Appendix Q save stuff PhL 3.26.05

Note that page numbering is turned on in this template.

Summary of results:

Summary of k and its limits (Q.8)

$k \equiv -j\sqrt{\mathbf{zy}} = -j\sqrt{(R+j\omega L)(G+j\omega C)}$ // definition of the wavenumber k(ω)

$$k = \sqrt{\frac{a^2-c}{2}} - j\sqrt{\frac{a^2+c}{2}} \qquad jk = \sqrt{\mathbf{zy}} = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{\frac{a^2+c}{2}} + j\sqrt{\frac{a^2-c}{2}}$$

$a \equiv (R^2+\omega^2L^2)^{1/4}\,(G^2+\omega^2C^2)^{1/4} = |k|$
$c \equiv RG - \omega^2LC$

Large ω: $\mathrm{Re}(k) = \omega\sqrt{LC} + \dfrac{(RC-LG)^2}{8\omega(LC)^{3/2}}$ // $\sqrt{LC} = 1/v_d$ and $\omega\sqrt{LC} = \omega/v_d = \beta_{d0}$

$$\mathrm{Im}(k) = -\frac{(RC+GL)}{2\sqrt{LC}} + \frac{(RC-GL)^2\,(RC+GL)}{16\omega^2\,(LC)^{5/2}} \qquad \omega >> (R/L),(G/C) \qquad (Q.5)$$

Small ω, G>0: $\mathrm{Re}(k) = (\omega/2)\dfrac{RC+GL}{\sqrt{RG}}$

$$\mathrm{Im}(k) = -\sqrt{RG} - (\omega^2/8)\,\frac{(RC-LG)^2}{(RC)^{3/2}} \qquad \omega << (R/L),(G/C) \qquad (Q.6)$$

Small ω, G=0: $\mathrm{Re}(k) = \sqrt{RC/2}\;\omega^{1/2} + \sqrt{C/8R}\;\omega^{3/2}$

$$\mathrm{Im}(k) = -\sqrt{RC/2}\;\omega^{1/2} + \sqrt{C/8R}\;\omega^{3/2} \qquad \omega << (R/L) \qquad (Q.7)$$

Comment: In principle, all four quantities R,C,L,G can be functions of ω. According to the analysis of Chapter 4 leading to summary box (4.11.34), if we stay away from the low frequency range, only parameter L shows a strong frequency dependence since $L = L_e + L_i(\omega)$. As shown in the comments below (4.11.34), $\lim_{\omega\to\infty} L_i(\omega) = 0$. This is due to the skin effect and result (C.6.8). Thus, we know that $\lim_{\omega\to\infty} L(\omega) = L_e$. But (4.11.34) says $L_eC = \mu_d\varepsilon_d = 1/v_d^2$ where v_d is the speed of light in the dielectric. Therefore, $\lim_{\omega\to\infty} L(\omega)C = L_eC = 1/v_d^2$ and thus *for large ω* we get $\mathrm{Re}(k) = \omega\sqrt{LC} = \omega/v_d$ and the phase velocity of the wave equals the dielectric speed of light. As ω drops from the high end, the identification of $\sqrt{LC}$ with $1/v_d$ becomes less accurate.