This was added to the new Appendix Q. Can dump this file soon.

Q.9 The general appearance of Re(Z_0) and Im(Z_0) for Belden 8281 cable

This section is very similar to Section Q.5 above concerning the appearance of k(ω). We are interested in viewing the real and imaginary parts of Re(Z_0) over the full frequency range, not just at the extremes of small ω and large ω. The expressions shown in (Q.6.1) are,

```
ReZ0 := (b/sqrt(2))*sqrt(1+(d/a^2));
```

$$ReZ0 := \frac{1}{2}\left(\frac{R^2+\omega^2 L^2}{G^2+\omega^2 C^2}\right)^{\left(\frac{1}{4}\right)} \sqrt{2}\sqrt{1+\frac{RG+\omega^2 LC}{\sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}}}$$

```
ImZ0 := -S*(b/sqrt(2))*sqrt(1-(d/a^2));
```

$$ImZ0 := -\frac{1}{2}S\left(\frac{R^2+\omega^2 L^2}{G^2+\omega^2 C^2}\right)^{\left(\frac{1}{4}\right)} \sqrt{2}\sqrt{1-\frac{RG+\omega^2 LC}{\sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}}} \qquad \text{(Q.9.1)}$$

When the full Heaviside model of Section Q.1 is inserted for the parameters R,L and G, one can see that ReZ_0 and ImZ^0 are complicated functions of ω. Rather than attempt to deal with generic special cases (such as small G), we shall again consider the Belden 8281 cable of Appendix R to be a "typical" transmission line with regard to the relative sizes of the parameters R_{dc}, L_e, L_{idc}, C, ω_d, and $\tan_L$. In our model the "low frequency" range will be taken to be ω = 1 to 500,000.

A new feature not present in the k(ω) case is the sign S which from (Q.6.1) is S = sign(RC-LG). Here are plots of RC-LG and S = sign(RC-LG) for the Belden cable, using the same wide-range ω plotting trick mentioned at the end of Section Q.5 (horizontal axis labeled by $\log_{10}(\omega)$):

```
omega := 10^w1:
plot((R*C-L*G),w1 = -8..11):
plot(S,w1 = -8..14);
```

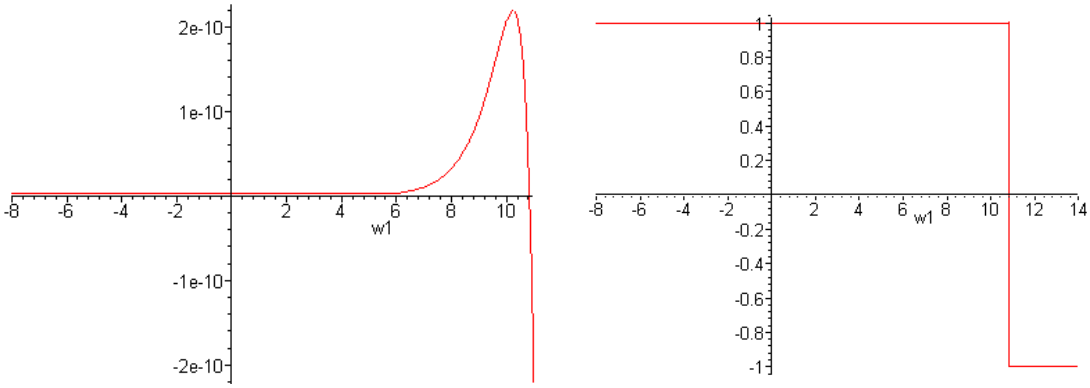


Fig Q.9.1

Basically $\sigma = +1$ up to about $\omega = 10^{11}$ then it becomes -1.

Now recall from (Q.6.1) that

$$\text{Rek} = (b/\sqrt{2}) [\sqrt{1+(d/a^2)} \ ] \qquad -\text{Imk} = \sigma (b/\sqrt{2}) [\sqrt{1-(d/a^2)} \ ] \ . \qquad \text{(Q.9.2)}$$

Here is a plot of the ratio d/a^2 (using our model and the Belden parameters) only up to $\omega = 500{,}000$:

```
semilogplot(d/a^2,omega = 1e0..5e5,thickness=2,numpoints = 2000);
```

