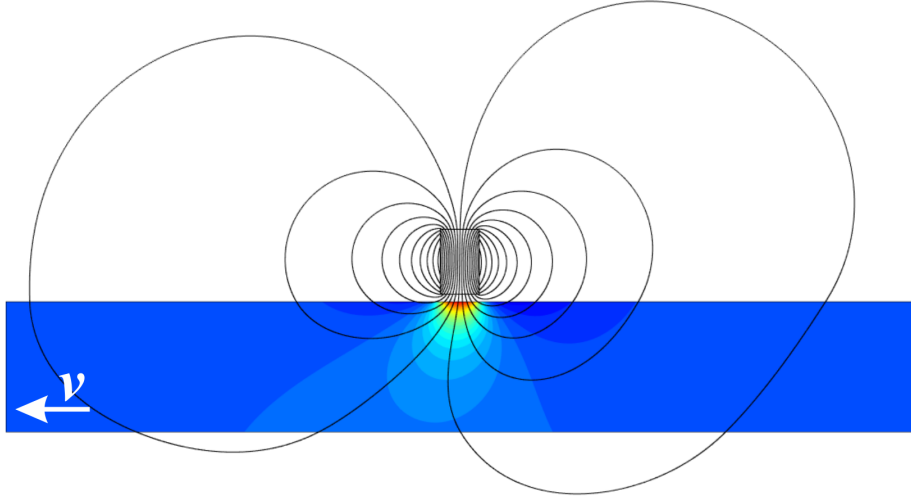
Figure 6.2: Lorentz force dependency on the value of R_m . Study is performed for conducting plate with thickness H_c . The corresponding velocity range is $v = 0..40$ m/s.

cylindrical shape of the magnet, an array of n parallel octagon shaped coils which are carrying the current I_0 equal to $nI_0 = B_r H_m / \mu_0$ were defined. Here $B_r = \mathbf{M}\mu_0$ and H_m are the remanence and the height of the used permanent magnet, respectively. This semi-analytical approach has been implemented in Matlab[®] and enables fast force calculations [137]. In order to account for an infinite plate, in the implementation of QSA the conductor is considered to be long and wide enough compared to the size of the magnet, i.e. $L_c, W_c \gg D_m, H_c$. The comparison between numerical and semi-analytical solutions for a wide range of the resulting magnetic Reynolds numbers, i.e. $R_m = 0..20$ ($v = 0..40$ m/s) is presented in Fig. 6.2. A very good agreement between both approaches is obtained in the whole range of R_m . In order to better understand the physical background of such a force behaviour it is useful to distinguish three characteristic regions presented in Fig. 6.2, namely (i) low- R_m , (ii) moderate- R_m , and (iii) high- R_m range.

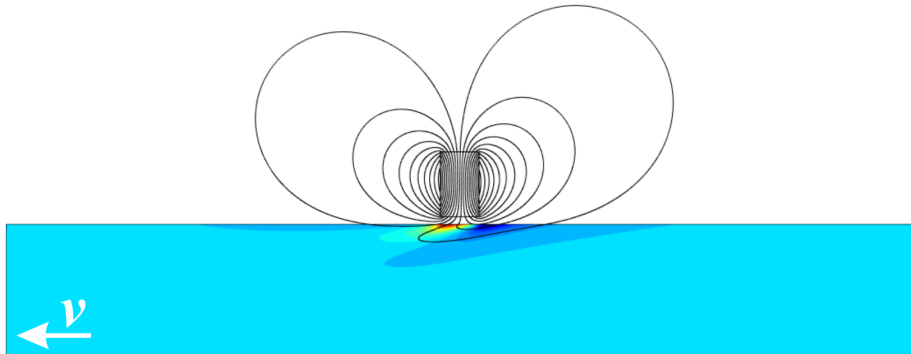
In the low- R_m range ($R_m < 1$) the dependency of the Lorentz force with R_m is well described. The drag component of the Lorentz force varies linearly with R_m ($F_D \sim R_m$), whereas the corresponding lift force component varies quadratically with R_m ($F_L \sim R_m^2$) [23, 133, 134]. This is mostly because the induced (secondary) magnetic field $\mathbf{B}_s$ is low compared to the applied (primary) magnetic field $\mathbf{B}_p$ (Fig. 6.3a). As a consequence, the total magnetic field $\mathbf{B}$ is equal to $\mathbf{B}_p$ due to which induced eddy currents have symmetric distribution and penetrate deep inside the conducting domain. Based in this observation, it is therefore expected that, under the similar testing conditions, LET could provide deeper defects detection compared to the traditional ECT technique.



(a) Low- R_m range where $\mathbf{B}_s \ll \mathbf{B}_p$ ($R_m = 0.01$).



(b) Moderate- R_m range where $\mathbf{B}_s < \mathbf{B}_p$ ($R_m = 1$)



(c) High- R_m range where $\mathbf{B}_s \sim \mathbf{B}_p$ ($R_m = 100$)

Figure 6.3: Influence of the induced (secondary) magnetic field $\mathbf{B}_s$ on the total magnetic field $\mathbf{B}$ distribution with respect to R_m .

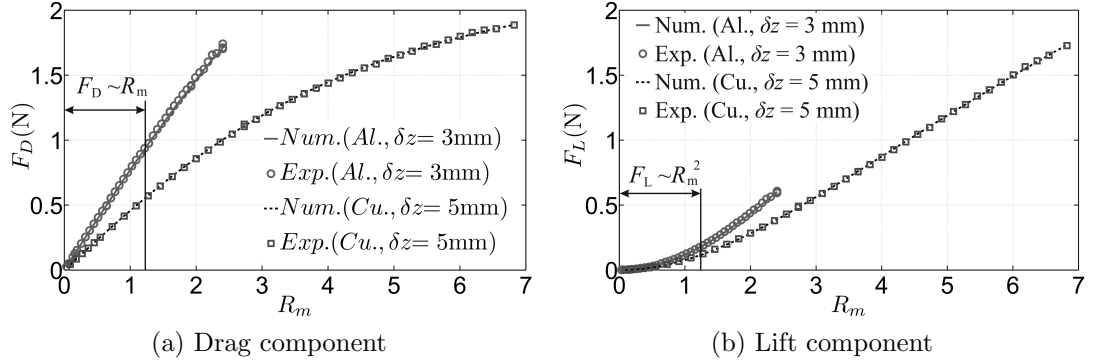


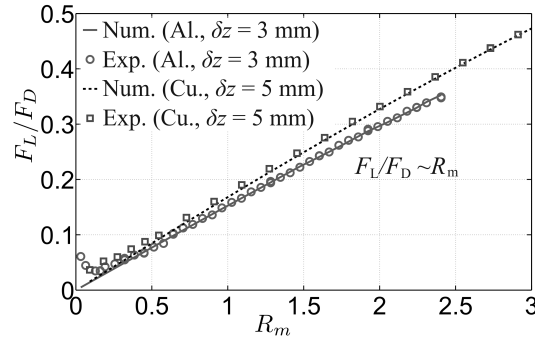
Figure 6.4: Lorentz force dependency on the value of R_m . Study is performed for two conductors made from aluminium ($\sigma_{Al} = 24.1$ MS/m) and copper ($\sigma_{Cu} = 57.9$ MS/m).

In the moderate- R_m range ($1 < R_m < 20$) the induced secondary magnetic field starts to influence the total magnetic field distribution. This so-called motional skin effect makes the distribution of induced eddy currents asymmetric and consequently reduces their penetration depth (Fig. 6.3b). As a result the non-linear dependency of drag and lift force components with R_m is observed (Fig. 6.2).

In the high- R_m range ($R_m > 20$) the motional skin effect is pre-dominant. The magnetic field is almost completely pushed out from the interior of the conductor inducing the current only close to the surface of the conductor (Fig. 6.3c). Thus, the drag component of the Lorentz force decreases with increasing R_m , whereas the corresponding lift force still continues to rise with R_m (Fig. 6.2). It should be mentioned that, for extreme values of R_m both forces reach their asymptotic limits [133].

The characteristic Lorentz force to R_m dependency (Fig. 6.2) has been additionally confirmed experimentally [15, 138]. To reach moderate values of R_m , two different conductors namely, from aluminium ($\sigma_{Al} = 24.1$ MS/m) and copper ($\sigma_{Cu} = 57.9$ MS/m) have been used. In both cases the conductor had the same geometry with a finite cross-section described by parameters given in Table 5.3. The analysis also helps to validate the results of QSA used in all numerical calculations. As expected, the obtained results confirm observations made above and clearly show the linear and the quadratic dependency of drag and lift Lorentz force components with R_m , respectively. It should be mentioned that the actual limit of low- R_m range ($R_m \sim 1$) strongly depends on the definition of the characteristic length scale L (section 2.2.4). Nevertheless, the experiments additionally justify the choice of $L = H_c/2$ for the LET benchmark problem under investigation (Fig. 5.1).

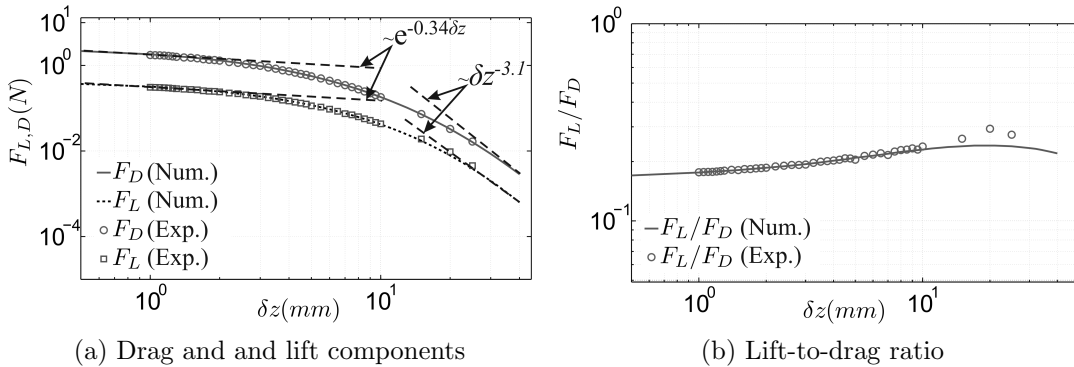
In order to measure the efficiency of magnetic levitation (MAGLEV) systems the so-called lift-to-drag ratio of the Lorentz force is often used [133, 134]. Based on the previous analysis, in low- R_m range the lift-to-drag force ratio in LET depends linearly

Figure 6.5: Dependency of the lift-to-drag ratio on the value of R_m .

on R_m , i.e. $F_L/F_D \sim R_m$ (Fig. 6.5). Moreover, it can be observed that the linearity is preserved even for moderate values of R_m ($R_m \sim 3$) which in case of a copper conductor ($\sigma_{Cu} = 57.9$ MS/m) corresponds to testing velocities of $v \sim 1.7$ m/s.

6.1.2 Lift-off Dependence Study

It is well known that the magnitude of the eddy currents induced inside the moving conductor vary with the relative position between the magnet and the conductor [37, 139]. For simplified LET configurations, when the magnet is described with a point dipole formula (4.1) and when the conductor has the form of an infinite plate, it can be shown analytically that the drag component of the Lorentz force varies monotonically with the so-called lift-off distance δz , i.e. $F_D \sim \delta z^{-3}$ [25, 26]. For realistic LET systems such as given LET problem (Fig. 5.1) the lift-off dependency can not be determined in the closed form analytically [67]. Thus, in this section, the effect of δz on the resulting Lorentz force acting on the magnet has been studied numerically. The validation of results with corresponding experiments has been performed as well (see Fig. 6.6a). Apart from a good agreement between numerical and experimental predictions it is more important to notice a similar dependency of both Lorentz force components on variations

Figure 6.6: Lift-off dependency of the Lorentz force. Testing velocity is $v = 2$ m/s.

in δz . For low lift-off distances ($\delta z < 5$ mm) the obtained decay is exponential, with small constant decay rate of $F_{L,D} \sim e^{-0.34\delta z}$. It has been shown before that actual the decay rate is not depending on the testing velocity or the size of the used magnets [138]. For larger distances the power decay of $F_{D,L} \sim \delta z^{-3.1}$ for both force components is reached (Fig. 6.6a). From the obtained results an obvious conclusion can be made that by using the lift-to-drag ratio of the Lorentz force (F_L/F_D) the effect of variations in δz can be considerably reduced or even completely avoided. This assumption has been demonstrated in Fig. 6.6b both, numerically and experimentally.

6.1.3 Magnetization Strength Study

The magnetization strength $|\mathbf{M}|$ of the permanent magnets is usually unknown or it is given in a wide range by the manufacturer. In practice, $|\mathbf{M}|$ is determined by measurements of the external magnetic field and then deducing $|\mathbf{M}|$ from this field. However, for design and practical realization of future LET systems it is necessary to estimate the sensitivity of the Lorentz force on possible variations in $|\mathbf{M}|$. For the given LET benchmark problem, the effect of the magnetization strength on both Lorentz force components has been calculated numerically (Fig. 6.7a). For the analysis it is assumed that the magnet is uniformly magnetized in the z -axis direction, i.e. $\mathbf{M} = M\mathbf{e}_z$. From the obtained results it can be observed that both Lorentz force components scale quadratically with $|\mathbf{M}|$. This observation was also confirmed in [25, 26], where instead of realistic permanent magnets the point dipole approximation has been used. This means that the characteristic lift-to-drag ratio of Lorentz force is independent on the actual magnetization of the used permanent magnets (Fig. 6.7b).

