Better expression for R and Li PhL 9.6.14

Here I develop the plots which now appear in App Q section "how good is the crude model". The Maple code for these plots is in "Plots of Li and R vs w Belden.mws "

After two major screw-up versions, I am back again to this question. Remember from lines doc (4.11.34) that.

$$R = \mathrm{Re}(Z_{s1} + Z_{s2}) \qquad L = L_e + (1/\omega)\,\mathrm{Im}(Z_{s1} + Z_{s2}) = L_e + L_i$$

and these are the R and L_i I am trying to make a better model for! For the round wire, I know from Chapter 2 that,

$$Z_s(\omega) = \frac{-j\omega\mu}{2\pi a\beta}\,\frac{J_0(\beta a)}{J_1(\beta a)} \qquad (2.4.6)$$

at least for the central conductor of a coax cable! So how does this give R_{dc} in the low ω limit? We can go back a little more to this point from (2.2.30),

$$E_z(r) = \frac{\mu I}{2\pi a}\,\frac{J_0(\beta r)}{J_1(\beta a)}\,(-j\omega/\beta) \quad (2.2.29) \qquad E_z(a) = \frac{\mu I}{2\pi a}\,\frac{J_0(\beta a)}{J_1(\beta a)}\,(-j\omega/\beta) \quad (2.2.27)$$

The small ω limit of the above would involve

$$J_0 = 1 \qquad J_1 = (z/2)$$

Then

$$E_z(r) = \frac{\mu I}{2\pi a}\,\frac{J_0(\beta r)}{J_1(\beta a)}\,(-j\omega/\beta) = \frac{\mu I}{2\pi a}\,\frac{1}{(\beta a/2)}\,(-j\omega/\beta) = \frac{-j\omega\mu I}{\pi a}\,\frac{1}{(\beta^2 a)} = \frac{-j\omega\mu I}{\pi a}\,\frac{1}{(-j\omega\mu\sigma a)}$$

$$= \frac{I}{\pi a}\,\frac{1}{(\sigma a)} = \frac{I}{\sigma\pi a^2} \qquad \Rightarrow \qquad J_z = \frac{I}{\pi a^2} = \text{quite reasonable}$$

Meanwhile, in the same limit

$$Z_s(\omega) = \frac{-j\omega\mu}{2\pi a\beta}\,\frac{J_0(\beta a)}{J_1(\beta a)} = \frac{-j\omega\mu}{2\pi a\beta}\,\frac{1}{(\beta a/2)} = \frac{-j\omega\mu}{\pi a^2\beta^2} = \frac{-j\omega\mu}{\pi a^2[-j\omega\mu\sigma]} = \frac{1}{\pi a^2[\sigma]} = \frac{1}{\sigma\pi a^2} = R_{dc}$$

and there is your answer! In Chapter 2 I plot Z_z versus δ instead of ω. I could make new plots versus ω and just see what they look like. But basically, here is my model for the center conductor,

$$R_1(\omega) = \mathrm{Re}\left[\frac{-j\omega\mu}{2\pi a\beta}\,\frac{J_0(\beta a)}{J_1(\beta a)}\right] \qquad \omega L_{i1}(\omega) = \mathrm{Im}\left[\frac{-j\omega\mu}{2\pi a\beta}\,\frac{J_0(\beta a)}{J_1(\beta a)}\right] \qquad \text{where } a = a_1$$

What about the *other* conductor of Belden cable? I guess I have never addressed this question. I could apply the above formula to widely spaced twin lead.

For the Belden, App R shows that the DC L_i of the shield is about 1/8th that of the center conductor.

.8e-8/.5e-7 = 8/50 = 0.16 = 1/8.

App R also shows that at high ω, both R and L_i shield/center have a ratio of $1/a_2 \,/\, 1/a_1 = a_1/a_2$ = 1/6.4 ≈ 1/6. So in both limits, L_i and R of the shield can be roughly neglected compared to the center conductor.

How would this be affected by changing the shield thickness? The hi ω results would be the same, since only the inside surface of the shield matters. But if we make t of the shield larger, we have

$$L_i(\text{shield}) = \frac{\mu_0}{8\pi}\,[\,(4/3)(t/a_2)\,] = \frac{\mu_0}{8\pi}\,[\,(4/3)\,\frac{1}{\sigma\, 2\pi a_2^2 R_{2DC}}\,]$$

so if we were to fatten the shield, its L_i does in fact increase as its R_{2DC} decreases and t increases.

