Appendix R: A case study: Belden 8281 Coaxial Cable

This is an older version of Appendix R. The *current* version is in file belden v3.doc and that current version has been installed. So THIS doc is now "reviewed".

Here we gradually work our way through the Belden data sheet for its 8281 cable, correlating the data presented there with the general theory of this document. The data sheet is available here

www.belden.com/techdatas/metric/8281.pdf

but we will be quoting most of it below.

(a) The Basic Parameters

We start with:

Physical Characteristics (Overall)

Conductor

AWG:

# Coax	AWG	Stranding	Conductor Material	Dia. (mm)
1	20	Solid	BC - Bare Copper	0.7874

Total Number of Conductors:

Insulation

Insulation Material:

Insulation Material	Dia. (mm)
PE - Polyethylene	5.0292

Outer Shield

Outer Shield Material:

Layer #	Type	Outer Shield Material	Coverage (%)
1	Braid	TC - Tinned Copper	95.000
2	Braid	TC - Tinned Copper	95.000

There is a two-layer braided copper shield which has a very thin tin plating to stop oxidation of the copper. Tin is a worse conductor than copper ($\sigma_{tin}/\sigma_{copper} = 0.16$), but we assume below that the plating is so thin that it does not affect our shield model.

So we start accumulating some numbers,

```
a1 :=  (1/2)*0.7874e-3;
```

$$a1 := .0003937000000$$

```
a2 :=  2.5146e-3;
```

$$a2 := .0025146$$

The dielectric constant for the polyethylene (PE) used in this cable we will take to be exactly

$\varepsilon_d = 2.30000\ \varepsilon_0$

although the reader will recall that this number varies for different types of PE product as shown in the discussion below (3.3.5). We shall also use the loss tangent quoted there (dissipation factor)

$\tan_L = .0002$

resulting in this frequency-dependent dielectric conductivity.

$\sigma_d = \varepsilon_d \tan_L \omega.$

We duly enter these expressions into our code for later use:

```
e0 := 8.8541877e-12;
```

$$e0 := .88541877\ 10^{-11}$$

```
ed := 2.3*e0;
```

$$ed := .2036463171\ 10^{-10}$$

```
tanL := .0002;
```

$$tanL := .0002$$

```
sd := ed*tanL*omega;
```

$$sd := .4072926342\ 10^{-14}\ \omega$$

The dimensionless constant K of Chapter 4 can be computed from (4.6.3)

$K = 2 \ln(a_2/a_1) = 3.709$ *(4.6.3)*

```
K := 2*ln(a2/a1);
```

$$K := 3.708559652$$

The capacitance per unit length may then be computed from (4.11.34),

$C = 4\pi\varepsilon_d/K = 69.005$ pF/m *(4.11.34)*

```
C := evalf(4*Pi*ed/K);
```

$$C := .6900509458\ 10^{-10}$$

which we find is quite close to Belden's advertised value,

Nom. Capacitance Conductor to Shield:

Capacitance (pF/m)
68.901

```
69.005/68.901;
```

$$1.001509412$$

so our error here is less than 2 tenths of one percent.

The external inductance L_e may also be computed from (4.11.34).

$$L_e = \frac{\mu_d}{4\pi} K = \frac{\mu_0}{4\pi} K = .3709\ \mu H/m \qquad (4.11.34)$$

```
mu0 := evalf(4*Pi*1e-7);
```

$$\mu 0 := .1256637062\ 10^{-5}$$

```
Le := evalf(mu0*K/(4*Pi));
```

$$Le := .3708559652\ 10^{-6}$$

At very high frequency, we expect from (Q.2) the phase velocity of a transmission line wave to be,

$v_d = 1/\sqrt{L_e C} = 1.977 \times 10^8$ m/sec

$v_d/c = .6594$

```
v := 1/sqrt(Le *C);
```

$$v := .1976772942\ 10^{9}$$

```
c := 2.998e8;
```

$$c := .2998\ 10^{9}$$

```
v/c;
```

$$.6593638899$$

At very high frequency, as shown below and as was discussed in (4.11.36), we know $L \rightarrow L_e$ and that is why we can use L_e (external inductance) in the above calculation.