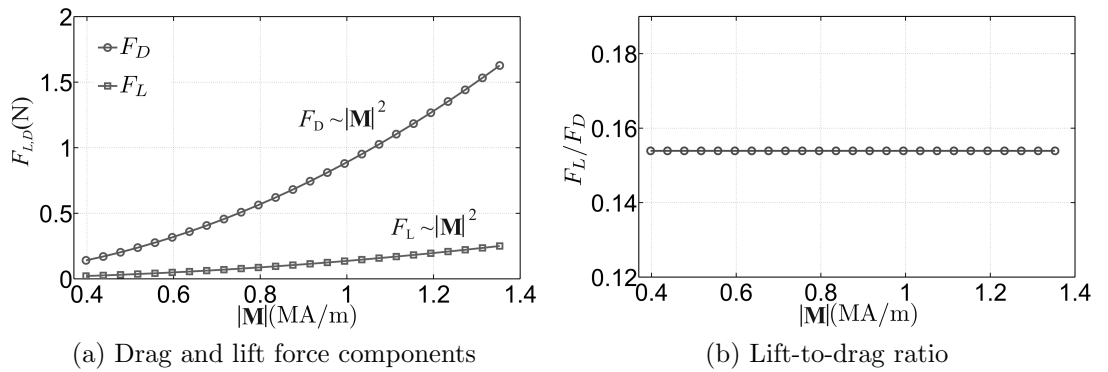


Figure 6.7: Influence of the magnet magnetization strength $|\mathbf{M}|$ on the Lorentz force ($R_m = 1$).

6.1.4 Magnet Size Study

In the previous section it has been shown that the absolute Lorentz force acting on the magnet increases quadratically with the magnetization strength $|\mathbf{M}|$ of the magnet (see Fig. 6.7). The main aim here is to study how the size of the magnet, namely its volume affects the intensity of the Lorentz force. Similar studies have been performed in magnetohydrodynamics (MHD) applications, in particular for Lorentz force velocimetry (LFV) investigations where Lorentz forces are used to measure the flow rates of liquid metals and even glass melts [25, 26]. Due to relatively low electrical conductivities, the main interest in LFV is to increase the intensity of the Lorentz force acting on the magnet system while minimising its weight [140]. In LET systems this information can help to choose the optimal force sensor or the magnet size for specific NDT&E application.

For the analysis, the LET benchmark problem (Fig. 5.1) and cylindrical permanent magnet with $D_m \times H_m = a \times a$ have been used. The magnet is magnetised along the z -axis with magnetization density $\mathbf{M} = M\mathbf{e}_z$ given in Table 5.3. From the obtained results the linear dependency between both Lorentz force components and the resulting volume of the used magnet ($V = \pi/4 a^3$) has been observed (Fig. 6.8). Similar linear dependency of the drag Lorentz force component with respect to the volume of the magnet has been reported for LFV applications involving liquid metals [141].

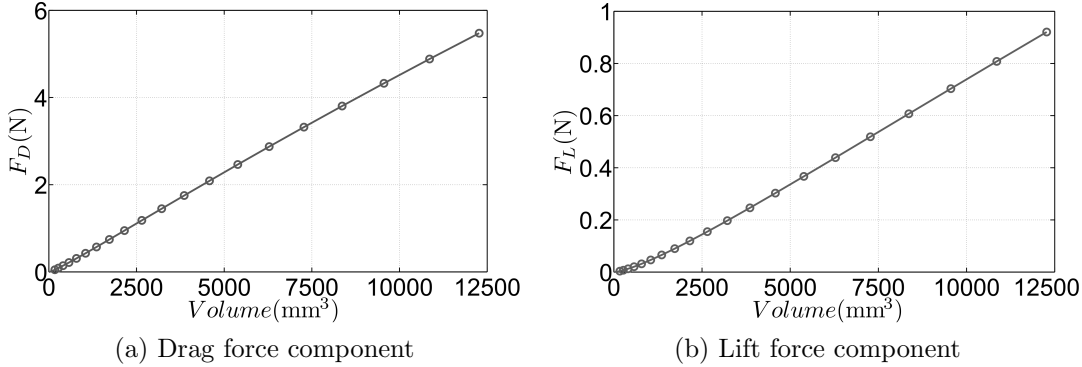


Figure 6.8: Influence of the magnet size (volume) magnetised with constant magnetisation density $\mathbf{M}$ on the Lorentz force ($R_m = 1$). The used magnet has cylindrical shape ($D_m \times H_m = a \times a$) and is placed at a lift-off distance $\delta z = 1$ mm.

6.1.5 Lorentz Force Sigmometry

Based on the previous observations the working principle of LET has found its practical application for contactless measurement of the specific electrical conductivity of a solid or an electrically conducting fluids [137, 142]. The technique is termed Lorentz Force Sigmometry (abbreviated LoFoS) where the neologism Sigmometry is derived from the Greek letter sigma, often used to denote the electrical conductivity. The main aim of this section is to give a brief theoretical description of LoFoS, whereas the measurement procedure can be found in [15, 137].

Using the definition of the magnetic Reynolds number (2.33) and assuming the low- R_m region ($R_m < 1$) it has been shown that dependency of Lorentz forces with R_m are as follows

$$F_D \sim R_m, \quad (6.1)$$

$$F_L \sim R_m^2. \quad (6.2)$$

An obvious idea to calculate the electric conductivity using only the drag or the lift Lorentz force component would lead to higher measurements uncertainties due to the high force dependency on the variations in lift-off distance and the strength of the used magnet [137] (Fig. 6.6a and Fig. 6.7a). To overcome these disadvantages a modified approach based on the lift-to-drag ratio has been applied. As a result of performed parametric studies which additionally confirm (6.1) and (6.2) it has been shown that the ratio F_L/F_D depends linearly on R_m

$$\frac{F_L}{F_D} \sim R_m. \quad (6.3)$$

Under the assumption that the conductivity is homogeneous, isotropic and the testing speed is constant, the lift-to-drag force ratio and the electrical conductivity are related as

$$\sigma = \alpha_s \frac{F_L}{F_D}, \quad (6.4)$$

where the calibration coefficient α_s depends on the geometry of the magnet system, on the translational velocity and (weakly) on the distance between the plate and the magnet (Fig. 6.6b). Since α_s is *a priori* unknown, the LoFoS system has to be calibrated. For simplified LoFoS configurations, to obtain the necessary calibration curves a semi-analytical model presented in section 6.1.1 can be used. When conductors have finite dimensions or measurements are based on more complex magnet systems a numerical model can be applied. For the given benchmark problem the calibration curves obtained numerically for different testing velocities are presented in Fig. 6.9a. In the

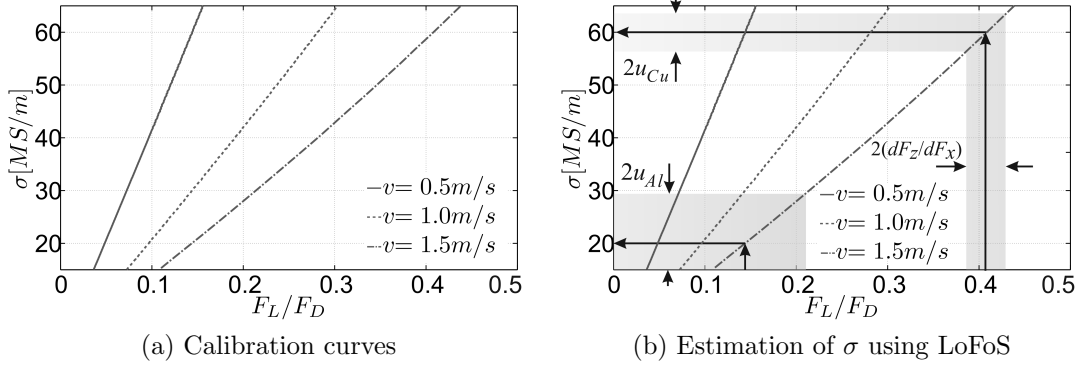


Figure 6.9: Numerically obtained calibration curves for lift-off distance $\delta z = 3$ mm

wide range of considered (unknown) electrical conductivities the results show fairly linear dependency between σ and the lift-to-drag Lorentz force ratio F_L/F_D . Nevertheless, this is only true if the condition $R_m < 1$ is satisfied, i.e. in the low- R_m regime. For higher R_m values the non-linearity caused by the motional skin effect has to be considered.

In order to prove the theoretical observations experimentally, the electrical conductivity of two solid specimens of known electrical conductivity, namely aluminum ($\sigma_{Al} = 20.4 \text{ MS/m}$) and copper ($\sigma_{Cu} = 57.92 \text{ MS/m}$) have been obtained by LoFoS. The given conductivities have been measured with the eddy current device *Sigmatest* 2.069 (Institut Dr. Foerster GmbH & Co. KG) [15, 137]. For measurements of the lift-to-drag ratio of the experimental setup described in section 5.8 has been used. By applying LoFoS conductivities of $\sigma_{Al} = 21.59 \text{ MS/m}$ and $\sigma_{Cu} = 60.08 \text{ MS/m}$ have been obtained (Fig. 6.9b). The combined measurement uncertainties of $u_{Al} = 9.82 \text{ MS/m}$ and $u_{Cu} = 3.46 \text{ MS/m}$ are based on the uncertainty of the force sensor of $dF_D = 15 \text{ mN}$ and $dF_L = 50 \text{ mN}$ (Table 5.6). These uncertainties can be reduced when the measurements are repeated several times or by applying a more accurate force sensor [15, 137].

The presented analysis showed that LoFoS is able to provide the electrical conductivity of a unknown specimen. In a nutshell, LoFoS has the following advantages (i) LoFoS is contactless, (ii) LoFoS can be applied continuously during production processes, (iii) LoFoS is a method that enables the user to measure conductivity beyond the surface and (iv) the proposed data processing makes LoFoS resistant to changes in the lift-off distance and the strength of the magnetic field source. LoFoS is suitable for specimen of any kind of physical condition, where the only limitation is given by the minimal measurable Lorentz force components of a multi-component force sensor [15].

6.2 Defective Conductor

In this section several parametric studies based on the LET benchmark problem and the conductor with defects (Fig. 5.1) have been performed. The main aim is to quantify the effect of various testing parameters on the resulting perturbations of the Lorentz force, such as: magnet shape and size, magnetization $\mathbf{M}$, lift-off distance δz , depth of defects d and the resulting magnetic Reynolds number R_m . The analysis provides reference results to better understand the feasibility and testing capabilities of LET. In order to provide validation of results the calculations have been performed assuming the laminated structure of the conductor under test.

6.2.1 Magnet Shape Study

It is reasonable to expect that the density and spatial distribution of eddy currents induced inside a conductor in motion, i.e. the resulting Lorentz force depends on the geometry of the used magnets. In the field of optics, the so-called point spread functions (PSF) are typically used in order to describe the quality of an imaging instrument [143]. They were first introduced by Sir Airy in 1835 to describe the best focused spot of light that a perfect circular lens can produce [144]. In principle, PSF can be understood as the three-dimensional diffraction pattern of light emitted from a point source, which is transmitted to an imaging plane of an instrument [143]. In traditional eddy current testing (ECT) a similar approach based on the PSF has been applied to test the sensitivity and performance of different pick-up coil configurations [145, 146]. In contrast to its first application in optics in ECT, PSF is defined as the spatial change of the impedance of the pick-up coil which results from a raster surface scan of the test conductor containing a small *point-like* defect (Fig. 6.10a).

The point-like defects in NDT applications should be understood as defects whose

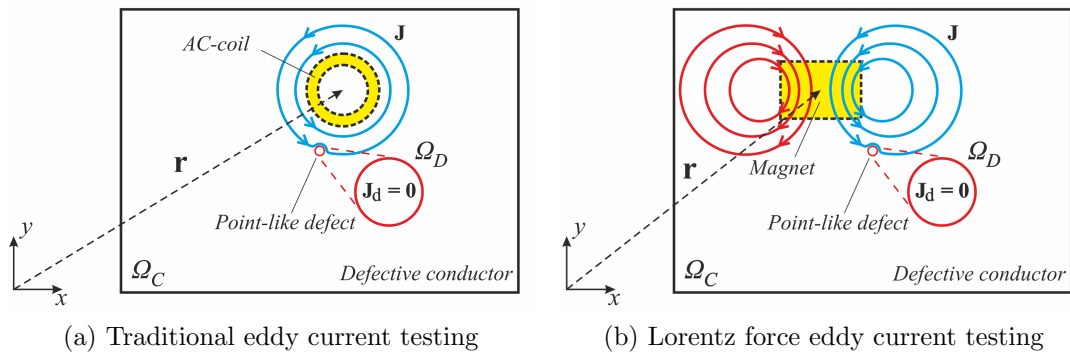


Figure 6.10: Calculation of the point spread function (PSF). The relative position between the point-like defect and the permanent magnet is denoted by $\mathbf{r}$.

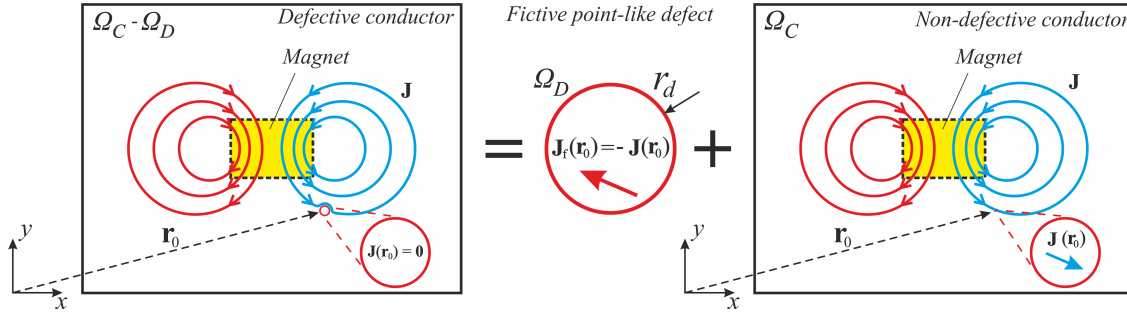


Figure 6.11: Fictive point-like defect modelling used for fast calculation of the point spread function (PSF). $\mathbf{r}_0$ is the relative position between the point-like defect and the permanent magnet.

size is small, so that the applied field $\mathbf{B}$ can be considered to be uniform over its entire volume [146, 147]. Additionally, the dimension of defects must be small compared with the distance from the defect to the nearest boundary of the conductor under test and small compared to the size of the used sensor (see Fig. 6.10). It should be mentioned that the shape of the particular PSF is not only a function of the coils geometry, but it also depends on the current lift-off distance δz and depth of the defect d [146].

