Note that page numbering is turned on in this template.

scraps from Appendix M work

Anomaly: [no longer relevant!] For a conducting dielectric ($\sigma_d > 0 \Rightarrow \omega_d > 0$) the ratio $|k/\beta|$ for small ω has the form

$$|k/\beta| = \sqrt{\frac{|(R+j\omega L)(G+j\omega C)|}{\omega\mu\sigma}} \rightarrow \sqrt{\frac{|R_{dc}(C\omega_d+j\omega C)|}{\omega\mu\sigma}} = \sqrt{\frac{R_{dc}C\omega_d}{\omega\mu\sigma}} = \sqrt{\frac{R_{dc}G_{dc}}{\omega\mu\sigma}} \qquad (M.13)$$

which becomes infinite at $\omega = 0$. The issue here is that as $\omega \rightarrow 0$ we have $\beta \rightarrow 0$ but $k \rightarrow -j\sqrt{R_{dc}G_{dc}} \neq 0$. Since the dielectric conducts current, there must be a DC loss going down the transmission line in z and this is associated with $\text{Im}(k) = -\sqrt{R_{dc}G}$, so it is *expected* that $k \not\rightarrow 0$. We then expect our 10^{-3} inequality rule of (M.12) to be violated when

$$10^{-6} < R_{dc}G_{dc}/(\omega\mu\sigma)$$

or

$$\omega < 10^{6}\, R_{dc}G_{dc}/(\mu\sigma) \approx 10^{6}\, R_{dc}G_{dc}/73 \; . \qquad \text{// copper conductors}$$

For the Belden 8281 cable of Appendix R, we find that $R_{dc} \approx .036$, $G_{dc} = \omega_d C \approx 0.8 \times 10^{-14}$ so our condition for violation becomes

$$\omega < 10^{6}\,(.036)(\,0.8 \times 10^{-14}/73\,) = 3.9 \times 10^{-12}\ \text{sec}^{-1}$$

It is not unreasonable that at such ultra-low frequencies near DC the transverse current J_r can be large relative to J_z. The network model is then that of Fig K.4. One could solve the DC magnetostatics problem for an infinite transmission line with round conductors to determine J_r and J_z and compare their magnitudes. Since a practical transmission line is not operated at such ultra low frequencies, we ignore the anomaly and state the claim (M.1) as if it were valid all the way down to $\omega = 0$.

NOTE in red: In (D.11.14) I claim to know J_z and J_r as $\omega \rightarrow 0$ for each partial wave. The ratio seems to be this

$$J_r/J_z \approx aG / \sqrt{G/R} = a\sqrt{GR}$$

For Belden 8281 center wire this is

$$J_r/J_z = a_1\sqrt{GR} = .00039 * \sqrt{(.036)(\,0.8 \times 10^{-14}/73\,)} = .7 \times 10^{-11}$$

which disagrees with the above discussion!! I do state there that $k \approx -j\sqrt{RG}$ which agrees with above. In box (D.11.7) the ratio E_r/E_z is clearly just (ak). What happened to (k/β) ? Aha! It is those Bessel functions that I ignored above !!!!