Since v_d is the speed of light in the dielectric medium, we should get the same number using (1.1.29) applied to the dielectric

$v_d = 1/\sqrt{\mu_0 \varepsilon_d}$

```
v := 1/sqrt(mu0*ed);
```

$$v := .1976772942\ 10^{9}$$

```
v/c;
```

$$.6593638899$$

and indeed the number is the same. These identical calculations in fact deliver Belden's official value,

Nominal Velocity of Propagation:

VP (%)
66

The conductance G of the dielectric can be found from (4.11.34) and (R.1) above,

$G = (\sigma_d/\varepsilon_d)\ C = .1380 \times 10^{-13}\ \omega$ mhos/m *(4.11.34)*

```
G := (sd/ed)*C;
```

$$G := .1380101892\ 10^{-13}\ \omega$$

Next we find that the conductor resistances are,

Nom. Conductor DC Resistance:

DCR @ 20°C (Ohm/km)
32.4819

Nominal Outer Shield DC Resistance:

DCR @ 20°C (Ohm/km)
3.6091

so

$R_1 = 32.4819 \times 10^{-3}$ ohms/m
$R_2 = 3.6091 \times 10^{-3}$ ohms/m

$R = R_1 + R_2 = 36.0910 \times 10^{-3}$ ohms/m

```
R1 := 32.4819e-3;
```

$R1 := .0324819$

```
R2 := 3.6091e-3;
```

$R2 := .0036091$

```
R := R1+R2;
```

$R := .0360910$

We have so far gathered up R,C, L_e and G, but L requires some work. Recall that the DC internal inductance L_i of a round wire was computed in (C.3.10) to be

$$L_i = \frac{\mu_0}{8\pi} = 50 \text{ nH/m} = .050 \ \mu\text{H/m} \ . \qquad (C.3.10)$$

This is fine for the round center conductor of a coaxial transmission line, but one must also consider the L_i of the shield. In the next section we shall derive the following simple model for L_i for the Belden 8281 cable,

$$L_i(\omega) = L_i = 1.16 \ \frac{\mu_0}{8\pi} \ [\ \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2} \ \theta(\omega>\omega_t)\ ]$$

where $\omega_t = (8/a_1^2)(1/\mu_0\sigma)$.

As shown below, at this frequency the skin depth is exactly half the radius of the center conductor,

$$\delta_t = (a_1/2)$$

In this *ad hoc* model, below frequency ω_t the inductance is given by $1.16 \ \frac{\mu_i}{8\pi}$. Comparing to ***, the extra 16% is due to the shield. Above ω_t this constant L_i tapers off as $(\omega_t/\omega)^{1/2}$ and eventually we get $L_i \rightarrow 0$ for very large ω. Obviously this model is not exact, but it seems reasonable.

The total inductance is then

$$L(\omega) = L_e + L_i = L_e + 1.16 \ \frac{\mu_0}{8\pi} \ [\ \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2} \ \theta(\omega>\omega_t)\ ] \ .$$

Since we already have a value for L_e, we can evaluate this $L(\omega)$ anywhere below the transition point to find

$$L(\omega_t) = L_e + 1.16 \ \frac{\mu_0}{8\pi} = .4289 \ \mu\text{H/m}$$

```
dL := evalf(1.16*mu0/(8*Pi));
```

$$dL := .5800000001\ 10^{-7}$$

```
Lwt := Le + dL;
```

$$Lwt := .4288559652\ 10^{-6}$$

Belden publishes the following value for L,

Nom. Inductance:

Inductance (µH/m)
0.429811

and we can compare their value with our calculation,

L_{Belden} / $L(\omega_t)$ = .429811/.42886 = 1.00221750688

so again our model error is 2 tenths of one percent. Belden gives no hints as to how it obtains the parameter values it claims, and the number of decimal places is admittedly suspicious. Our agreement with their data is really better than one should expect.