Using the same principle as in traditional ECT, the sensitivity of the Lorentz force perturbations on the particular shape and size of the used magnet has been studied by means of PSF. In LET systems, PSF can be defined as the spatial distribution of the Lorentz force perturbation caused by raster surface scanning of the test conductor containing a point-like defect (Fig. 6.10b). In contrast to ECT which have only one PSF for particular pick-up coil configuration [145, 146] in LET systems each Lorentz force component has different and independent PSF. They can be calculated by applying the Lorentz force formula (2.2) for each relative position between the magnet and the point-like defect. Since PSF describes the perturbation of the Lorentz force only due to defects, its calculation involves two separate analysis, i.e. for defective and non-defective conductor

$$\Delta \mathbf{F}(\mathbf{r}) = - \underbrace{\iiint_{\Omega_C - \Omega_D} \mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) d\Omega}_{\text{Defective cond.}} + \underbrace{\iiint_{\Omega_C} \mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) d\Omega}_{\text{Non-defective cond.}}, \quad (6.5)$$

where Ω_C is the volume of the test conductor and Ω_D the volume of the small point-like defect. For each relative position between the magnet and the defect the solution of (6.5) would involve separate FEM calculations, which in 3D LET investigations can be very time consuming.

Fortunately, in contrast to ECT the calculation of PSF in LET systems can be considerably simplified (Fig. 6.11). In order to solve (6.5) for each relative position between the magnet and the point-like defect, it is more efficient to move the

defect instead ($\mathbf{r} \rightarrow \mathbf{r}_0$). Furthermore, applying the superposition principle an equivalent model of the conductor can be used (Fig. 6.11). The model assumes fictive current inside the point-like defect $\mathbf{j}_f$ which has the same amplitude but opposite direction as the current in the non-defective conductor at $\mathbf{r}_0$, i.e. $\mathbf{j}_f(\mathbf{r}_0) = -\mathbf{j}(\mathbf{r}_0)$. This makes the total current inside the real point-like defect to zero (Fig. 6.11). Taking all this into account the left integral in (6.5) can be calculated as

$$\iiint_{\Omega_C - \Omega_D} \mathbf{j}(\mathbf{r}_0) \times \mathbf{B}(\mathbf{r}_0) d\Omega = \iiint_{\Omega_C} \mathbf{j}(\mathbf{r}_0) \times \mathbf{B}(\mathbf{r}_0) d\Omega - \iiint_{\Omega_D} \mathbf{j}(\mathbf{r}_0) \times \mathbf{B}(\mathbf{r}_0) d\Omega, \quad (6.6)$$

Applying (6.6) in (6.5) PSF can be obtained by

$$\Delta \mathbf{F}(\mathbf{r}_0) = \iiint_{\Omega_D} \mathbf{j}(\mathbf{r}_0) \times \mathbf{B}(\mathbf{r}_0) d\Omega. \quad (6.7)$$

Since, inside the point-like defect the magnetic field $\mathbf{B}(\mathbf{r}_0)$ and fictive current $\mathbf{j}(\mathbf{r}_0)$ is essentially uniform, the integral (6.7) can be substituted by simple multiplication

$$\Delta \mathbf{F}(\mathbf{r}_0) = (\mathbf{j}(\mathbf{r}_0) \times \mathbf{B}(\mathbf{r}_0)) \cdot \Omega_D, \quad (6.8)$$

which needs to be performed for each relative position $\mathbf{r}_0$ between the magnet and the defect. This can be obtained by ease evaluating the Lorentz force density $\mathbf{f} = \mathbf{j} \times \mathbf{B}$ inside the conductor under test at the xy -plane determined by depth of the defect d

$$\Delta \mathbf{F} = (\mathbf{j} \times \mathbf{B}) \cdot \Omega_D = \mathbf{f} \cdot \Omega_D. \quad (6.9)$$

The particular shape of PSF also depends on the lift-off distance δz of the magnet. As it will be shown later, typically symmetrical shapes of PSF can be significantly deformed when magnetic Reynolds number (R_m) is high.

Without any loss of generality, to test the sensitivity of the Lorentz force perturbations on the particular shape and size of magnets the low- R_m regime has been considered ($R_m = 0.1$). The effect of R_m has been investigated in a separate section using realistic artificial defects described in experiments (see section 6.2.4). PSF have been calculated for spherical, cylindrical and cubic magnets which are typically applied in LET. All magnets have been placed at the fixed lift-off distance $\delta z = 1$ mm and magnetised along z -axis with magnetization density $\mathbf{M}$ defined in Table 5.3. To provide a comprehensive comparison the analysis has been performed for (i) magnets having the same size determined by the length parameter a (Fig. 6.12a) and (ii) magnets having the same characteristic volume Ω_M (Fig. 6.12b). Figure 6.13 shows the characteristic

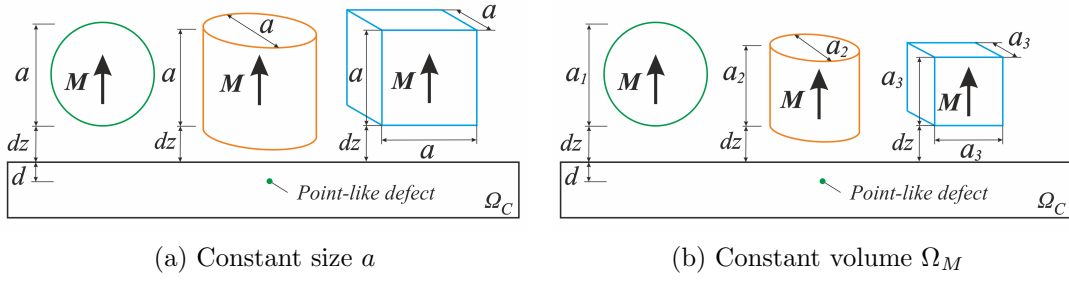
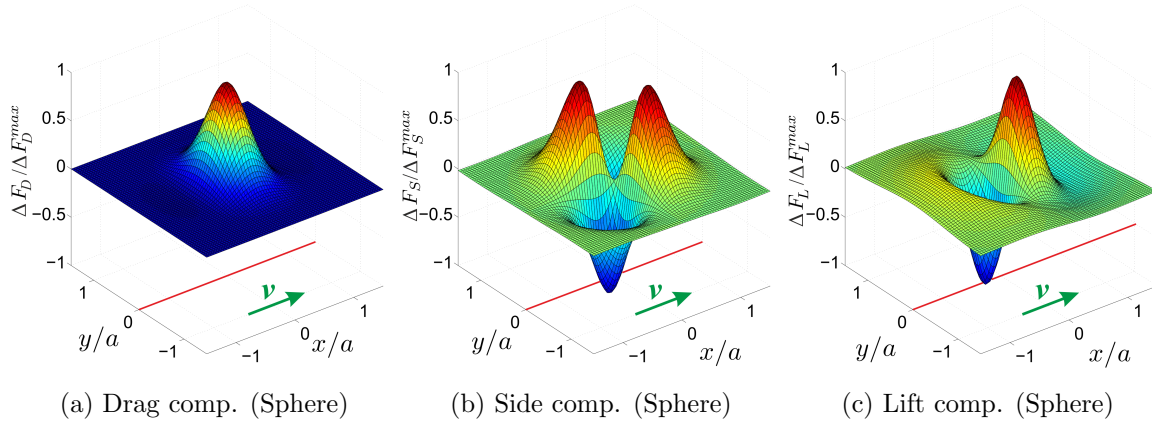


Figure 6.12: Three characteristic shapes of permanent magnets used for in LET systems.


 Figure 6.13: Typical shape of the PSF obtained for a spherical magnet having diameter a at lift-off distance $\delta z = 1$ mm and $R_m = 0.1$. The depth of the point-like defect is $d = 1.5$ mm.

shape of PSF for all three Lorentz force components of a spherical magnet. The radius of the point-like defect used for PSF calculations was $r_D = a/30$. For generality and to provide better comparison, the obtained results have been scaled to the resulting maximum force perturbation $\Delta F_{i,sphere}^{max}$ ($i \in D, L$) of a spherical magnet. The shapes of PSF resulting from cylindrical and cubic magnets are almost identical to those shown in Fig. 6.13. Thus, they are not presented for simplicity. More informative direct comparison between the considered magnet shapes is provided by taking a linear cut of the corresponding PFS at $y = 0$ along the x -axis, i.e. in the motion direction (Fig. 6.13). Since at $y = 0$ the side component of the Lorentz force F_S is zero, it was not used for comparison.

When magnets have the same characteristic size a (Fig. 6.12a) the strongest Lorentz force perturbations have been obtained for a cubic magnet (Fig. 6.14). The increase in the force perturbation with respect to the spherical magnet is ~ 2.5 and ~ 3 times for drag and lift Lorentz force components, respectively. The force perturbation resulted from a cylindrical shape was lower compared to cubic magnet shape as well (see Fig. 6.12a).

When magnets have the same volume Ω_M (Fig. 6.12b) similar results have been

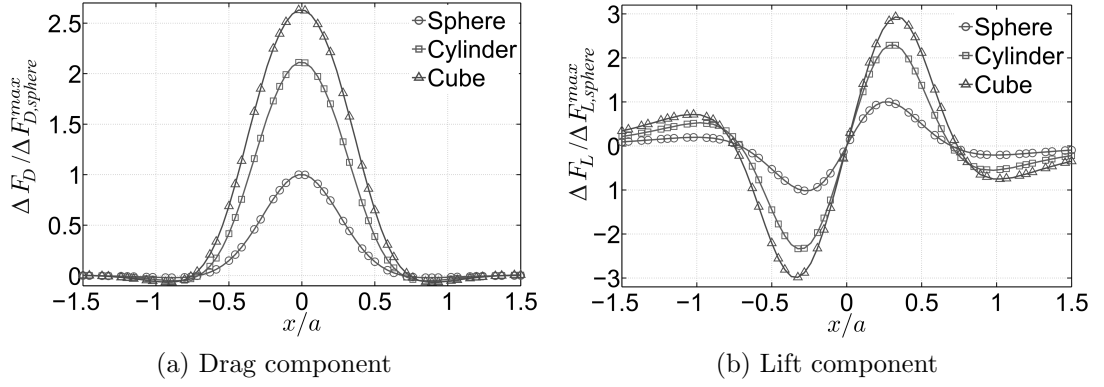


Figure 6.14: Direct comparison of normalized PSF for three typical shapes of permanent magnets. All magnets have the same characteristic size determined by length the parameter a_i . The maximum perturbation of the spherical magnet $\Delta F_{i,sphere}^{max}$ ($i \in D, L$) is used as a reference.

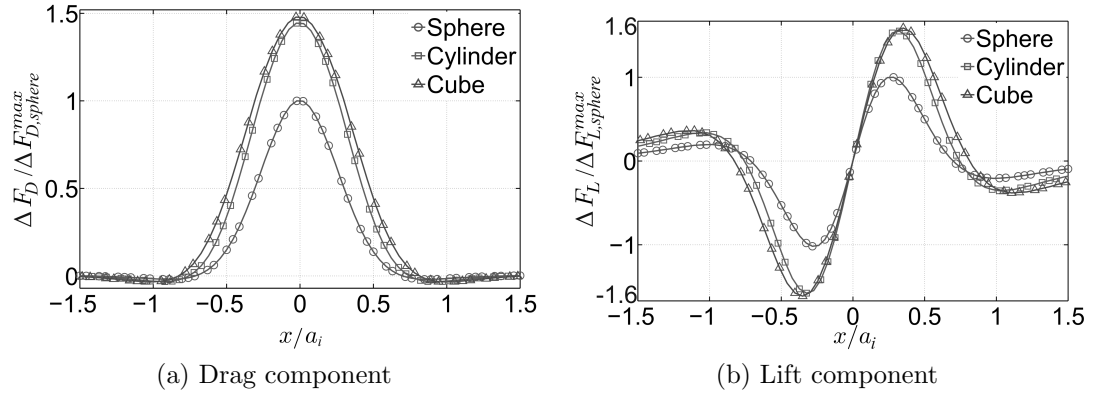


Figure 6.15: Direct comparison of normalized PSF for three typical shapes of permanent magnets. All magnets have the same volume Ω_M which results in different characteristic length a_i ($i \in a_1, a_2, a_3$). The maximum perturbation of the spherical magnet $\Delta F_{i,sphere}^{max}$ ($i \in D, L$) is used as a reference.

obtained. Nevertheless, in contrast to the previous case the difference in the maxima of force perturbations was not so expressed (Fig. 6.15b). The force perturbations resulting from cubic and cylindrical magnets are almost identical for both Lorentz force components, but still ~ 1.5 larger with respect to the spherical magnet.

Thus, independent on the type of the defect to be detected, in order to increase the resulting Lorentz force perturbations it is advisable to use larger magnets of cubic shape. On the other hand, in the LET applications based on absolute force measurements, larger magnets will result in higher absolute forces which scale linearly with the resulting volume of the magnet (see section 6.1.4). Even though this increases the force perturbations as well they are still 2 or even 3 orders of magnitude lower than the absolute forces acting on the magnet. This actually means that, it could be very difficult task to resolve the resulting force perturbations using the absolute force sensors today commercially available. Furthermore, if the size of the magnet is increased the weight of the whole magnet system which is carried by the force sensor needs to

be taken into account [15]. Nonetheless, independent on the shape and type of defects PFS based approach described above can be used to test and optimize different magnet configurations to further increase force perturbations.

6.2.2 Magnet Size Study

Increasing the size of the magnet with respect to the size of defects would result in higher Lorentz force perturbations. This is of great interest if only the detection of defects within the conductor is required. However, in non-destructive evaluation (NDE) applications it is not only enough to detect defects, but the shape reconstruction is required as well [11, 148]. As it will be shown below, with increasing the magnet size the information content of the actual shape of defects is considerably reduced. Apart from allowing the fast estimation of the intensity of force perturbation for different shapes of magnets, PSF are very useful to determine the sensitivity of force signals on the actual shape of defects as well. To this end, based on Fig. 6.13 for each Lorentz force component an effective radius of the PSF can be defined as the distance from the center of the magnet to the point in space where the intensity of the force perturbation drops to zero ($\Delta \mathbf{F}(r_i^{PSF}) = 0$). The effective radius is denoted by r_i^{PSF} where $i \in D, S, L$ represents the drag, side and lift Lorentz force component, respectively. For the characteristic shapes of the magnets typically used in LET (sphere, cylinder and cube) all three force components have approximately the same effective PSF radius $r_i^{PSF} \approx a$ (see Fig. 6.13). This is expected since all magnets under investigation have the same characteristic size determined by a . In principle this means that, independent on the particular shape of defects, PSF allow to estimate the information content of the force signals by only one parameter given by a .

