Problem with k(ω) expansion for small ω PhL 9.5.14

The problem: In Appendix Q as it currently stands, I have this expansion of k(ω) for small ω, where I use only my low-ω range model for R,L,G,C : [this is in "Q2 lo omega k version 3 wd GT 0.mws"]

```
> Rek := sqrt((a^2-c)/2);
```

$$Rek := \frac{1}{2}\sqrt{2\sqrt{(Rdc^2+w^2\,Ldc^2)\,(C^2\,(wd+t\,w)^2+w^2\,C^2)}-2\,Rdc\,C\,(wd+t\,w)+2\,w^2\,Ldc\,C}$$

```
> Rek1 := series(Rek,w=0,7): Rek1 := simplify(collect(%,w));
```

$$Rek1 := \frac{1}{2}\frac{(Rdc+Ldc\,wd)\sqrt{C}}{\sqrt{Rdc}\,\sqrt{wd}}w-\frac{1}{4}\frac{(Rdc-Ldc\,wd)\,t\,\sqrt{C}}{wd^{\left(\frac{3}{2}\right)}\sqrt{Rdc}}w^2+\frac{1}{16}\frac{\sqrt{C}\,(3\,Rdc^3\,t^2-Rdc^3-Rdc^2\,t^2\,Ldc\,wd+Rdc^2\,Ldc\,wd+Ldc^2\,wd^2\,Rdc-Ldc^3\,wd^3)}{Rdc^{\left(\frac{5}{2}\right)}\,wd^{\left(\frac{5}{2}\right)}}w^3-$$

$$\frac{1}{32}\frac{\sqrt{C}\,(5\,Rdc^3\,t^2-5\,Rdc^3-Rdc^2\,t^2\,Ldc\,wd+3\,Rdc^2\,Ldc\,wd+Ldc^2\,wd^2\,Rdc+Ldc^3\,wd^3)\,t}{Rdc^{\left(\frac{5}{2}\right)}\,wd^{\left(\frac{7}{2}\right)}}w^4+$$

$$\frac{1}{256}\frac{\sqrt{C}\,(35\,Rdc^5\,t^4-70\,Rdc^5\,t^2+7\,Rdc^5-5\,Rdc^4\,t^4\,Ldc\,wd+30\,Rdc^4\,t^2\,Ldc\,wd-5\,Rdc^4\,Ldc\,wd+6\,Rdc^3\,t^2\,Ldc^2\,wd^2-2\,Rdc^3\,Ldc^2\,wd^2+2\,Rdc^2\,t^2\,Ldc^3\,wd^3-2\,Rdc^2\,Ldc^3\,wd^3-5\,Ldc^4\,wd^4\,Rdc+7\,Ldc^5\,wd^5)}{Rdc^{\left(\frac{9}{2}\right)}\,wd^{\left(\frac{9}{2}\right)}}w^5-$$

$$\frac{1}{512}\frac{\sqrt{C}\,(63\,Rdc^5+63\,Rdc^5\,t^4-210\,Rdc^5\,t^2-35\,Rdc^4\,Ldc\,wd-7\,Rdc^4\,t^4\,Ldc\,wd+70\,Rdc^4\,t^2\,Ldc\,wd-10\,Rdc^3\,Ldc^2\,wd^2+10\,Rdc^3\,t^2\,Ldc^2\,wd^2-6\,Rdc^2\,Ldc^3\,wd^3+2\,Rdc^2\,t^2\,Ldc^3\,wd^3-5\,Ldc^4\,wd^4\,Rdc-7\,Ldc^5\,wd^5)\,t}{Rdc^{\left(\frac{9}{2}\right)}\,wd^{\left(\frac{11}{2}\right)}}w^6+O(w^7)$$

from "Q2 lo omega k version 3 wd GT 0.mws". I know it is illegible, but the problem is this: it seems that this is an expansion in powers of (ω/ω_d) where $\omega_d = 10^{-4}$. It seems then that the radius of convergence of the expansion is something like 10^{-4} in ω space, and this makes the series useless (or at least, not too interesting). I want to know something about k(ω) at perhaps ω = 100, 1000 or 10,000, not just $\omega < 10^{-4}$. I can use my "low ω model" up to at least ω = 500,000 so I would like to see a plot of things up to that frequency, but my low-ω expansion cannot do that!

