Repair Section D.11 PhL 9.9.14

This small repair to D.11 (b) only was installed today 9.9.14.

I will start with section (b) and come back later to Section (a).

(b) Low frequency values for β'

For low ω and G > 0: We seek an expression for β' at low ω. Since $\beta'^2 = \beta^2 - k^2$, we need to know about β and k. For any ω, we know first that

$$\beta^2 = -j\omega\mu\sigma \qquad \textit{(1.5.1d)} \text{ for good conductor}$$

so $\beta^2 \to 0$ as $\omega \to 0$. Meanwhile, the ultra-low frequency limit of k is known from (Q.4.6) to be,

$$\begin{aligned} \mathrm{Re}(k) &\approx (\omega/2)\,(R_{dc} + \omega_d L_{dc})\sqrt{C/(\omega_d R_{dc})} + O(\omega^2) \qquad \omega < \omega_d = (\sigma_d/\varepsilon_d) \\ \mathrm{Im}(k) &\approx -\sqrt{R_{dc} C \omega_d}\,[\,1 + (\tan_L/2)\,(\omega/\omega_d)] + O(\omega^2)\ . \end{aligned} \qquad \textit{(Q.4.6)}$$

As $\omega \to 0$, we find $\mathrm{Re}(k) \to 0$ and $\mathrm{Im}(k) \to -\sqrt{R_{dc} C \omega_d} = -\sqrt{R_{dc} G_{dc}}$. Thus we have at low ω,

$$\beta^2 \approx 0$$

$$k \approx -j\sqrt{R_{dc} G_{dc}} \qquad \Rightarrow \qquad k^2 = -R_{dc} G_{dc}$$

$$\beta'^2 = \beta^2 - k^2 = -k^2 \approx R_{dc} G_{dc}\ . \qquad \text{(D.11.1)}$$

For any reasonable transmission line $R_{dc}G_{dc}$ will very small so the Bessel argument $x_a = \beta' a \ll 1$.

Low Frequency Example:

For Belden 8281 coaxial cable, Appendix R below Fig R.6 gives $R_{dc} = .036$ and $G_{dc} = 0.338 \times 10^{-14}$ so $\beta'^2 \approx R_{dc}G_{dc} \approx 1.22 \times 10^{-16}$ and then $\beta' \approx 1.1 \times 10^{-8}\ \mathrm{m}^{-1}$. The central conductor has $a = 3.94 \times 10^{-4}$ m so $\beta' a \approx 5 \times 10^{-12} \ll 1$.

Since β'a is very small, we shall need to evaluate f_m, g_m and h_m for small $x_a = \beta' a$.

For low ω and G= 0: When $G_{dc} = 0$, the ultra-low ω behavior of k is given by (Q.4.9),

$$\begin{aligned} \mathrm{Re}(k) &\approx +\sqrt{R_{dc} C/2}\ \sqrt{\omega}\,(\,1 - \tan_L/2) + O(\omega^{3/2}) \\ \mathrm{Im}(k) &\approx -\sqrt{R_{dc} C/2}\ \sqrt{\omega}\,(\,1 + \tan_L/2) + O(\omega^{3/2})\ . \end{aligned} \qquad \textit{(Q.4.9)}$$

As $\omega \to 0$ we then have

$k \approx \sqrt{R_{dc}C/2}\ \sqrt{\omega}\ (1 - j)$

$k^2 = [\ R_{dc}C/2]\ \omega\ [\ -2j\ ] = -j\omega\ R_{dc}C$

$\beta^2 = -j\omega\mu\sigma$ *(1.5.1d)* for good conductor

$\beta'^2 = \beta^2 - k^2 \approx -j\omega\mu\sigma + j\omega R_{dc}C = -j\omega(\mu\sigma - R_{dc}C)$. (D.11.2)

Again β' is very small at low frequency and in fact $\beta' \rightarrow 0$ as $\omega \rightarrow 0$. Thus again we have $\beta'a << 1$ so we shall need to evaluate f_m, g_m and h_m for small $x_a = \beta'a$.

(c) Low frequency evaluation of f_m, g_m and h_m

No changes needed for this section (c).

(d) Low frequency E fields

I think the rest of this section is all OK. The anomaly remains!