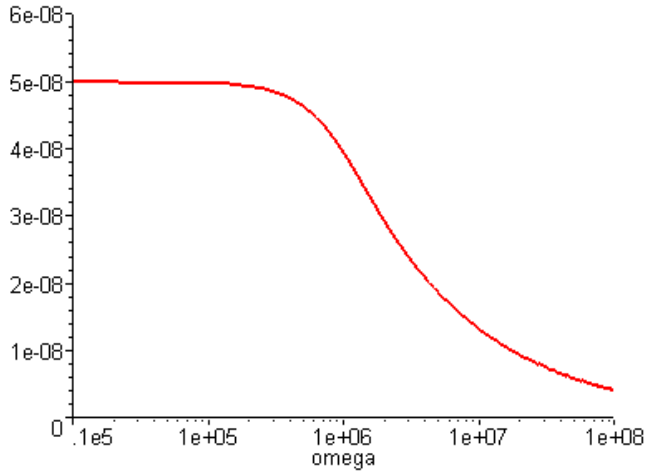
I could do a Chapter 2 style solution for the shield and I probably could get some kind of Bessel function formula for its Z_s at all ω. But maybe I will just assume we can neglect it more or less, or that I am dealing with widely spaced twin lead. I have done enough detail work I think.

So, here are my new plots for the center of Belden 8281.

$$R_1(\omega) = \text{Re}\,[\frac{-j\omega\mu}{2\pi a\beta}\,\frac{J_0(\beta a)}{J_1(\beta a)}] \qquad \omega\, L_{i1}(\omega) = \text{Im}\,[\frac{-j\omega\mu}{2\pi a\beta}\,\frac{J_0(\beta a)}{J_1(\beta a)}] \qquad \text{where } a = a_1$$

OK, here are some good plots:

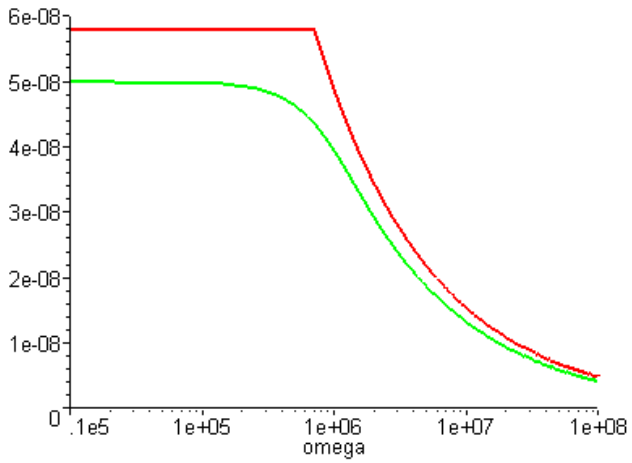
```
wlo := 1e4: whi := 1e8:
semilogplot(Li1,omega = wlo..whi, view = [wlo..whi,
0..6e-8],numpoints=800,thickness=2);
```



This one shows how L_i remains constant at $\mu/8\pi$ out to about 500KHz and then drops off in the expected $1/\sqrt{\omega}$ fashion. It is not so different from my Heaviside plot!

Here is a comparison of the above plot to my original Heaviside model,

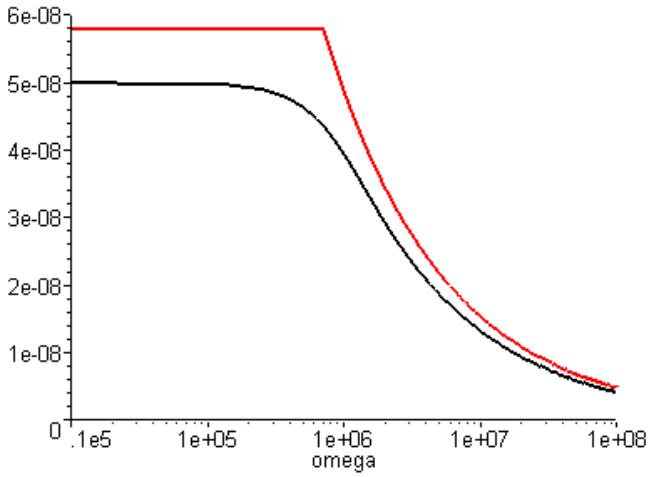
```
semilogplot([Li_,Li1],omega = wlo..whi, view = [wlo..whi,
0..6e-8],numpoints=800,thickness=2);
```