If defects are larger than the characteristic size of the magnet a , the force perturbation will be strongly influenced by the shape of the defect. This means that the force signal carries enough information about the particular defect which should simplify its reconstruction and localization. When defects are small compared to the size of the magnet ($w, h, l < a$) the resulting force perturbations will have the similar form as the PSF shown in Fig. 6.13. Thus, the information about the defect shape will be considerably reduced or even lost completely. For example, if there is more than one defect in the area determined by parameter a , it would be very difficult to determine whether the force is perturbed by only one larger defect or by several smaller defects. Strictly speaking when $w, h, l < a$, the resulting force perturbations have always the same form determined by the shape of the particular PSF, i.e. by the size and shape of the used magnets.

A very intuitive example confirming the observations mentioned above can be pre-

sented in form of direct defect imaging. Thus, the LET benchmark problem defined in Fig. 5.1 has been applied and solved using weak reaction approach (WRA) approach (see section 5.4). Due to the strongest force perturbation the analysis has been performed for the cubic permanent magnet with $a = 1, 6, 12$ and 24 mm. For imaging the cross ("+") defect lying at $d = 2$ mm under the surface of the conductor has been considered. As expected, for relatively small magnet sizes ($a = 1$ mm) the Lorentz force perturbations are strongly influenced by the shape of the defect. Due to different characteristic PSF shapes (see Fig. 6.13) each Lorentz force component carries different and independent information about the particular defect (Fig. 6.16a, 6.16b and 6.16c). This is of great interest in NDE where defects have to be reconstructed. Furthermore, due to the characteristic Mexican hat shape of the PSF (Fig. 6.13a), the drag force component can even provide the direct image of the cross ("+") defect without additionally solving an inverse problem.

The quality of an image is not only influenced by the effective radius of the PSF ($r_i^{PSF} \approx a$), i.e. the size of the magnet, but it also depends on the depth of the defect d and lift-off distance δz . Defects lying deeper inside the material and magnets at higher δz will result in somehow blurry images of the defect as seen in Fig. 6.16a. It is important to notice that, due to the small size of the magnet the intensity of the force perturbation is very low. Increasing the size of the magnet the intensity of Lorentz force perturbations increases, while their information content about the particular defect shape is considerably reduced (Fig. 6.16). As mentioned above, when magnet is larger than the used cross ("+") defect ($a = 24$ mm) the obtained image is mostly determined by the shape of the magnet (Fig. 6.16j, 6.16k and 6.16l). In this case the shape reconstruction of defects would be very difficult or even impossible to perform.

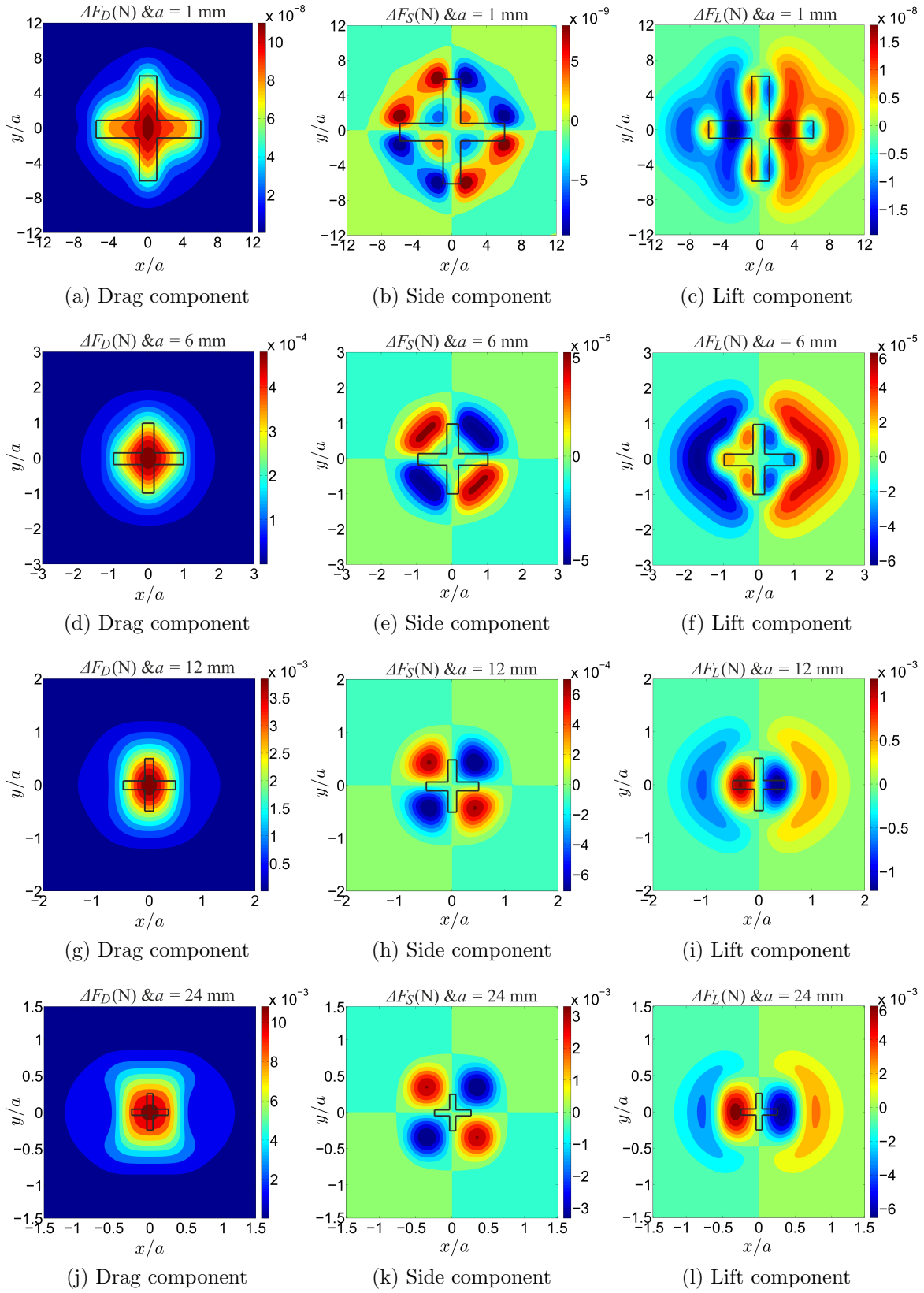


Figure 6.16: LET direct imaging of cross ("+") defect lying at $d = 2$ mm under the conductor surface. Cubic permanent magnet with $a = 1, 6, 12$ and 24 mm is located at lift-off distance $\delta z = 1$ mm.

6.2.3 Influence of the Magnetisation Direction and Intensity

Magnetisation Direction

In the previous section the effect of the magnet size and shape on the resulting Lorentz force perturbations has been discussed. The main aim in this section is to investigate the effect of the magnetisation vector on the intensity of Lorentz force perturbations.

To study the effect of the magnetization direction PSF procedure has been applied again. Thus, the same point-like defect $r_d = a/30$ and material parameters described in section 6.2.1 have been used. Without any loss of generality the magnet is considered to be of spherical shape with $a = 5$ mm. In order to simplify the comparison between different orientations of $\mathbf{M}$, the corresponding PSF have been normalised with reference to $\mathbf{M} = M\mathbf{e}_z$ (see Fig. 6.17). From the obtained results, a very strong dependency of the Lorentz force perturbations with respect to the actual magnetization direction has been observed. The particular direction of $\mathbf{M}$ not only affects the shape of the PSF for each force component, but the intensity of the force perturbations is considerably reduced as well. For this test example the amplitude of the PSF can be up to 4 times lower for both x - and y -axis orientations of $\mathbf{M}$, with respect to its z -axis direction. Based on this results, it is desirable to design LET systems with magnets magnetized along the z -axis, i.e. normal to the direction of motion.

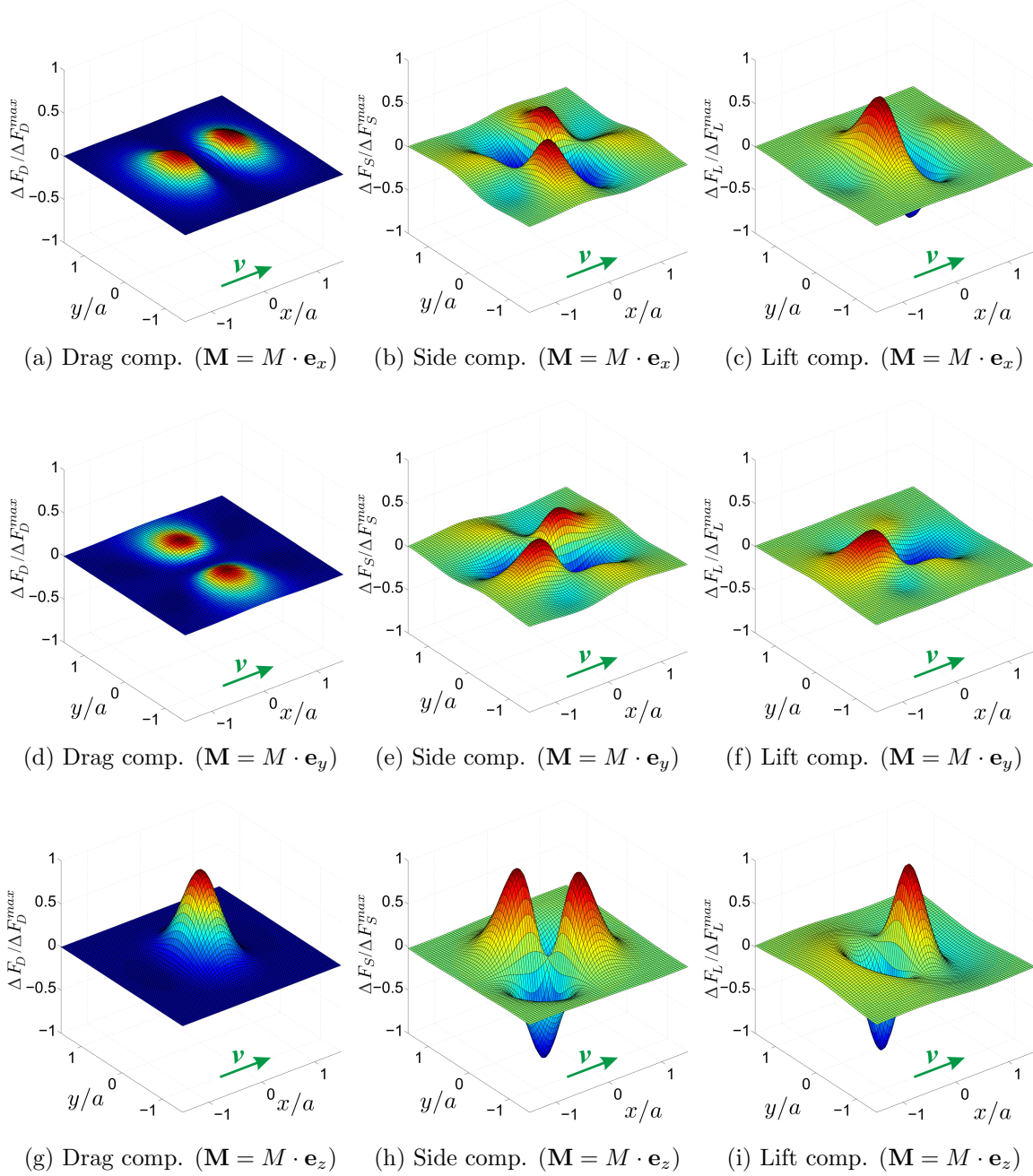


Figure 6.17: Influence of the magnetisation direction $\mathbf{M}$ on the shape of PSF for $R_m = 0.1$. The spherical magnet has a diameter $a = 5$ mm and it is placed at lift-off distance $\delta z = 1$ mm. The depth of the point-like defect is $d = 1.5$ mm. The magnetisation direction along the z -axis has been used as a reference.

Magnetization Intensity

Another important parameter which affects the force perturbations is the strength of magnets. It is well known that the absolute values of the Lorentz force increase quadratically with $|\mathbf{M}|$ (see section 6.1.3). For working with LET systems it is very useful to know how the force perturbations are affected by the particular value of $|\mathbf{M}|$. Thus, the LET benchmark problem with all material and geometrical parameters given in Table 5.3 has been solved using the MDA approach (see section 5.2.2). For this analysis $R_m = 0.48$ ($v = 0.5$ m/s) and the long (") defect lying $d = 2$ mm under the surface of the conductor has been considered. The cylindrical magnet is placed at $\delta z = 1$ mm above the conductor.

To simplify the comparison of the resulting characteristic force perturbation profiles (see Fig. 5.20), the so-called peak-to-peak (P2P) force perturbations can be used (Fig. 6.18). The P2P Lorentz force perturbation for each force component, denoted

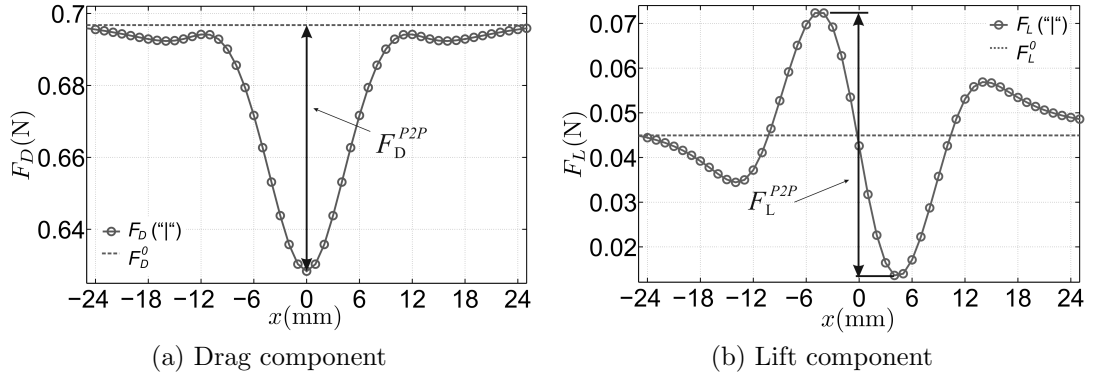


Figure 6.18: Definition of the Lorentz force peak-to-peak (P2P) perturbations. The results have been obtained for the long (") defect.

