Belden and Power Examples Data PhL 7.3.14

This is basically my first work on the Belden Appendix R. I gather the basic facts, but things ended up differently in the final Appendix R from the way they are presented here. Keep this stuff for the record, it has a list of metal conductivities including tin; gold worse than silver and copper!

Data is file www.belden.com/techdatas/metric/8281.pdf which I have stored as a file 8281.pdf.
Maple support file is called "belden and power.mws".

First batch of data:

Physical Characteristics (Overall)

Conductor

AWG:

# Coax	AWG	Stranding	Conductor Material	Dia. (mm)
1	20	Solid	BC - Bare Copper	0.7874

Total Number of Conductors:

Insulation

Insulation Material:

Insulation Material	Dia. (mm)
PE - Polyethylene	5.0292

Outer Shield

Outer Shield Material:

Layer #	Type	Outer Shield Material	Coverage (%)
1	Braid	TC - Tinned Copper	95.000
2	Braid	TC - Tinned Copper	95.000

Here we learn that both conductors are basically copper, improved by tinning, with these dimensions:

a_1 = .7874/2 = 0.3937 mm inner radius
a_2 = 5.0292/2 = 2.5146 mm
dielectric = PE

We can compute K from $K = 2 \ln(a_2/a_1)$ which is (4.6.3) to get

$$K = 3.709$$

Next data group:

Electrical Characteristics (Overall)

Nom. Characteristic Impedance:

Impedance (Ohm)
75

Nom. Inductance:

Inductance (µH/m)
0.429811

Nom. Capacitance Conductor to Shield:

Capacitance (pF/m)
68.901

So

$C = 68.901 \times 10^{-12}$ farad/meter
$L = 0.429811 \times 10^{-6}$ henry/meter

Comment added later:

Recall that maybe fix comment about μ_0 for a metal!

$$L_e = \frac{\mu_d}{4\pi} K$$

If I put in the K value above along with μ_0 this gives

```
mu0 := evalf(4*Pi*1e-7);
```

$$\mu 0 := .1256637062\ 10^{-5}$$

```
Le := evalf((mu0/(4*Pi))* K);
```

$$Le := .3708559652\ 10^{-6}$$

which says

$L_e = .3709$ µH/m [ends up as .3714 in final App R due to a_2 adjustment]

and this differs from the nominal inductance they give of 0.429811 uH/m. That is a good thing because I expect to have $L > L_e$ due to the effect of L_i.

From these two numbers I can calculate v_d as $1/\sqrt{LC}$,

```
C := 68.901e-12;
```

$$C := .68901\ 10^{-10}$$

```
L :=  0.429811e-6;
```

$$L := .429811\ 10^{-6}$$

```
vd1 := 1/sqrt(L*C);
```

$$vd1 := .1837589389\ 10^{9}$$

```
c := 3e8;
```

$$c := .3\ 10^{9}$$

```
vd1/c;
```

$$.6125297963$$

This **disagrees** with the 66% shown below, I wonder why? Here is another calculation:

[The disagreement is because Belden gives v = 66%c at high frequency, and that involves L_e which is not the datasheet L quoted above which appears on their data sheet. Their L is the DC value of L. This matter is resolved.]

$v_d = 1/\text{sqrt}(\mu\varepsilon)$

If I assume that $\mu = \mu_0$, then

$v_d/c = \sqrt{\varepsilon_0/\varepsilon} = \sqrt{1/(2.3)} = .66$

```
sqrt(1/2.3);
```

$$.6593804734$$

and that is where their 66% comes from.

Again, why this discrepancy with $1/\sqrt{LC}$? Look at Appendix Q. There you see this for large ω

$$\text{Re}(k) = \omega\sqrt{LC} + \frac{(RC-LG)^2}{8\omega(LC)^{3/2}}$$

Losses cause a shift in the phase velocity. I think v_d is the lossless value, so I really do think

$v_d = 1/\sqrt{LC}$

Now what is L here? It is the L appearing in all the Z_0 type formulas, and that is the L taken from **z** . But all we really know is that

$\mathbf{z} = R + j\omega L = Z_{s1}(\omega) + Z_{s2}(\omega) + j\omega L_e$

So the total L is really a function of ω.

$$\mathrm{Im}(\mathbf{z}) = \omega L = \mathrm{Im}(Z_{s1} + Z_{s2}) + \omega L_e$$

$$L = (1/\omega)\,\mathrm{Im}(Z_{s1} + Z_{s2}) + L_e$$

I don't off hand have a formula for Z_{s2} for the sheath, but I do have an exact formula for Z_{s1} for the central conductor. At large ω, I would use the Chapter 2 result which says for the central wire,

$$Z_{s1}(\omega) \approx \frac{1}{\sigma(2\pi a)\delta}(1+j) \qquad \delta << 4a \ . \tag{2.4.16}$$

and this is for the central conductor.