We already have R,G and C, and we can now add our model for L:

```
wt := (8/a1^2)*(1/(mu0*sigma));
```

$$wt := 706924.7074$$

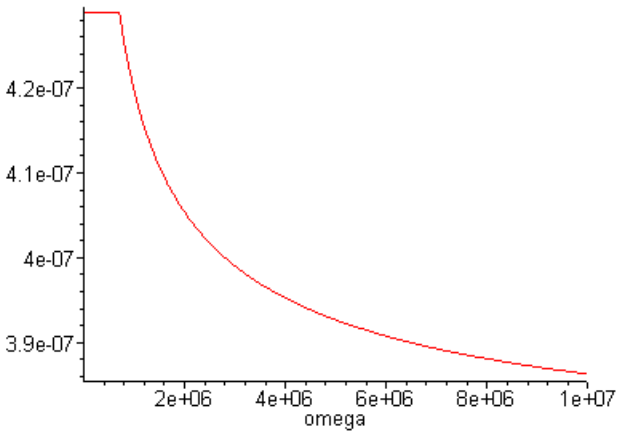
```
ft := evalf(wt/(2*Pi));
```

$$ft := 112510.5615$$

```
L := Le + dL*( Heaviside(wt-omega) + Heaviside(omega-wt)*sqrt(wt/omega) );
```

Here we see that that our somewhat arbitrary transition frequency is ω_t = 112 KHz. Here is a simple plot of L(ω) versus ω

```
plot(L,omega=1..1e7);
```



which has the expected model shape, shown here from 0 to 100 MHz. In a true model the sharp corner would be smoothed off in some fashion.

We come next to the matter of Z_0, the characteristic impedance. From (4.11.34),

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (4.11.34)$$

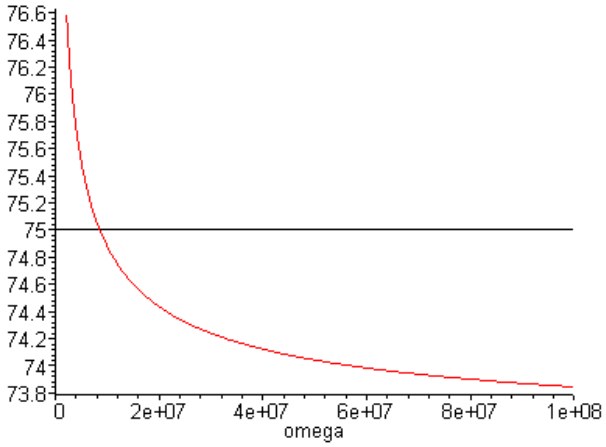
All the pieces are now in place for plot of this function:

```
Z0 := sqrt((R+j*omega*L)/(G + j*omega*C));
```

$$Z0 := \sqrt{\frac{R + j\,\omega\,L}{G + j\,\omega\,C}}$$

(this line was entered at the very start, before R,L,G,C were known). Plotting only the real part we find

```
plot([Re(Z0),75], omega = 0..1e8, color=[red,black]);
```



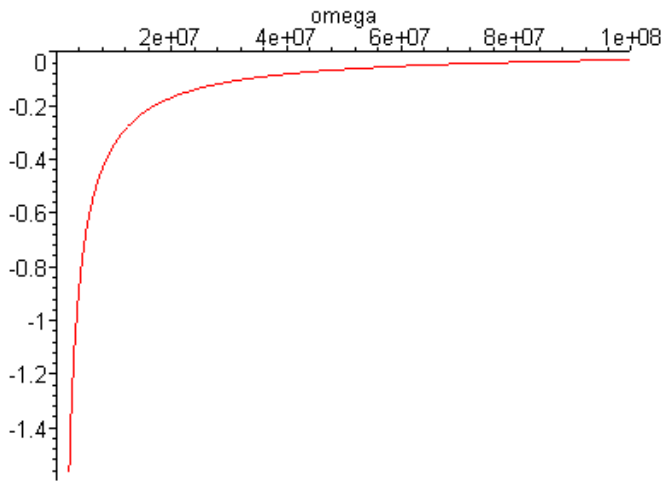
where we have marked the official Z_0 of the cable,