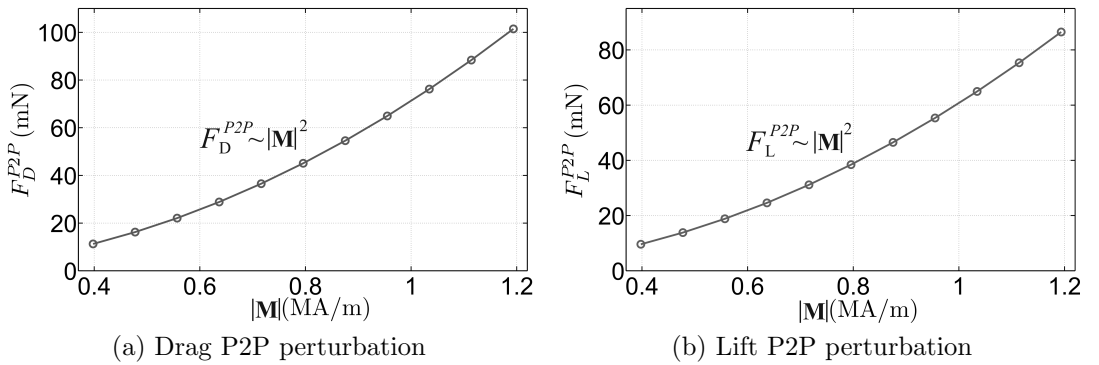


Figure 6.19: Influence of the magnetization strength $|\mathbf{M}|$ on the P2P Lorentz force perturbations. The cylindrical magnet is magnetised in the z -axis direction.

as F_D^{P2P} and F_L^{P2P} can be described as

$$F_D^{P2P} = \max(F_D - F_D^0) - \min(F_D - F_D^0), \quad (6.10)$$

$$F_L^{P2P} = \max(F_L - F_L^0) - \min(F_L - F_L^0), \quad (6.11)$$

where F_D^0 and F_L^0 represent the drag and the lift Lorentz force components for conductor free of defects, respectively. Since the magnet was positioned centrally in the middle of the conductor under test the side component F_S of the Lorentz force was zero. The obtained results show, that the P2P perturbations of both Lorentz force components, as well as their corresponding absolute values are increasing quadratically with the strength of applied magnets (Fig. 6.19). Thus, to increase the testing depth of LET systems stronger magnets should be applied.

6.2.4 Influence of the Magnetic Reynolds Number

A detailed analysis describing the influence of the magnetic Reynolds number (R_m) on the absolute Lorentz forces acting on the magnet for non-defective conductor is given in section 6.1.1. It has been observed that different Lorentz force components

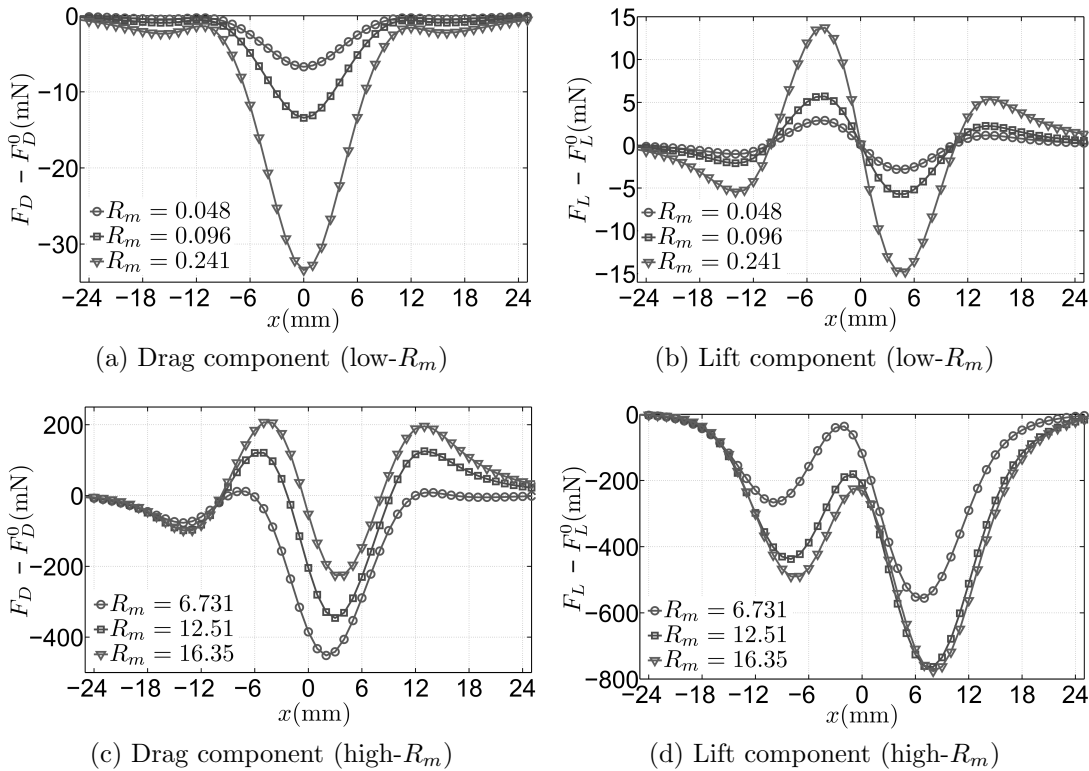


Figure 6.20: Influence of the magnetic Reynolds number R_m on the characteristic Lorentz force perturbations. Calculations have been performed for the long (") defect located at $d = 2$ mm using MDA. The magnet is located at a lift-off distance of $\delta z = 1$ mm.

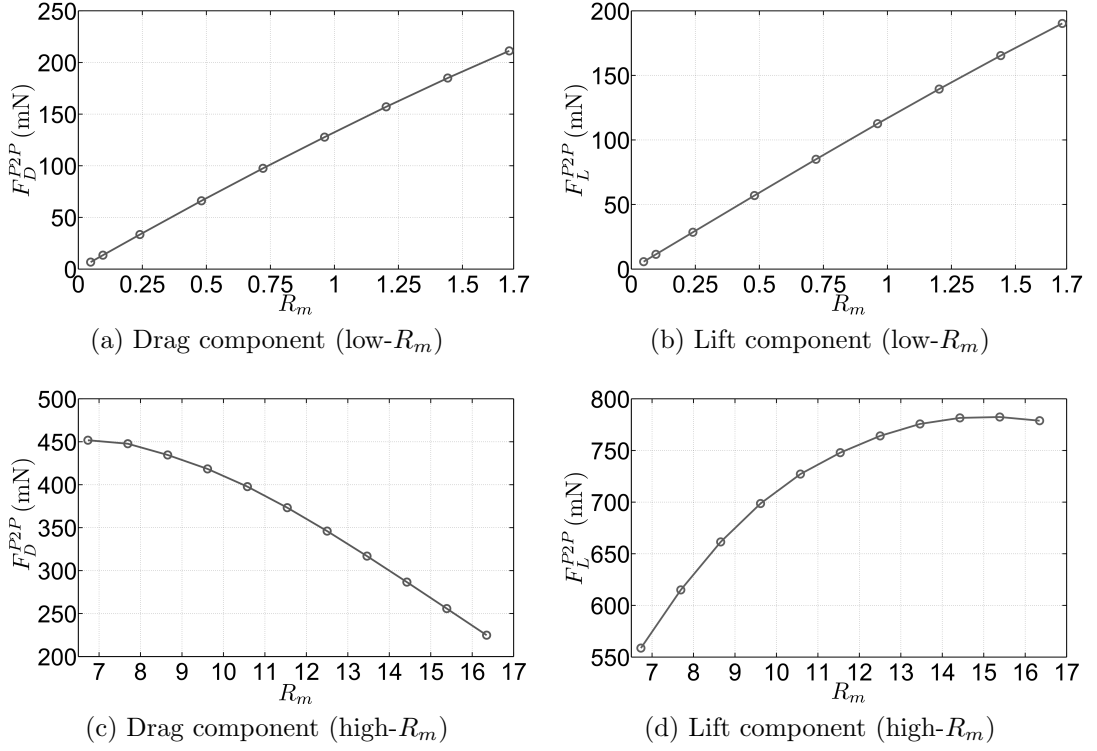


Figure 6.21: Influence of the magnetic Reynolds number R_m on the P2P Lorentz force perturbations. Calculations have been performed for long (") defect located at $d = 2$ mm using MDA. The magnet is located at lift-off distance $\delta z = 1$ mm.

have a different dependency on R_m . Moreover, when R_m is small ($R_m < 1$) it is observed that the drag component of Lorentz force depends linearly on R_m , and that the corresponding lift force depends quadratically on R_m , i.e. $F_D \sim R_m$ and $F_L \sim R_m^2$ (see Fig. 6.2).

As already mentioned in section 6.2.2, when defects are smaller than the used magnet, the characteristic perturbation of the Lorentz force in LET systems is strongly determined by the shape of the resulting point-spread functions (PSF). Fig. 6.13 shows the characteristic PSF for all three Lorentz force components which has been calculated for low R_m values ($R_m = 0.1$). Thus, apart from testing the influence of R_m on the resulting P2P force perturbations, the additional aim here was to study the effect of R_m on the characteristic shape of the Lorentz force perturbations, i.e. on the characteristic shape of the PSF. Therefore, the LET benchmark problem (see section 5.1) has been solved using MDA by considering the long (") defect placed at $d = 2$ mm and lift-off distance of the magnet $\delta z = 1$ mm.

In low- R_m range ($R_m < 1$) the characteristic shapes of the Lorentz force perturbations are practically independent on the actual value of R_m (see Fig. 6.20a and Fig. 6.20b). Nonetheless, the intensity of the P2P Lorentz force perturbations for both force components is increasing linearly with R_m , i.e. $F_{D,L}^{P2P} \sim R_m$.

In the moderate- R_m range ($R_m > 7$) the characteristic shapes of the Lorentz force perturbations are strongly influenced by the value of R_m . The so-called bell shape of the drag force perturbation has now very similar bipolar form to the lift force component in low- R_m range (compare Fig. 6.20c and Fig. 6.20b). The shape of the lift force perturbation in the moderate- R_m range tends to the shape of the corresponding drag force perturbation in the low- R_m range. However, the resemblance is not so expressed as in the drag force component (compare Fig. 6.20d and Fig. 6.20a).

In contrast to low- R_m range the resulting P2P Lorentz force perturbations in moderate- R_m range are decreasing with R_m . For the given benchmark problem the decrease of the P2P force perturbations starts earlier for the drag force component ($R_m \approx 7$) compared to the corresponding lift force component ($R_m \approx 15$). The decrease of the P2P force perturbations is caused by the so-called motional skin effect which occurs in the moderate- and high- R_m range [149]. Due to this effect the induced currents are being pushed towards the surface of the conductor which then reduces the force perturbations (see Fig 6.3). The actual value of R_m for which the P2P force perturbations start to decrease is therefore a function of the actual depth of the defect. This effect can be exploited in order to determine the actual depth of defects. A similar strategy has been applied in traditional ECT systems, where defects depth has been estimated by performing material testing with different excitation frequencies. This technique is also referred to as multi-frequency ECT [150].

6.2.5 Testing Depth Study

The first estimation of the testing capabilities of LET can be obtained by varying the depth d of the defects within the test conductor. Therefore, the LET benchmark problem has been applied and the depth of all three artificial defects, i.e. cross ("+"), long ("|") and wide ("—") has been changed in range of $d = 0$ to 12 mm. In section 5.1 the depth of defects has been defined as a distance between the upper surface of the conductor and the upper surface of the defect (see Fig. 5.1). Thus, the zero depth ($d = 0$ mm) should be understood as the defect lying on the top surface of the conductor under test. Fig. 6.22 shows the resulting Lorentz force P2P perturbations as a function of defect depth obtained by the moving defect approach (MDA). In order to validate the obtained results LET problem has been solved assuming a testing velocity of $v = 0.5$ m/s ($R_m = 0.48$) and a lift-off distance of $\delta z = 1$ mm. Due to the fact that in experiments the solid conductor is approximated with a set of aluminum sheets (each 2 mm thick) the conductor has been modelled using the anisotropic conductivity model (ACM) described in section 5.8.1.

Apart from a good agreement between numerical and experimental results, it is

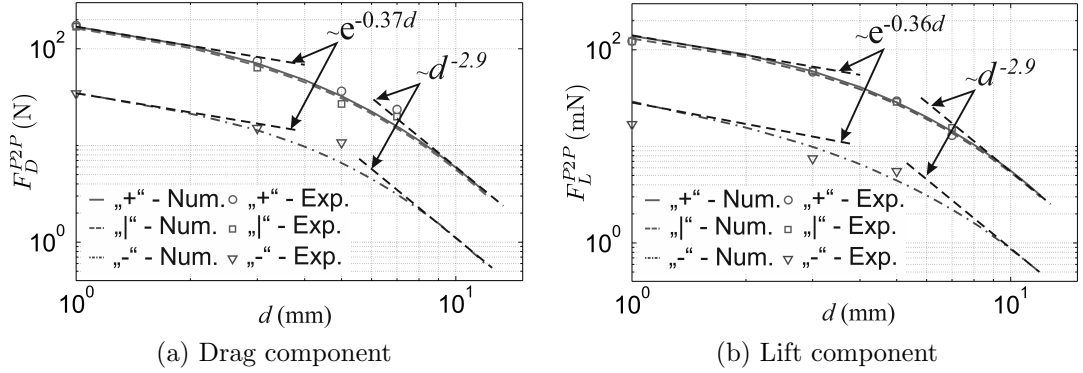


Figure 6.22: Influence of the defect depth d on the P2P Lorentz force perturbations. The calculations have been performed using MDA for $R_m = 0.48$ ($v = 0.5$ m/s) and lift-off distance $\delta z = 1$ mm.

more important to determine the actual dependency between the resulting P2P Lorentz force perturbations and d . When defects are relatively close to the magnet ($d < 3$ mm), an exponential decay of P2P force perturbations for both force components is observed. The obtained decay constant is rather small ($\Delta F_D \sim e^{-0.37d}$ and $\Delta F_L \sim e^{-0.35d}$) and it is independent on the particular shape of the used defects (Fig. 6.22). For larger defect depths ($d > 7$ mm) the exponential decay increases to power decay of $\sim d^{-2.9}$, for both force components. This is expected since the magnetic field relatively far away from the magnet has the decay rate of d^{-3} , i.e. the field distribution corresponds more to the field created by a magnetic point dipole (see section 4.1). This also means that in the low- R_m range ($R_m < 1$) the testing capabilities of LET systems are only limited by the natural diffusion of the applied magnetic field $\mathbf{B}$ which is in order of $\sim d^{-3}$.