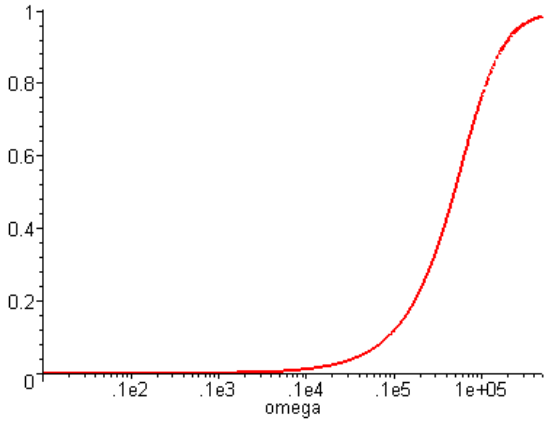


Fig Q.9.2

This shows that $d << a^2$ on the left side of the graph, so for that range we would expect to find that ReZ_0 and $-ImZ_0$ are about the same. That fact is born out in this plot of ReZ_0 and $-ImZ_0$ for ω in (10,500,000):

```
loglogplot([ReZ0,-ImZ0],omega = 1e1..5e5,numpoints=2000,thickness=2, color =
[red,black],scaling = constrained);
```

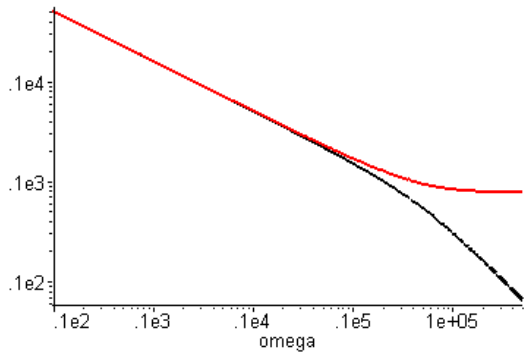


Fig Q.9.3

The red curve is ReZ_0, while the black curve is $-ImZ_0$. This plot then gives a good view of ReZ_0 and ImZ_0 for what one would normally call "the low frequency range" of this Belden cable, roughly below 1 MHz. However, this is not the low frequency range for which (Q.8.3) applies. Recall that (Q.8.3) only applies for $\omega < \omega_d$ (and $\omega_d = \sigma_d/\varepsilon_d \approx 5 \times 10^{-5}$ for the Belden 8281 cable), which we call "ultra low frequencies". We can redo the above plot in the ultra-low range $\omega = 10^{-7}$ to 10^{-2} sec^{-1} :

```
loglogplot([ReZ0,-ImZ0],omega = 1e-7..1e-2,numpoints=2000,thickness=2, color
= [red,black],scaling = constrained);
```

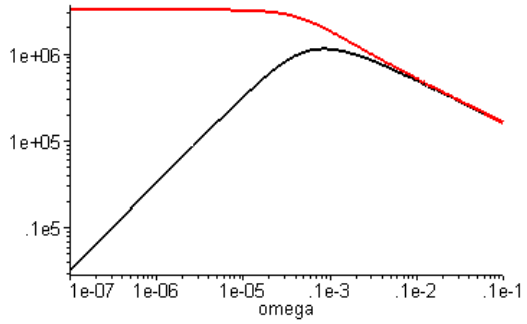


Fig Q.9.4

This shows what happens as we go off the left edge of the previous graph. We find that ReZ_0 goes to a constant, while $-ImZ_0$ has slope 1 so is proportional to ω. This is consistent with the low ω limit (Q.8.3),

$$Re(Z_0) \approx \sqrt{R_{dc}/(C\omega_d)} \; - \; \sqrt{R_{dc}/(C\omega_d)} \; \tan_L/2 \; (\omega/\omega_d) \; + O(\omega^2) \qquad \textit{(Q.8.3)}$$

$$Im(Z_0) \approx - \; (1/2) \; (\; R_{dc} \; - \omega_d L_{dc} \;) \; \; (\omega/\omega_d) \; / \; \sqrt{R_{dc} C \; \omega_d} \; \; + O(\omega^2) \qquad \omega_d \equiv (\sigma_d/\varepsilon_d)$$

Specifically, $\sqrt{R_{dc}/(C\omega_d)} = \sqrt{R_{dc}/G_{dc}} = \sqrt{.036 \; /.34 \text{ x } 10^{-14}} = 3.25 \text{ x } 10^6$ as the plot shows.