Nom. Characteristic Impedance:

Impedance (Ohm)
75

The graph shows the impedance is close to 75 ohms at about 9 MHz, but varies somewhat as one moves to either side of this frequency. Roughly in the range 2 MHz to 100 MHz Z_0 ranges from 76.6 to 73.8 ohms. The imaginary part of Z_0 over the same ω range is quite small,

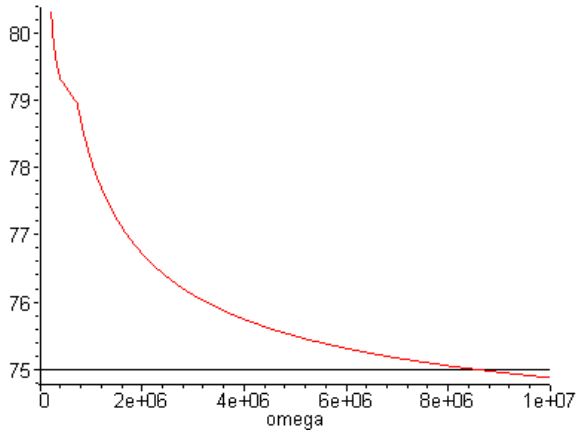
```
plot(Im(Z0), omega = 0..1e8);
```



being typically less than 1 ohm.

Here is another plot showing more detail on Re(Z_0) for the lower frequency range 0 to 10 MHz,

```
plot([Re(Z0),75], omega = 0..1e7, color=[red,black]);
```



where the kink is due to our simple model for L(ω).

Next, we examine the cable attenuation. With the traveling wave form $e^{j(\omega t - kz)}$ the amplitude z dependence is $e^{-jkz} = e^{-az}\ e^{-jbz}$ where

a = -Im(k) = Re[$\sqrt{(R+j\omega L)(G+j\omega C)}$] // attenuation per distance

b = Re(k) = Im[$\sqrt{(R+j\omega L)(G+j\omega C)}$] . // phase *(5.3.6)*

Just to head off any confusion here, look at these details:

$e^{-jkz} = e^{-az}\ e^{-jbz}$

$e^{-jkz} = e^{-j[Re(k)+jIm(k)]z} = e^{[-jRe(k)+Im(k)]z} = e^{-jRe(k)z}\ e^{Im(k)z}$

$= e^{-jRe(k)z}\ e^{-[-Im(k)]z} = e^{-jbz}\ e^{-az}$

and therefore

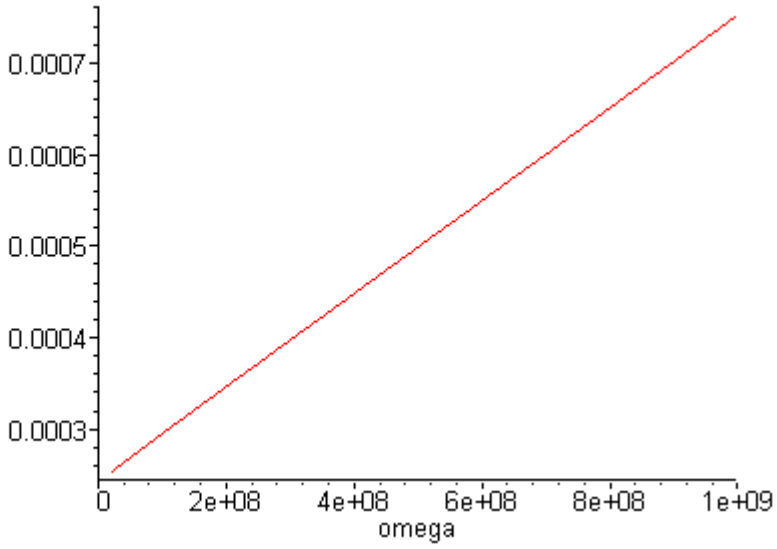
b = Re(k)
a = -Im(k)

Here then is a plot of the attenuation factor a,

```
k := -j*sqrt((R+j*omega*L)*(G+j*omega*C));

a := -Im(k);

plot(a,omega = 0..1e9);
```