Due to the limited accuracy of the force sensor used in experiments, defects lying up to $d = 6$ mm under the surface of the conductor could be detected ($v = 0.5$ m/s). The detection of defects lying deeper inside the material requires a force sensor with higher precision.

6.2.6 Lift-off Dependence Study

Another important issue of LET is the dependency of the Lorentz force on the lift-off distance of the magnet. In order to increase the total force acting on the magnet small lift-off distances δz are preferable (see Fig. 6.6a). The disadvantage of decreasing the lift-off distance is the increase of the sensitivity to surface dependent lift-off distance changes, e.g. surface roughness. For non-defective conductors (see section 6.1.2) it has been observed that the intensity of both force components have similar exponential dependency on variations in δz . Thus, the so-called lift-to-drag ratio of the Lorentz force has been introduced which is only weakly dependent on variations in δz .

In order to study the sensitivity of the Lorentz force perturbations caused by ma-

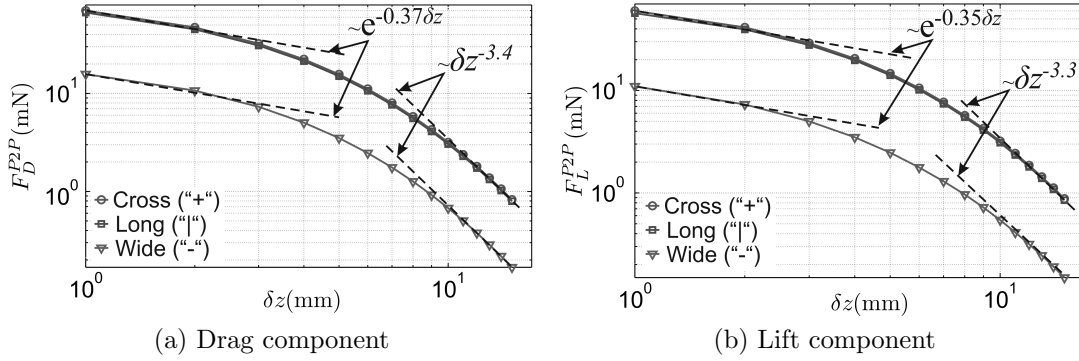


Figure 6.23: Influence of the lift-off distance on the P2P Lorentz force perturbations. The calculations have been performed for $R_m = 0.48$ ($v = 0.5$ m/s) and depth of defect $d = 2$ mm.

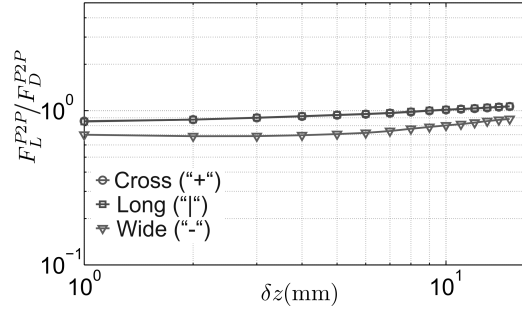


Figure 6.24: Dependency of the P2P lift-to-drag ratio of the Lorentz force on the changes in the lift-off distance δz .

terial defects on changes in δz the peak-to-peak (P2P) force perturbations (6.11) have been applied. Thus, an additional parametric study based on the LET benchmark problem has been performed. The lift-off distance of the magnet has been changed in the range of $\delta z = 1$ mm to 15 mm. To test if there is any influence of the defect shape on the P2P force variations with δz , the simulations have been performed for all three types of artificial defects, i.e. cross ("+"), long ("|") and wide ("−") defect (Fig. ??). As in the case of the non-defective conductor, for low values of δz ($\delta z < 3$ mm) the P2P perturbations of both Lorentz force components have similar exponential decay with increasing δz ($F_D^{P2P} \sim e^{-0.37\delta z}$ and $F_L^{P2P} \sim e^{-0.35\delta z}$). Even though, for higher values of δz ($\delta z > 7$ mm) the power decay rate of $F_{D,L}^{P2P} \sim \delta z^{-3}$ is expected, due to the finite cross-section of the conductor its edges additionally influence the force perturbations. As a result the power decay is slightly increased to $F_D^{P2P} \sim \delta z^{-3.4}$ and $F_L^{P2P} \sim \delta z^{-3.3}$. Similarly to the conductor free of defects, it is therefore expected that the so-called lift-to-drag ratio of the P2P perturbations is only weakly or completely independent on δz variations. Fig. ?? shows the sensitivity of the lift-to-drag ratio of the Lorentz force P2P perturbations resulting from given variations in δz . Compared to single force components (Fig. ??) a considerable decrease in the lift-off distance dependency can be observed. Furthermore, it has been shown previously in [30] that the decay rates of

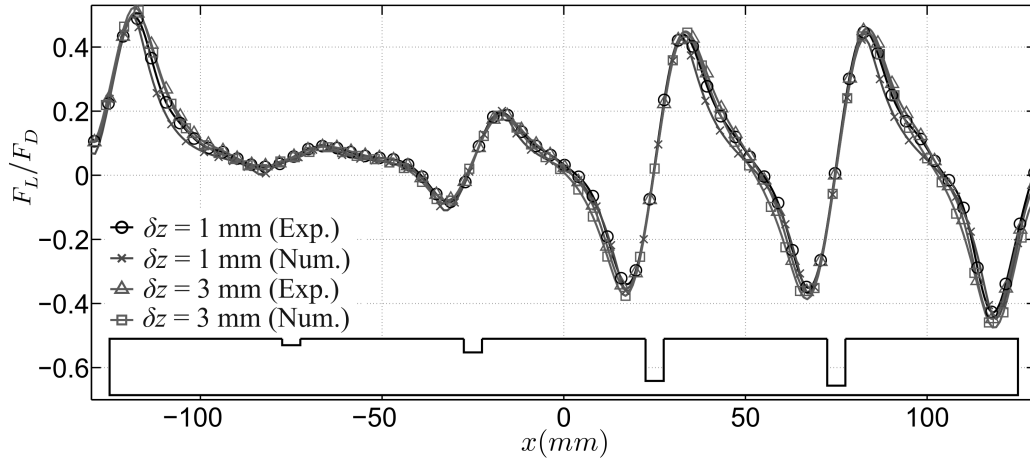


Figure 6.25: Lift-to-drag ratio of the Lorentz force obtained for the solid (isotropic) test conductor with idealised surface defects in form of linear slits.

the Lorentz force with respect to variations in δz are independent on the actual depth of defects as well [121].

To prove the above statements an experiment has been performed [15, 31]. A solid electrically conducting bar with 4 artificial surface defects in form of linear slits of different sizes has been used (see section 5.8). The cylindrical magnet, moving with constant velocity $v = 0.5$ m/s, has been placed at $\delta z = 1$ mm and $\delta z = 3$ mm above the surface of the conductor. The obtained results show that the resulting lift-to-drag ratio of the total Lorentz force (F_L/F_D) is practically independent on variations in δz (Fig. ??). In contrast to single force components, the perturbation of the lift-to-drag ratio is therefore only influenced by the presence of defects which is extremely important in NDT&E applications. The same experimental configuration has been also solved numerically by applying the fast MMA approach (see section 5.2.1). Apart of good agreement with experiments it additionally confirms the observations made above.

6.2.7 Differential Lorentz Force Eddy Current Testing

The previous parametric studies have confirmed that LET represents a promising NDT&E technique which could provide fast and relatively deep testing of electrically conducting materials. Even though the used absolute force sensor was not optimal for the presented experimental setup (see section 5.8), its resolution of 15 mN for drag and side force components and 50 mN for the corresponding lift force component could still provide the detection of defects lying up to 6 mm deep under the surface of the conductor (see Fig. 6.22) [15, 30, 31]. Additionally to the low resolution, the relatively low dynamics of the force sensor (up to 10 KHz), which is based on the strain-gauge

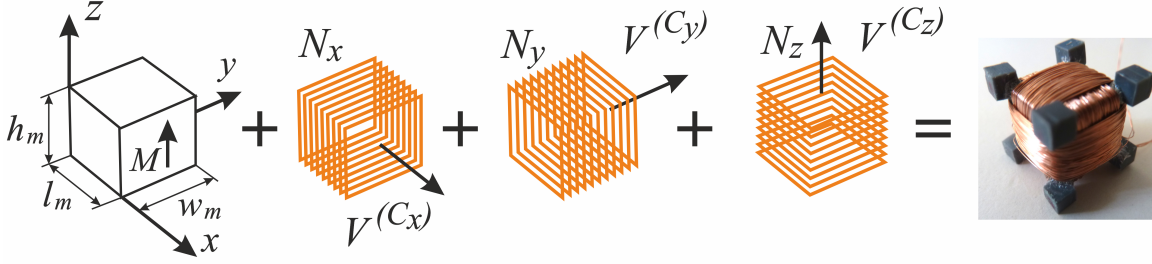


Figure 6.26: Schematic of possible differential Lorentz force sensor for DiLET applications.

technology limited the LET investigations for conductors with defects to velocities up to $v = 0.5$ m/s or $R_m = 0.48$ (see section 5.8 and [15] for more informations).

Based on the LET analysis presented above, the testing should be performed with higher velocities v (R_m) and using larger quadratically shaped permanent magnets placed at low lift-off distances δz . Apart from increasing the force perturbations this would also increase the absolute values of the Lorentz force which can be even few orders of magnitudes larger than the perturbations caused by defects (compare Fig. 6.4 and Fig. ??). Unfortunately, precise measurements of small force variations in relatively large range of applied forces is very difficult [15]. Thus, in LET systems there is a strong demand for the differential force measurements. Similar tendency is observed in traditional ECT systems as well. In ECT, various differential pick-up probe configurations offering higher testing sensitivity have already been designed and successfully implemented [146, 151]. Currently there are several possibilities to obtain the differential force signals resulting from LET systems. In principle, it is also possible to measure the acceleration or the velocity of the magnet [152]. However, the application of commercially available differential force sensors would lead to higher spatial integration requirements and considerably higher costs. This would be even more important when designing sensor arrays for LET which could simplify and advance defects detection and reconstruction. The main aim in this section is to propose the simple and low-cost modification of LET setup which could be used for measurements of differential Lorentz force signals caused by material defects. The proposed modification affects only the used magnet where three independent and passive pick-up coils have been wound on its outer surface (see Fig. 6.26). The principal idea is to use voltages induced in the additional coil system and correlate the voltage signals with the corresponding differential Lorentz force signals acting on the magnet. In fact, the resulting magnet system can be directly applied to the existing LET experimental setup, i.e. it can be used as a complete differential Lorentz force sensor. Since, this specific modification of LET systems allows differential Lorentz force measurements, the proposed technique has been termed as differential Lorentz force eddy current testing, abbreviated DiLET.

The main idea of using simple coil system to obtain the differential Lorentz force

signals can be traced back to Lorentz (Kelvin) force law (2.2). Assuming a cubic permanent magnet uniformly magnetised along the z -axis ($\mathbf{M} = M \cdot \mathbf{e}_z$) the Lorentz force acting upon it can be described as

$$\mathbf{F} = \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} (M \frac{\partial}{\partial z}) \mathbf{B} dx dy dz, \quad (6.12)$$

where w_m , l_m and h_m represents the width, length and height of the magnet, respectively (see Fig. 6.26). Differentiating (6.12) with respect to time and by considering all three force components separately the differential force signals can be described as

$$\frac{\partial F_D}{\partial t} = M \frac{\partial}{\partial t} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_x}{\partial z} dx dy dz, \quad (6.13)$$

$$\frac{\partial F_S}{\partial t} = M \frac{\partial}{\partial t} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_y}{\partial z} dx dy dz, \quad (6.14)$$

$$\frac{\partial F_L}{\partial t} = M \frac{\partial}{\partial t} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_z}{\partial z} dx dy dz, \quad (6.15)$$

If dimensions of the magnet (w_m, l_m, h_m) are small enough the following approximation applies

$$\frac{\partial B_i}{\partial z} \approx \frac{B_i^{z=h_m} - B_i^{z=0}}{h_m} \quad (6.16)$$

where $i \in \{x, y, z\}$. By using (6.16) directly in (6.13) - (6.15) the differential Lorentz force acting on the magnet can be well approximated as

$$\frac{\partial F_D}{\partial t} \sim \frac{M}{h_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \left[\frac{\partial B_x^{z=h_m}}{\partial t} - \frac{\partial B_x^{z=0}}{\partial t} \right] dx dy dz, \quad (6.17)$$

$$\frac{\partial F_S}{\partial t} \sim \frac{M}{h_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \left[\frac{\partial B_y^{z=h_m}}{\partial t} - \frac{\partial B_y^{z=0}}{\partial t} \right] dx dy dz, \quad (6.18)$$

$$\frac{\partial F_L}{\partial t} \sim \frac{M}{h_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \left[\frac{\partial B_z^{z=h_m}}{\partial t} - \frac{\partial B_z^{z=0}}{\partial t} \right] dx dy dz, \quad (6.19)$$

In DiLET the pick-up coils are moving together with the primary magnetic field $\mathbf{B}_p$, i.e.