So here I have a major problem! Here are the unanswered questions:

(1) How do they arrive at their official value of L? Is this measured at some particular ω?

(2) What is Z_{s2} for the sheath?

(3) Therefore, what is my model for L(ω) to be used in the k and Z_0 formulas?

For the moment, I will defer these questions.

Next data group:

Nominal Velocity of Propagation:

VP (%)
66

Nominal Delay:

Delay (ns/m)
5.05274

Nom. Conductor DC Resistance:

DCR @ 20°C (Ohm/km)
32.4819

Nominal Outer Shield DC Resistance:

DCR @ 20°C (Ohm/km)
3.6091

so then

$$R = (32.4819+3.6091) = 36.091 \text{ ohms/km} = 36.091 \times 10^{-3} \text{ ohms/m} = 3.6091 \times 10^{-2} \text{ ohm/m}$$

= .036091 ohm/m

The only other data is attenuation which I will attend to soon.

Nothing is said about ε, so I will use my own PE data: (from lines 3.3.5)

$\varepsilon'_{d} \approx 2.3$ $\qquad$ $\tan_{L} \approx 2 \times 10^{-4}$

This causes the σ_{eff} to be a function of frequency from my table

f(GHz)	sigma_eff(mho/m)
.0,	2.7e-14
.1,	2.6e-06
1.0,	2.6e-05
10.0,	2.6e-04
100.0,	2.6e-03

but I will just use the formula

$\sigma_{d,eff}(\omega) \approx = 4.07 \times 10^{-15}\ \omega$ $\qquad$ // PE $\qquad$ (3.3.6)

We can now compute G from (4.11.34)

$G/C = \sigma_{d,eff}/\varepsilon_{d}$

and here are some calcs:

```
tanL := 2e-4;
```

$$tanL := .0002$$

```
sigma := ed*tanL*omega;
```

$$\sigma := .4072926342\ 10^{-14}\ \omega$$

```
GoverC := sigma/ed;
```

$$GoverC := .0002000000000\ \omega$$

```
G := GoverC*C;
```

$$G := .1378020000\ 10^{-13}\ \omega$$

```
R*C;
```

$$.2486705991\ 10^{-11}$$

```
L*G;
```

$$.5922881542\ 10^{-20}\ \omega$$

```
sqrt(L/C);
```

$$78.98161329$$

and the last result is off the official 75 ohms by quite a bit. Of course I used "their" value of L even though I don't know what it means.

The L Problem.

Looking at Chapter 2, I conclude that the exact formula given there for the center wire Z_s is correct and applicable to Belden. If I could argue that L from the sheath is much smaller, maybe I can get a model going.

Let's look at the DC situation for inductances. For a round wire I show in App C that the internal inductance is given by simply

$$L_i = \frac{\mu_i}{8\pi} = \frac{\mu_i}{\mu_0}\frac{\mu_0}{8\pi} = \frac{\mu_i}{\mu_0}\frac{4\pi \times 10^{-7}}{8\pi} = \frac{\mu_i}{\mu_0} * 50 \text{ nH/m} \qquad \text{(C.3.10)}$$

Meanwhile, the DC inductance of a thin sheath of radius a_2 given by

$$L_i = \mu_i \frac{1}{6\pi}(d/a_2) = \frac{\mu_i}{8\pi}[(4/3)(d/a_2)] \qquad \text{thin shell, valid for } d << a_2 \qquad \text{(C.6.8)}$$

where d is the sheath thickness and a_2 the inner radius of the shield. What are the Belden numbers?

$a_2 = (1/2)$ 5.0292 mm

d = two layers of braid

They don't indicate the braid thickness but I can to look at a sample? Cannot find a sample, and hard to measure anyway. Another idea is this: They quote the shield resistance. If I model the shield as a thin copper shell (but it is tinned), then

$R = \rho/A = 1/(\sigma A) = 1/(\sigma\, 2\pi a_2 d)$.