The *power* attenuation is given by

dB(z) = 20 $\log_{10}$ (e^{-az}) = 20(-az) $\log_{10}$e = 20(-az) .4342944819

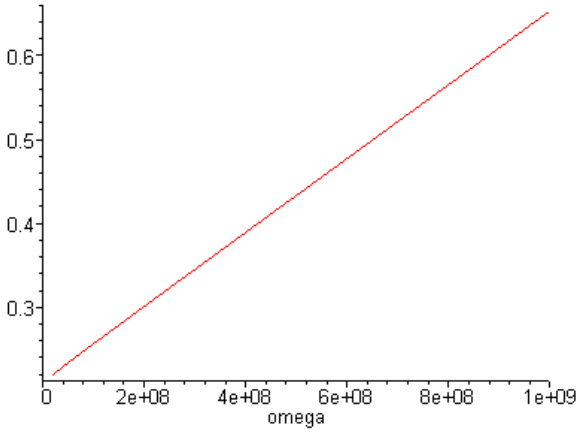
= - 8.685889638 az

The loss per 100 meters is obtained by setting z = 100, then

-dB(100m) = 868.5 a

and here is a plot of the power loss per 100 meters,

```
plot(868.5*a,omega = 0..1e9);
```



But Belden gives much larger numbers, for example,

Nom. Attenuation:

Freq. (MHz)	Attenuation (dB/100m)
1	0.9843
3.6	1.6405
10.0	2.6248
71.5	6.8901
135	9.843
270	14.1083
360	16.7331
540	20.6703
720	24.2794
750	24.9356
1000	30.1852

so the machine once again grinds to a halt while I hunt down this bug. Now 10:15AM 7.6.14.

Plan A: If I replace L by L_e in the k formula, results are still much too small.

Plan B: Consider the high ω limit. Then we know that

$$a = -\mathrm{Im}(k) = \mathrm{Re}[\sqrt{(R+j\omega L)(G+j\omega C)}\,]$$

But both L and G are functions of ω in my model.

$$\sigma_d = \varepsilon_d \tan_L \omega$$

$$G = (\sigma_d/\varepsilon_d)\ C = ([\ \varepsilon_d \tan_L \omega]/\varepsilon_d)C = \tan_L \omega C$$

$$R = 36.0910 \times 10^{-3}$$

$$C = 69.005 \times 10^{-12}$$

$$L = L_e + 1.16\ \frac{\mu_0}{8\pi}\ [\ \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2}\ \theta(\omega>\omega_t)\ ]$$

$$= 0.37 \times 10^{-6} + 0.58 \times 10^{-7}\ [\ \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2}\ \theta(\omega>\omega_t)\ ]$$

$$= 0.37 \times 10^{-6} + 0.58 \times 10^{-7}\ (\omega_t/\omega)^{1/2} \qquad \text{/ / large } \omega$$

Now at large ω we know that $L \rightarrow L_e$ so just write then

$$L = 0.37 \times 10^{-6} = \text{fixed}$$

Then

$$(R+j\omega L) = 36.0910 \times 10^{-3} + j\omega\ 0.37 \times 10^{-6}$$

At large ω, the jωL term wins and we juat have

$$(R+j\omega L) = j\omega\ 0.37 \times 10^{-6}$$

Meanwhile,

$$(G+j\omega C) = \tan_L \omega C + j\omega C \approx j\omega C$$

So we then have

$$(R+j\omega L)\ (G+j\omega C) \approx j\omega\ 0.37 \times 10^{-6} * j\omega C = -\ 0.37 \times 10^{-6}\ C\ \omega^2$$

$$= -\ 0.37 \times 10^{-6} \times 69 \times 10^{-12}\ \omega^2$$

$$= -\ [0.37 * 69] \times 10^{-18}\ \omega^2 = -\ 25.53 \times 10^{-18}\ \omega^2$$

Then

$\sqrt{(R+j\omega L)(G+j\omega C)} \approx j * 5 \times 10^{-9}\ \omega$

$k = -j\ \sqrt{(R+j\omega L)(G+j\omega C)} = 5 \times 10^{-9}\ \omega$ = real with no imaginary part

so in this limit, I lose the attenuation completely.