$\partial \mathbf{B} / \partial t = \partial (\mathbf{B}_p + \mathbf{B}_s) / \partial t = \partial \mathbf{B}_s / \partial t$. Using the reasonable assumption $\partial B_i^{z=h_m} / \partial t \ll \partial B_i^{z=0} / \partial t$, where $i \in \{x, y, z\}$ (6.17) - (6.19) can be simplified as

$$\frac{\partial F_D}{\partial t} \sim -\frac{M}{h_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_x^{z=0}}{\partial t} dx dy dz, \quad (6.20)$$

$$\frac{\partial F_S}{\partial t} \sim -\frac{M}{h_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_y^{z=0}}{\partial t} dx dy dz, \quad (6.21)$$

$$\frac{\partial F_L}{\partial t} \sim -\frac{M}{h_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_z^{z=0}}{\partial t} dx dy dz. \quad (6.22)$$

From (6.20) - (6.22) it can be observed that in order to obtain the differential Lorentz force signals it is sufficient to measure the time variation of the magnetic flux density ($\partial B_i / \partial t$) at $z = 0$, i.e. at the bottom surface of the magnet. As it will be shown, this can be very effectively accomplished by a simple pick-up coil system presented in Fig. 6.26. Voltages induced in each of the coils can be calculated as follows

$$V^{(C_x)} = \sum_1^{N_x} \left[\int_0^{w_m} \int_0^{h_m} \frac{\partial B_x}{\partial t} dy dz \right] \sim \frac{N_x}{h_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_x}{\partial t} dx dy dz, \quad (6.23)$$

$$V^{(C_y)} = \sum_1^{N_y} \left[\int_0^{w_m} \int_0^{h_m} \frac{\partial B_y}{\partial t} dx dz \right] \sim \frac{N_y}{w_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_y}{\partial t} dx dy dz, \quad (6.24)$$

$$V^{(C_z)} = \sum_1^{N_z} \left[\int_0^{w_m} \int_0^{h_m} \frac{\partial B_z}{\partial t} dx dy \right] \sim \frac{N_z}{l_m} \int_0^{w_m} \int_0^{l_m} \int_0^{h_m} \frac{\partial B_z}{\partial t} dx dy dz, \quad (6.25)$$

where $V^{(C_x)}$, $V^{(C_y)}$ and $V^{(C_z)}$ are the voltages induced in the x , y and z -axis oriented coils, respectively. The total number of windings of each coil is denoted by N_x , N_y and N_z , respectively.

By direct comparison of (6.23) - (6.25) with (6.20) - (6.22) the differential Lorentz force acting on the magnet, which is caused by the presence of material defects can be well approximated as

$$\frac{\partial F_D}{\partial t} \sim -\frac{M}{h_m} \frac{h_m}{N_x} V^{(C_x)} \quad (6.26)$$

$$\frac{\partial F_S}{\partial t} \sim -\frac{M}{h_m} \frac{w_m}{N_y} V^{(C_y)} \quad (6.27)$$

$$\frac{\partial F_L}{\partial t} \sim -\frac{M}{h_m} \frac{l_m}{N_z} V^{(C_z)} \quad (6.28)$$

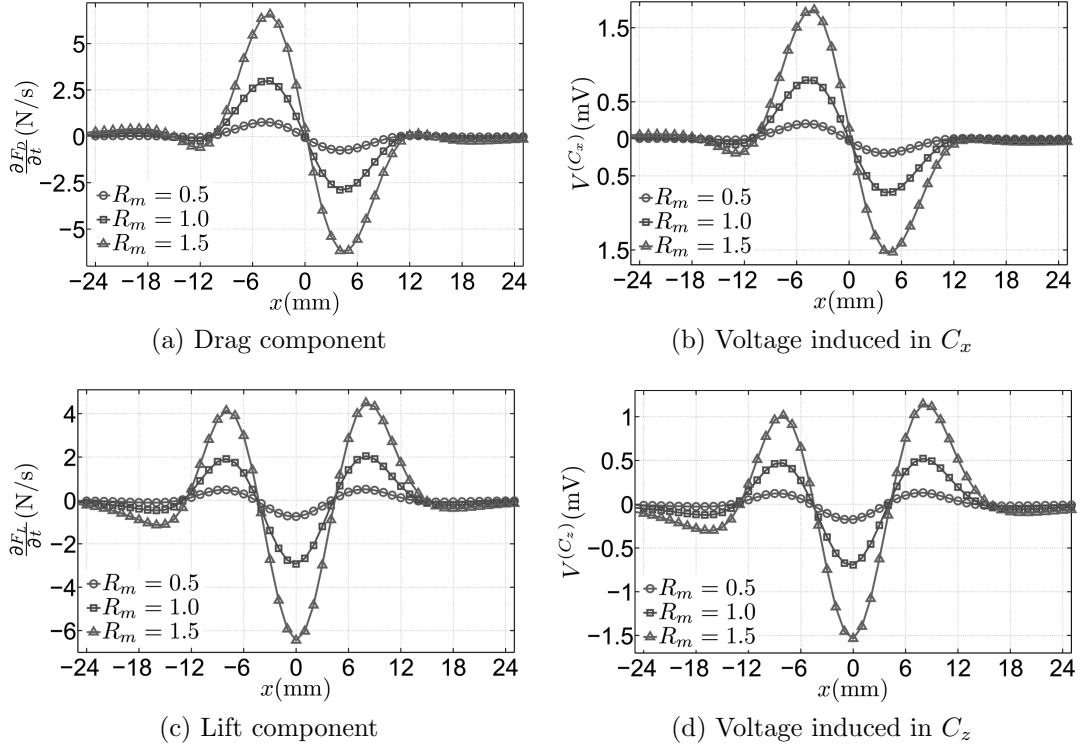


Figure 6.27: Direct comparison of the differential Lorentz force signals acting on the square magnet ($a = 1\text{cm}$) for different values of R_m . Calculations have been performed for long ("|") defect located at $d = 2\text{ mm}$ using MDA. The magnet is located at lift-off distance $\delta z = 1\text{ mm}$.

In order to test the proposed DiLET methodology the LET benchmark problem presented in Fig. 5.1 (see section 5.1) has been solved using MDA. Thus, the cylindrical permanent magnet has been replaced by a cubic magnet with finite dimensions $w_m = l_m = h_m = 1\text{ cm}$ placed at lift-off distance $\delta z = 1\text{ mm}$. All other geometrical and material parameters are described in Table 5.3. According to DiLET experiment which is currently in preparation (see Fig. 6.26), the total number of windings in each coil was fixed to $N_x = N_y = N_z = 300$. Furthermore, an additional parametric study assuming the wide range of testing velocities up to $v \sim 11\text{ m/s}$ ($R_m = 10$) has been performed to estimate the working range of this specific DiLET implementation. Fig. ?? and Fig. ?? show the differential Lorentz force signals calculated directly by time differentiation of (2.2), whereas Fig. ?? and Fig. ?? show the corresponding voltages induced in coils oriented along x -axis and z -axis. From the obtained results a very good correlation between the differential Lorentz force signals and the corresponding voltages can be observed (Fig. ??). Due to the axial symmetry of the model $\delta y = 0$ (see Fig. 5.11) the side component of the Lorentz force ($\mathbf{F}_S$) is equal to zero. To compare the results quantitatively the so-called Pearson's correlation coefficient (PCC) has been used [153]. PCC, here denoted as r_i where $i \in \{D, L\}$, is defined as the covariance of

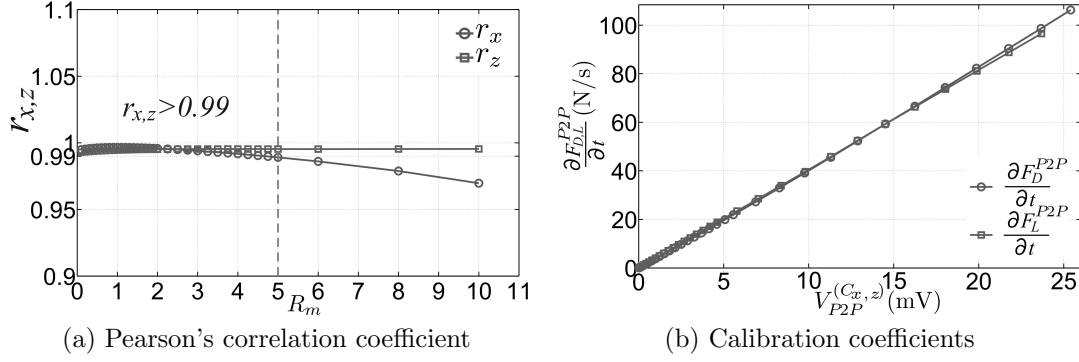


Figure 6.28: Comparison of differential Lorentz force signals with respect to the induced voltages. Calculations have been performed for long (") defect located at $d = 2$ mm using MDA. The magnet is located at lift-off distance $\delta z = 1$ mm.

the two variables under comparison divided by the product of their standard deviations

$$r_D = \frac{\text{cov}\left(\frac{\partial F_D}{\partial t}, V^{C_x}\right)}{\sigma\left(\frac{\partial F_D}{\partial t}\right) \sigma(V^{C_x})} \text{ and } r_L = \frac{\text{cov}\left(\frac{\partial F_L}{\partial t}, V^{C_z}\right)}{\sigma\left(\frac{\partial F_L}{\partial t}\right) \sigma(V^{C_z})}. \quad (6.29)$$

The value of PCC should be interpreted as follows. If $r_i = +1$ then the two variables under comparison have perfect increasing linear correlation. If $r_i = -1$ the corresponding variables have perfect decreasing linear correlation [153]. As r_i approaches to zero the variables are completely uncorrelated. Fig. 6.28a shows the PCC values for both differential Lorentz force components as a function of R_m (testing velocity v). It can be observed that even for relatively large size of magnets ($w = l = h = 1$ cm), the induced voltages are in very good correlation to the corresponding differential Lorentz force signals. Moreover, in the testing range of up to $R_m = 5$ the calculated PCC was more than 0.99. Since magnetisation of the magnet is considered to be along the z -axis ($\mathbf{M} = M \cdot \mathbf{e}_z$), the voltage induced in the z -axis oriented coil (V^{C_z}) correlates well with the corresponding differential lift force component in the whole range of R_m .

The presented analysis confirms the validity of simplifications assumed in order to derive (6.26) and (6.28). Nevertheless, to determine differential force signals directly from voltages induced in the corresponding coils an additional calibration of the proposed differential force sensor has to be performed. For this the P2P perturbations of the obtained differential force signals have been compared directly with the corresponding P2P perturbations of the induced voltages (see Fig. 6.28b). As directly follows from (6.26) - (6.28) the dependency between P2P values is linear. Moreover, due to the cubic shape of the magnet ($w_m = l_m = h_m$) and coils with the same number of windings ($N_x = N_y = N_c$), the calibration curves for both force components are almost identical.

Chapter 7

Summary and Outlook

In this chapter the major results from the present study are summarised and concluded. Additionally, suggestions for future work involving numerical and theoretical study of LET problems are presented.

7.1 Summary

The non-destructive testing technique, namely Lorentz force eddy current testing (LET) was thoroughly investigated in this work. In particular, the focus of the work was on the theoretical and numerical analysis of the problem of motion of a defective conductor in the presence of a permanent magnet. The following objectives have been accomplished:

- A creeping magnet problem was considered to provide insights into the fundamental principles of LET. This problem refers to the Faraday experiment of a magnet falling through an electrically conducting pipe. However, in this work, the motion of a free-falling permanent magnet in a pipe containing an idealized defect was considered. This problem represents a highly simplified yet enlightening version of Lorentz force eddy current testing. A combination of analytical theory, numerical simulation and experimental validation was used to investigate this problem. The analytical theory consists of the derivation and analysis of a non-linear ordinary differential equation for the time-dependent position of the falling magnet. The analytical theory allowed a rigorous prediction of the relation between the width of the defect and the change in falling time. The numerical simulation helped to overcome three limitations inherent in the analytical theory and to take into account (i) the finite size of the falling magnet, (ii) the finite width of the pipe and (iii) the finite value of the magnetic Reynolds number describing the deformation of the magnetic field lines. The predictions were tested

by a comparison with a series of experiments. It was observed that the difference between the measured values of the change in falling time and the predictions of the numerical and analytical models is in the range between 0.14% and 36.15%, respectively. Therefore, it can be concluded that the theory properly captures the essence of Lorentz force eddy current testing in spite of its apparent simplicity.

- Unlike the creeping magnetic problem, a real LET problem deals with completely three-dimensional and arbitrary defects. This requires effective numerical simulations to solve 3D transient LET problems involving parts in relative motion. Therefore, a new logical expressions approach (LEA) was proposed that permits the fast analysis of moving eddy current problems on a fixed computational grid. The proposed method provided accurate results for both small and large magnetic Reynolds numbers (R_m). Depending on the type of the object in motion, two different LEA implementations were presented, namely, the moving magnet approach (MMA) and the moving defect approach (MDA). The MMA is very time efficient, but it is restricted to simple magnet shapes. The MDA extends the applicability of the LE approach to complex magnet systems whilst maintaining their computational efficiency and accuracy.
- To additionally reduce the simulation time several fast numerical models have been developed based on the certain model simplifications. In the so-called low- R_m regime it has been shown that the instantaneous magnetic field reaction to any change in eddy current distribution can be considered. This resulted in the quasi-static approach (QSA). If conductors are free of defects, with QSA only one stationary analysis can be performed, providing the steady Lorentz forces acting on the magnet. If the conductor contains a defect, QSA can only be applied to LET problems within the regime of low R_m , i.e. $R_m < 1$. On the other hand, if the secondary magnetic field associated with the eddy currents is low, compared to the primary magnetic field, the QSA can be even further simplified. This is referred to as the weak reaction approach (WRA). It enables general 3D LET systems to be described by only one scalar electric potential, resulting in a considerable reduction of computational requirements. Nevertheless, the application range of the WRA is restricted to very low values of R_m ($R_m \ll 1$).
- All numerical results were verified with the sliding mesh technique (SMT), which requires the use of computationally expensive direct solvers. The results of the simulations also agreed well with the experiments, which additionally underlines the ability of the LET to detect deep lying defects with relatively high testing velocities.