Having dealt with low and ultra-low frequencies, we turn now to higher frequencies. For ω in the range 10^3 to 10^{10} one finds,

```
logplot([ReZ0,-Re(ImZ0)],w1 = 3..10,thickness=2, color = [red,black],scaling =
constrained);
```

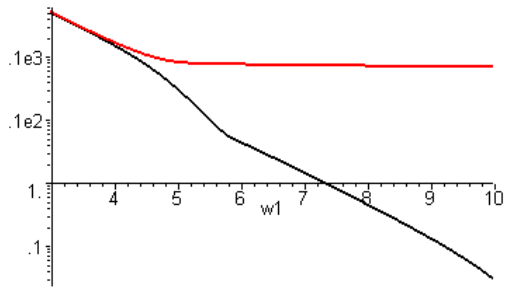


Fig Q.9.5

Recall that ImZ_0 experiences a sign change in the region of $\omega = 10^{11}$ (sign S goes from +1 to -1) as shown in this non-log plot of just $-ImZ_0$ for $\omega=10^9$ to 10^{18}:

```
plot(-ImZ0,w1 = 9..18,thickness=2, color = [red,black]);
```

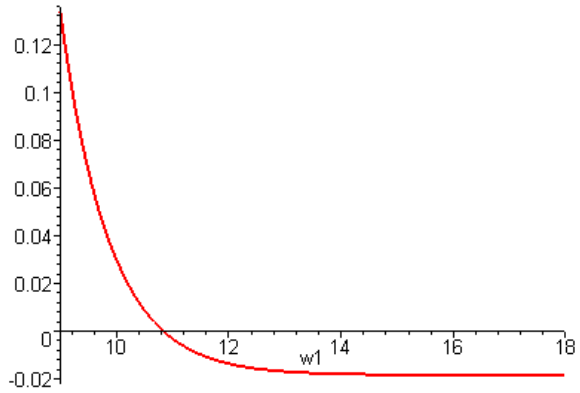


Fig Q.9.6

In fact, $-\mathrm{Im}Z_0$ approaches the constant value indicated in our large ω limit given in (Q.7.4),

$$\mathrm{Re}(Z_0) \approx [1/(v_d C)] + (v_d \kappa/2)/\sqrt{\omega} + O(1/\omega)$$
$$\mathrm{Im}(Z_0) \approx [1/(v_d C)] \tan_L/2 - (v_d \kappa/2)/\sqrt{\omega} + O(1/\omega) \qquad \textit{(Q.7.4)}$$
$$\text{where} \qquad \kappa \equiv \left(\frac{1}{p_1} + \frac{1}{p_2}\right)\sqrt{\frac{\mu}{2\sigma}}$$

that value being about

$$\mathrm{Im}Z_0 \rightarrow [1/(v_d C)] \tan_L/2 = [\sqrt{\varepsilon_d/\varepsilon_0}/(cC)] \tan_L/2 = [\sqrt{2.3}/(3\times10^8 \times 69 \times 10^{-12})] .0005/2$$

```
(sqrt(2.3)/(3e8*69e-12))*.0005/2;
                                    .01831612427
```

in agreement with the above plot.

Because $-\mathrm{Im}Z_0$ goes negative, we cannot do a full log plot of $-\mathrm{Im}Z_0$ without adding a small positive offset. This problem did not arise when dealing with k(ω) in Section Q.5 because Imk must always be negative to insure a loss at any ω.

Adding an offset of .02, we then make our plot over a full range of ω:

```
logplot([ReZ0,-ImZ0 + .02],w1 = -8..17,thickness=2,color =
[red,black],numpoints=200);
```