Here is the function I am looking at:

The real part of k(ω) is given from (Q.2.1) as follows:

```
Rek := sqrt((a^2-c)/2);
```

$$Rek := \frac{1}{2}\sqrt{2\sqrt{(Rdc^2+w^2\,Ldc^2)\,(C^2\,(wd+t\,w)^2+w^2\,C^2)}-2\,Rdc\,C\,(wd+t\,w)+2\,w^2\,Ldc\,C}$$

```
Imk := - sqrt((a^2+c)/2);
```

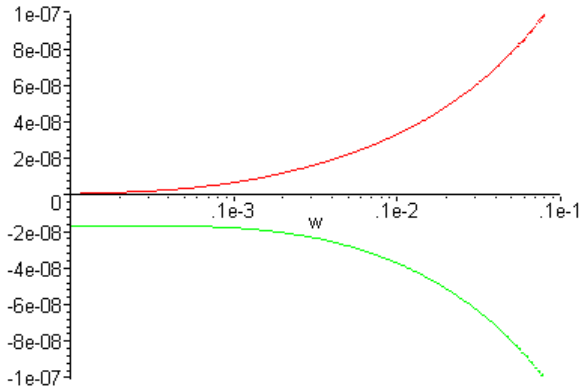
$$Imk := -\frac{1}{2}\sqrt{2\sqrt{(Rdc^2+w^2\,Ldc^2)\,(C^2\,(wd+t\,w)^2+w^2\,C^2)}+2\,Rdc\,C\,(wd+t\,w)-2\,w^2\,Ldc\,C}$$

and here are numbers for Belden 8281 cable: [again, the above uses only the low-ω model]

$R_{dc} = .036$
t = .0005
$L_{dc} = .428 \times 10^{-6}$
$C = 69 \times 10^{-12}$
wd = .00011

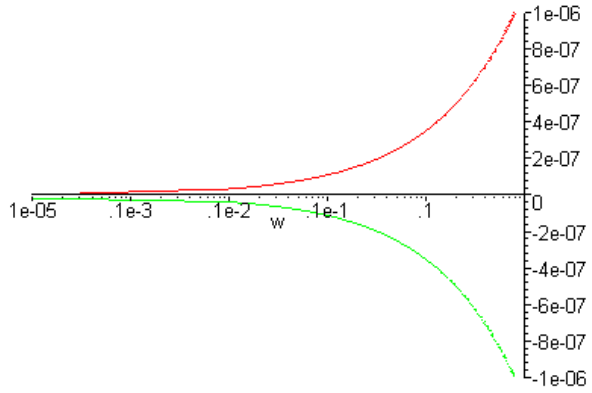
I would certainly like some way to simplify the expression, but everything depends on ω ! I suppose I could just plot the whole thing as is. Just plotting the above functions is a difficult task because I want such a large range of ω. Perhaps it is best to do it in small pieces. For example,

```
semilogplot([Rek,Imk],w=1e-5..1e-2,numpoints=1000, view=[1e-5..1e-2,-1e-7..1e-7]);
```



This gives a general idea of what things look like I think from $\omega = 0$ up to $\omega = 10^{-2}$. This does agree with my small ω expansion in that Imk $\rightarrow$ a negative constant, and Rek$\rightarrow$ 0. I can now extend the above picture to the $\omega = 1$ range,

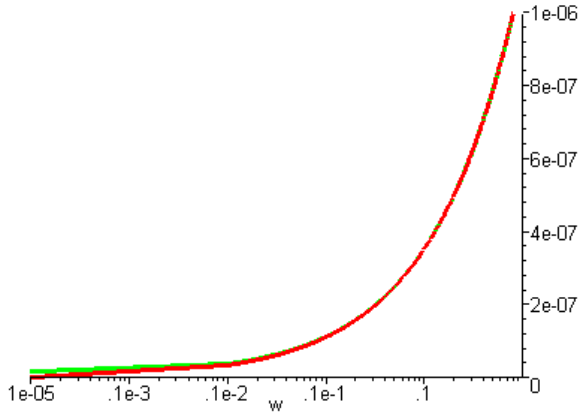
```
semilogplot([Rek,Imk],w=1e-5..1e0,numpoints=1000, view=[1e-5..1e-1,-1e-6..1e-6]);
```



You lose the detail going to $\omega = 0$, but that appears in the previous plot.

Discovery: Once you get above a few Hz, -Imk and Rek are identical! This is an important fact that I have completely missed.

```
semilogplot([Rek,-Imk],w=wlo..whi,numpoints=1000,
view=[wlo..whi,0..1e-6],thickness=3);
```



But then they split apart at higher ω

```
with(plots): wlo:=1e0; whi:=1e6;
```

$$wlo := 1.$$

$$whi := .1\ 10^{7}$$

```
semilogplot([Rek,-Imk],w=wlo..whi,numpoints=1000,
view=[wlo..whi,0..1e-3],thickness=3);
```