Plan C: OK, let's try Appendix Q:

$$\text{Im}(k) \approx -\frac{(RC + GL)}{2\sqrt{LC}} \quad \text{for large } \omega$$

Then numerator is

$RC + GL = RC + GL_e$

$= RC + \tan_L\ \omega C\ L_e$

$= C(R + \tan_L\ \omega\ L_e)$

$= C(36.0910 \times 10^{-3} + .0002\ \omega\ 0.37 \times 10^{-6}\ )$

$= C(36.0910 \times 10^{-3} + .2\ \omega\ 0.37 \times 10^{-9}\ )$

Now well above 1 MHz the second term wins so we have

$RC + GL \approx C * .2\ \omega\ 0.37 \times 10^{-9}$

$= 69 \times 10^{-12} * .2\ \omega\ 0.37 \times 10^{-9}$

$= 5.1 \times 10^{-12}\ \omega \times 10^{-9}$

$= 5.1 \times 10^{-21}\ \omega$

Meanwhile, $1/\sqrt{L_eC} = .2 \times 10^{9}$ = 2/3 speed of light, so then

$$-\frac{(RC + GL)}{2\sqrt{LC}} \approx -5.1 \times 10^{-21}\ \omega\ * (1/2) * .2 \times 10^{9}$$

$= -2.5 \times 10^{-12}\ \omega\ * .2$

$= -0.5 \times 10^{-12}\ \omega$

So I have shown that at high ω,

$$- \text{Im}(k) \approx \frac{(RC + GL)}{2\sqrt{LC}} = 0.5 \times 10^{-12}\,\omega$$

At $\omega = 10^9$, we then get

$$-\text{Im}(k) = 0.5 \times 10^{-3} = a = .0005$$

My graph above shows this to be .00075 or so we are in the ballpark. So even this was a difficult limit to compute by hand, the limit agrees with my plot and therefore not with Belden.

Plan D: Do Plan C in Maple

```
> (R*C + G*L)/(2*sqrt(L*C));
```

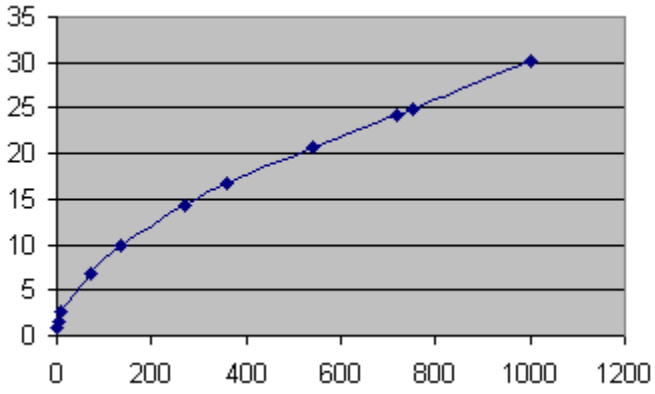
$$\frac{1}{2}\left(.2490462868\,10^{-11} + .1380101892\,10^{-13}\,\omega\left(.3708559652\,10^{-6} + .5800000001\,10^{-7}\,\text{Heaviside}(706924.7074 - \omega) + .00004876571251\,\text{Heaviside}(-706924.7074 + \omega)\sqrt{\frac{1}{\omega}}\right)\right) \Big/ \text{sqrt}\left(.2559095095\,10^{-16} + .4002295486\,10^{-17}\,\text{Heaviside}(706924.7074 - \omega) + .3365082604\,10^{-14}\,\text{Heaviside}(-706924.7074 + \omega)\sqrt{\frac{1}{\omega}}\right)$$

```
> eval(%,omega=1e9);
```

$$.0007525694650$$

and there once again is Im(k) = .00075 in agreement with my graph.

Plan E: Here is an Excel plot of the Belden data



At least it is more or less linear. The plot I get is this:

```
plot(-dB,f=0..1e9);
```