- In order to understand the feasibility and testing capabilities of LET systems, the benchmark problem was defined and solved by the proposed numerical methodology. In particular, parametric studies were performed to quantify the effect of various parameters on the total value of the Lorentz force acting on the magnet. Simulations were performed for two characteristic LET testing configurations, namely (i) a conductor without defects and (ii) a conductor with pre-defined material defects. In case of a non-defective conductor, the parametric studies have been used to quantify the effect of the lift-off distance, size and magnetization of the magnet, and the magnetic Reynolds number on the resulting Lorentz forces. It has been observed that all force components have the same dependency in terms of magnet size, its magnetization and lift-off distance. Furthermore, a linear dependency between the Lorentz force and the volume of the magnet has been observed. In case of magnetization intensity the dependency is quadratic. However, the dependency of the Lorentz force on the lift-off distance is non-monotonic. In particular, two regimes were identified. For low lift-off distances force decay exponentially with distance with small decay constant. By increasing the lift-off distance further, a regime where the force decays cubically has been observed. This corresponds to the actual decay of the magnetic field of the permanent magnet itself. For a conductor with material defects identical studies were performed to determine the influence of the given parameters on the Lorentz force. However, unlike in the case of a non-defective conductor, only Lorentz force perturbations were evaluated. Furthermore, the effect of the shape, size and position of the defect within the conductor on the characteristic Lorentz force perturbations were investigated. The analysis provided reference results to understand the feasibility and testing capabilities of LET.
- Another important parameter considered in the work was the magnetic Reynolds number, R_m . The dependency of individual force components on R_m is different. In the low- R_m regime it has been observed that the drag force is linearly dependent on R_m , whereas the corresponding lift force increases quadratically. This behaviour resulted, rather serendipitously, in a so-called Lorentz force sigmometry (LoFoS). LoFoS is based on the lift-to-drag ratio of the Lorentz force and enables contactless measurement of electrical conductivity of the conductor under test. The proposed technique LoFoS has the following advantages: (i) it is contactless, (ii) it can be applied continuously during production processes, (iii) it enables the user to measure the conductivity beyond the oxidised surface and (iv) the proposed data processing makes LoFoS resistant to changes in lift-off distances, velocity and strength of the magnetic field source. LoFoS is suitable

for specimen of any kind of physical condition. The only limitation is given by the minimal measurable Lorentz force components of the multi-component force sensor.

- To increase the testing range of LET, i.e. to detect defects deeper inside the material, the testing should be performed with low lift-off distances, large cubic magnets and higher Reynolds numbers. However, if the size of the magnet is larger than the size of the defect then the information content contained in the Lorentz force signals is reduced. In this case the shape of the force perturbation is merely an image of the magnet itself. In principle, this means that the solution of the inverse problem providing the identification of the defect would be very difficult or even impossible to perform. Therefore, the magnet size needs to be closely monitored and should be chosen based on the required testing resolution.
- Generation of high Lorentz force perturbations has a rather unfortunate side effect of increasing the absolute force as well. In such cases, the absolute values of Lorentz force is a few orders of magnitudes larger than the perturbations caused by defects. Unfortunately, precise measurements of such small force variations in relatively large range of applied forces is very difficult. Therefore, there is a strong demand for differential force measurements in LET systems. To this end, in this work a simple and low-cost modification to the initial LET configuration was proposed enabling three component differential Lorentz force measurements. Such a technique, has been referred to as Differential Lorentz Force Eddy Current Testing (DiLET). The proposed modification affects only the magnet system, whose outer surface was wound by three independent and passive pick-up coils. The principal idea is to use voltages induced in the additional coil system and correlate the obtained voltage signals with the corresponding differential Lorentz force signals acting on the magnet. In fact, the resulting magnet system can be directly applied to the existing LET experimental setup as a complete differential Lorentz force sensor. Numerical results demonstrated that the correlation between the differential Lorentz force and voltage induced in corresponding coils was more than 0.99 for $R_m < 5$. The correlation of the force component in the direction of the magnetization of the magnet was independent of R_m in the given range ($R_m < 12$).

7.2 Outlook

Theoretical and numerical modelling of LET problems is fairly well investigated in this work. However, some important questions remain still unanswered. Therefore, in this section, the focus is on addressing these questions to provide a good foundation for any future work involving LET. To this end, the section is divided into two parts, namely (i) numerical study and (ii) theoretical study.

7.2.1 Numerical Study

Suggestions for future numerical work concerning LET problems are as follows

- Extension of numerical analysis of LET benchmark problem by combining finite element and boundary element numerical procedures. This can help to additionally reduce the computational costs. Furthermore, the modelling can be considerably simplified since only the conducting domain needs to be considered.
- Logical expression approach (LEA) employed in this work requires permanent control over the time-step size used in the numerical simulations. In the commercial code Comsol Multiphysics[®] this was only possible by using the implicit generalized- α transient solver. However, it was observed that this solver is rather sensitive to the *a priori* specified damping factor α . Therefore, implementation of the LET problem (with and without LEA) in other commercial codes utilising different transient solvers would increase the robustness of the numerical methodology.
- Verification of the numerical results has been obtained by applying the SMT together with Lagrange multipliers. According to many numerical tests performed with the commercial FEM code Comsol Multiphysics[®], the resulting system of equations could only be solved by applying direct solvers. Another technique, namely the field interpolation technique, enabled the use of iterative solvers as well. However, even the finite element grid on the interface was conforming, the obtained solution was numerically noisy. To find out more about this error, it is advisable to solve the given LET problem by applying SMT and the interpolation method in other commercial codes.

7.2.2 Theoretical Study

The suggestions for future theoretical work in LET problems are as follows

- In this work only one permanent magnet was included in the analysis. However, it was apparent from the parametric studies that in order to increase the testing capabilities of general LET systems, strong and highly localised magnetic field distributions are required. Therefore, any future work could involve the investigation of complex magnet systems including an optimised array of magnets with ferromagnetic shielding materials or even superconductors.
- Most industrial applications employ ferromagnetic materials and therefore, there is a strong need to extend LET for testing such materials as well. However, such an endeavour creates additional challenges due to the non-linearity in magnetic permeability. In particular, extremely high reluctance forces between these materials and the permanent magnets makes the actual measurement of Lorentz forces very difficult or even impossible. This problem could be overcome by DiLET methodology. This, however requires thorough numerical and experimental investigation of DiLET systems proposed in this work.

Appendix A

To simplify the analysis of the general LET systems, the magnetic Reynolds number R_m has been extensively used throughout the work. In [32] R_m is derived using the magnetic field diffusion-convection equation described only by the magnetic flux density $\mathbf{B}$. Since, here the potential functions $\mathbf{A}$ and V have been used instead, an equivalent derivation of R_m based on [32] is performed.

Assuming a linear and isotropic conducting material the magnetic field diffusion-convection equation given by (2.31) can be written as

$$\frac{1}{\mu_0\sigma}\nabla \times (\nabla \times \mathbf{A}) = - \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla(\mathbf{v} \cdot \mathbf{A}) - \mathbf{v} \times \nabla \times \mathbf{A} \right), \quad (\text{A.1})$$

where the electric scalar potential has been replaced by $V = \mathbf{v} \cdot \mathbf{A}$ [62]. Since R_m is used to describe the effect of the secondary magnetic field only inside the conducting region, the external current density and the magnetisation of a permanent magnet are equal to zero ($\mathbf{j}_e = \mathbf{0}$ and $\mathbf{M} = 0$).

Based on the scales of length L , velocity $|\mathbf{v}|$, time $L/|\mathbf{v}|$ and vector potential $|\mathbf{A}|$, which are characteristic for the considered conductor in motion, equation (A.1) can be written as follows

$$\frac{1}{\mu_0\sigma L^2} |\mathbf{A}| \tilde{\nabla} \times (\tilde{\nabla} \times \tilde{\mathbf{A}}) = -\frac{1}{L} |\mathbf{A}| |\mathbf{v}| \left(\frac{\partial \tilde{\mathbf{A}}}{\partial \tilde{t}} + \tilde{\nabla}(\tilde{\mathbf{v}} \cdot \tilde{\mathbf{A}}) - \tilde{\mathbf{v}} \times \tilde{\nabla} \times \tilde{\mathbf{A}} \right), \quad (\text{A.2})$$

where the non-dimensional quantities have been defined as

$$\nabla = \frac{1}{L} \tilde{\nabla}, \quad (\text{A.3})$$

$$\frac{\partial}{\partial t} = \frac{|\mathbf{v}|}{L} \frac{\partial}{\partial \tilde{t}}, \quad (\text{A.4})$$

$$\mathbf{A} = |\mathbf{A}| \tilde{\mathbf{A}}, \quad (\text{A.5})$$

$$\mathbf{v} = |\mathbf{v}| \tilde{\mathbf{v}}. \quad (\text{A.6})$$

Using the given definition of the magnetic Reynolds number [32]

$$R_m = \mu_0 \sigma |\mathbf{v}| L \quad (\text{A.7})$$

the final non-dimensional form of the magnetic field diffusion-convection equation (2.31) can be obtained

$$\tilde{\nabla} \times (\tilde{\nabla} \times \tilde{\mathbf{A}}) = -R_m \left(\frac{\partial \tilde{\mathbf{A}}}{\partial \tilde{t}} + \tilde{\nabla}(\tilde{\mathbf{v}} \cdot \tilde{\mathbf{A}}) - \tilde{\mathbf{v}} \times \tilde{\nabla} \times \tilde{\mathbf{A}} \right). \quad (\text{A.8})$$

Appendix B

The MMA and SMT are defined in the fixed frame of reference K (see Section 2.2 and Section 5.2). In this reference frame the governing equation of LET system voids the additional velocity term which accounts for the induced eddy currents inside the conductor under test. In K the eddy currents are actually described by $\mathbf{j} = -\sigma \partial \mathbf{A} / \partial t$, which is strongly influenced by the applied time-discretisation scheme. In order to provide direct control over the time-step size the generalized- α transient solver has been applied in both, MMA and SMT implementations. However, it has been observed that the particular choice of the control parameter α strongly affects the stationary values of the Lorentz force. In order to numerically estimate the influence of α , the stationary value of the Lorentz force, i.e. its step response has been calculated using MMA. Thus, the LET benchmark problem for conductor free of defects has been applied with all geometrical and material parameters given in Table 5.3. The cylindrical magnet was placed in the middle of the conductor, whose length L was considered to be long enough not to influence the obtained solution. The results have been compared with respect to quasi-static solution (5.4) and (5.5), which has been extensively validated

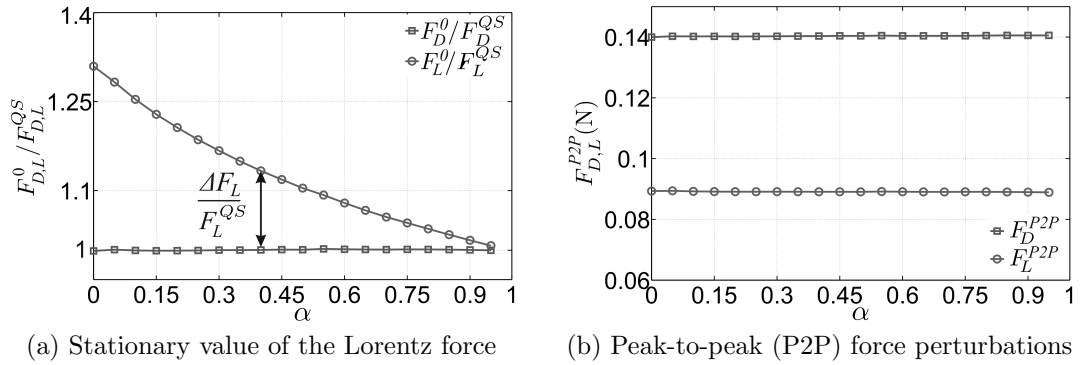


Figure B.1: Influence of the damping parameter α of the transient generalized- α solver on the stationary value of Lorentz force ($F_{D,L}^0$) and on the resulting Lorentz force peak-to-peak perturbations ($\Delta F_{D,L}^{P2P}$). The calculations have been performed for $R_m = 1$ ($v = 1$ m/s) and $\Delta X = 1$ mm ($dt = 0.5 \cdot 10^{-3}$ s)

by experiments (see Section 6.1). From the obtained results, a very strong dependency of the stationary Lorentz force lift component F_L^0 on α has been observed (Fig. B.1a). For the given LET parameters, the discrepancy of the lift force was up to 30% ($\alpha = 0$) and it was monotonically reducing to $\sim 0.7\%$ ($\alpha = 0.95$). The dependency of the corresponding drag force component F_D^0 was very weak ($< 0.2\%$) in the whole range of applied α . This is mostly because in the assumed velocity range the drag force component is almost two orders of magnitude larger than the corresponding lift force. However, for higher values of α ($\alpha > 0.85$) LET solution obtained by MMA and SMT tends to be unstable. This practically limited the application of the generalized- α solver to $\alpha \leq 0.85$. Fortunately, peak-to-peak (P2P) Lorentz force perturbations for both force components are independent on the used value of α (Fig. B.1b).

The actual quantification of the discrepancy between transient and quasi-static solution with respect to α could not be performed in the given time. Nevertheless, taking into consideration all the observations listed above and Fig. B.1 simple and constant correction of the transient Lorentz force signals obtained by MMA and SMT can be used

$$F_D^{corr} = F_D - \Delta F_D, \quad (B.1)$$

$$F_L^{corr} = F_L - \Delta F_L, \quad (B.2)$$

where $F_{D,L}^{corr}$ represents the corrected values of the Lorentz forces, $F_{D,L}$ is the transient solution obtained by MMA or SMT and $\Delta F_{D,L} = F_{D,L}^0 - F_{D,L}^{QS}$ is the constant correction factor (see Fig. B.1a). It should be mentioned that the proposed correction of the numerical results should only be performed when a direct comparison with experiments is required. Otherwise, since the characteristic perturbation of the Lorentz force is not influenced by values of α , the LET systems can be studied using unchanged MMA or SMT solutions.

The MDA is defined in the moving frame of reference K' (see section 2.2). In this reference frame the induced eddy currents are described by the additional velocity term $\mathbf{j} = \sigma(\mathbf{v} \times \nabla \times \mathbf{A})$. Due to this, the transient LET solution obtained by the MDA was independent on particular values of damping parameter α .

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Ilmenau, den 27.11.2012

Mladen Zec