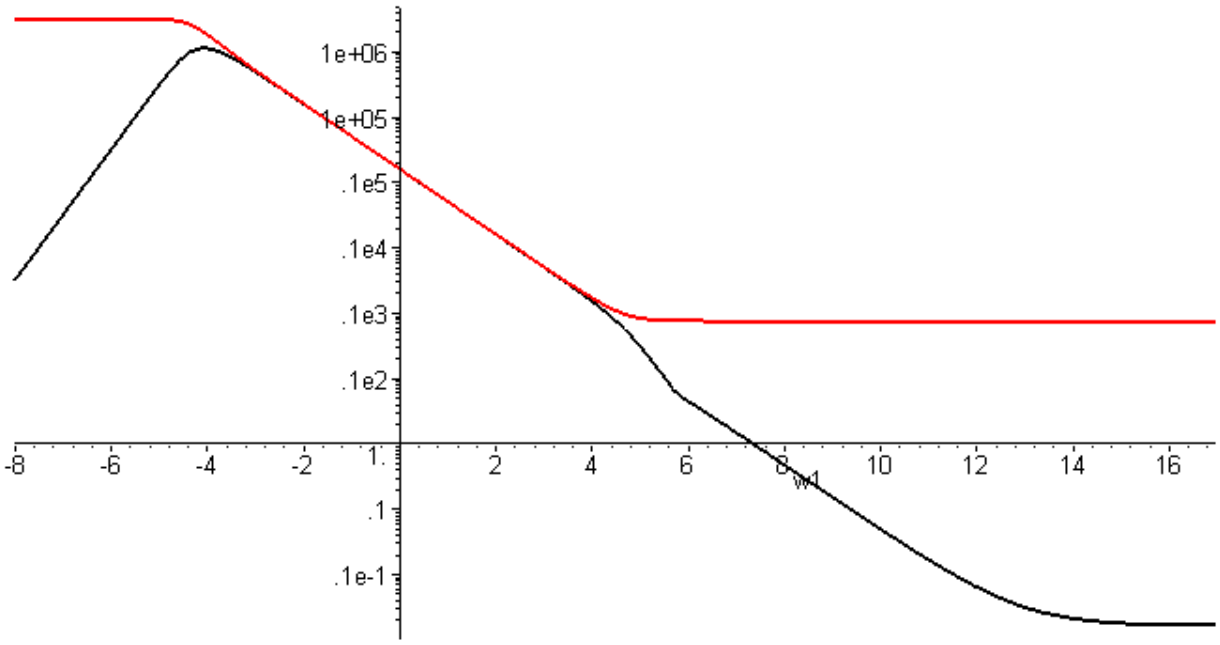


Fig Q.9.7

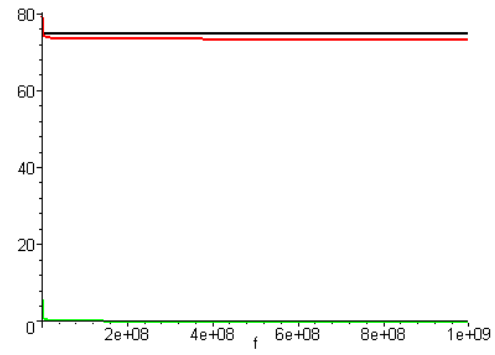
We have just shown that, for large ω, ImZ_0 approaches the constant - .18 Ω shown in (Q.7.4). We now see that ReZ_0 also approaches a constant value $[1/(v_d C)] = [\sqrt{\varepsilon_d/\varepsilon_0}/(cC)]$ which is

```
(sqrt(2.3)/(3e8*69e-12));
```

73.26449705

This value of 73.26Ω is slightly less than the nominal cable impedance of 75 Ω. The following plot shows ReZ_0 and $-ImZ_0$ for 100 KHz to 1 GHz on the left, and 10 KHz to 100 KHz on the right

```
omega := 2*pi*f:
plot([ReZ0,75,-ImZ0],f = 1e5..1e9, thickness
= 2, color = [red,black,green]);
```




```
omega := 2*pi*f:
plot([ReZ0,75,-ImZ0],f = 1e4..1e5, thickness =
2, color = [red,black,green]);
```

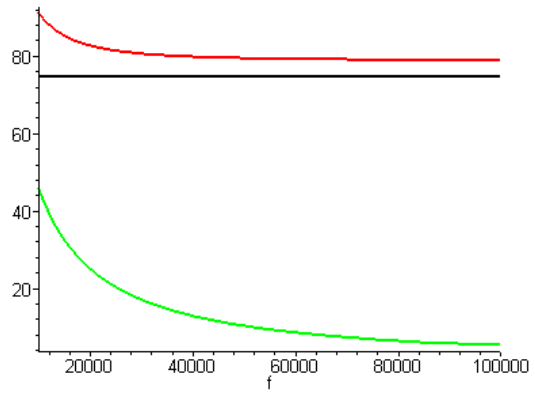


Fig Q.9.8

Above 100 KHz $-ImZ_0$ can be neglected, but at 10KHz it jumps up to 44 Ω.