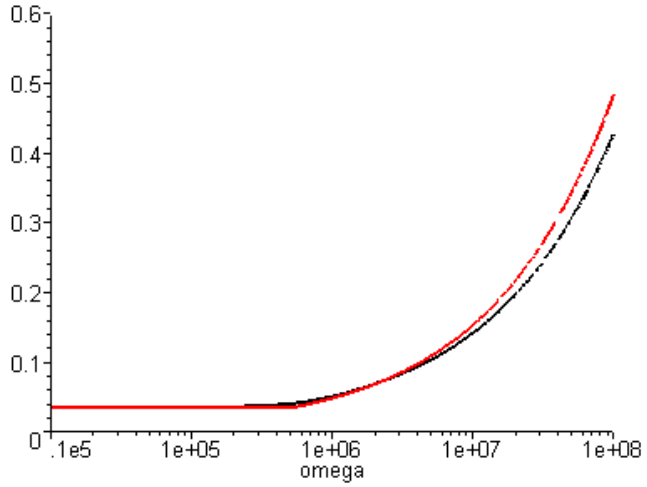
Question: Why is the DC value higher in the Heaviside model?
Answer: because I include the effect of the shield.

Here is the same plot and a corresponding one for R:

```
semilogplot([Li_,Li1],omega = wlo..whi, view = [wlo..whi,
0..6e-8],numpoints=800,thickness=2, color = [red,black]);
```



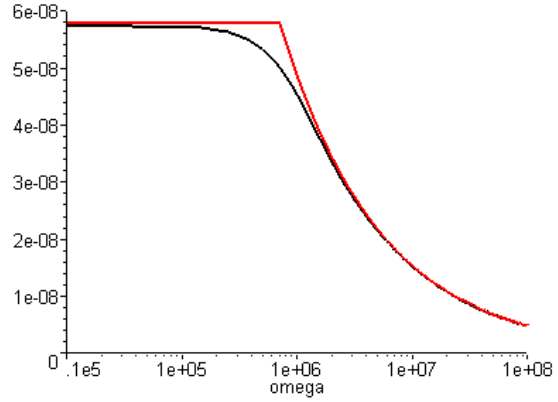

```
semilogplot([R_,R1],omega = wlo..whi, view = [wlo..whi,
0..6e-1],numpoints=800,thickness=2, color = [red,black]);
```



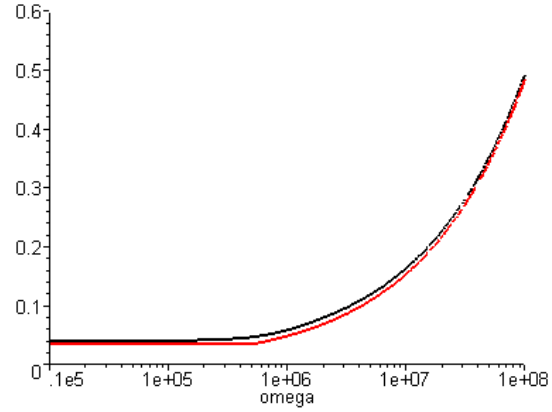
Apart from the issue just noted, the Heaviside model is not too bad!

Here are the same plots if I just boost the single-wire result by 1.15 to account for the shield.

```
semilogplot([Li_,Li1],omega = wlo..whi, view = [wlo..whi,
0..6e-8],numpoints=800,thickness=2, color = [red,black]);
```




```
semilogplot([R_,R1],omega = wlo..whi, view = [wlo..whi,
0..6e-1],numpoints=800,thickness=2, color = [red,black]);
```



These plots I think show just how good the Heaviside model really is at least for the Belden cable case.

This is now incorporated in Appendix Q.