Then solve to get

$\sigma\, 2\pi a_2 d = 1/R$ $\qquad$ $d = (1/R)(1/2\pi a_2\sigma) = 1/(2\pi a_2 \sigma R_{sh})$

We could then use $a_2 = (1/2)$ 5.0292 mm and σ for copper and R_{sh}= 3.6091 ohms and solve for d

$\sigma = 5.81 \times 10^7$ mho/m for copper (1.1.29)

Doing this I get that

d = .0003 m = 0.3 mm // recall a_1 = 0.3937 mm, a similar number

which seems ballpark OK. Then consider

$$L_i = \frac{\mu_i}{8\pi} \left[(4/3)(d/a_2) \right]$$

The factor [..] is unity for the central conductor, whereas for the shield we get

$$L_i = \frac{\mu_i}{8\pi} \left[(4/3)(d/a_2) \right] = 0.1600 \frac{\mu_i}{8\pi}$$

so at DC, the shield makes perhaps 1/8th the contribution to L_i that the center wire makes.

So yes, that is at DC. As you raise ω, skin effect occurs on both.

Note: Tin is a much worse conductor than copper and things the shield is tinned to stop oxidation.

http://chemistry.about.com/od/moleculescompounds/a/Table-Of-Electrical-Resistivity-And-Conductivity.htm

Material	ρ (Ω•m) at 20 °C Resistivity	σ (S/m) at 20 °C Conductivity
Silver	1.59×10^{-8}	6.30×10^{7}
Copper	1.68×10^{-8}	5.96×10^{7}
Annealed copper	1.72×10^{-8}	5.80×10^{7}
Gold	2.44×10^{-8}	4.10×10^{7}
Aluminum	2.82×10^{-8}	3.5×10^{7}
Calcium	3.36×10^{-8}	2.98×10^{7}
Tungsten	5.60×10^{-8}	1.79×10^{7}
Zinc	5.90×10^{-8}	1.69×10^{7}
Nickel	6.99×10^{-8}	1.43×10^{7}
Lithium	9.28×10^{-8}	1.08×10^{7}
Iron	1.0×10^{-7}	1.00×10^{7}
Platinum	1.06×10^{-7}	9.43×10^{6}
Tin	1.09×10^{-7}	9.17×10^{6}
Carbon steel	(10^{10})	1.43×10^{-7}
Lead	2.2×10^{-7}	4.55×10^{6}
Titanium	4.20×10^{-7}	2.38×10^{6}
Grain oriented electrical steel	4.60×10^{-7}	2.17×10^{6}
Manganin	4.82×10^{-7}	2.07×10^{6}
Constantan	4.9×10^{-7}	2.04×10^{6}
Stainless steel	6.9×10^{-7}	1.45×10^{6}

So gold is a WORSE conductor than copper, I did not realize that! Silver is the only better metal. Gold is put on surfaces for corrosion and wear resistance! Tin is much worse than copper at 9 x 10^{6} . A nice table to have.

So I have the center/shield comparison at DC. As ω increases, skin effect gets into both. At large ω, perhaps we can model both as shells of thickness δ. Then

L_i (center) = $\frac{\mu_i}{8\pi}$ [(4/3)(δ/a_1)] // these models are at high ω

L_i (shield) = $\frac{\mu_i}{8\pi}$ [(4/3)(δ/a_2)]

a_2 = (1/2)* 5.0292 mm = 2.5146 mm
a_1 = 0.3937 mm

Then at large ω my model says

L_i (shield) / L_i (center) $= (\delta/a_2) / (\delta/a_1) = a_1/a_2 = 0.3937/\ 2.5146 = 0.15656566$

which is almost the exact same number. So here is my model

$$L_i = 1.16\ \frac{\mu_i}{8\pi}$$

and I think this is valid at any ω. This includes the shield and the center wire.

In this little effort, I have assume that

$$L_i \text{ (center)} = \frac{\mu_i}{8\pi}\ [\ (4/3)(\delta/a_1)\ ]$$

What is the exact result here? I already quoted for the center wire,

$$Z_{s1}(\omega) \approx \frac{1}{\sigma(2\pi a_1)\delta}\ (1+j) \qquad \delta << 4a_1\ . \qquad \text{// center} \tag{2.4.16}$$

The contribution to inductance of this last formula is

$$L_i = (1/\omega)\ \mathrm{Im}(Z_{s1}) = (1/\omega)\ \frac{1}{\sigma(2\pi a_1)\delta}$$

But now use $\delta \equiv \sqrt{2/\omega\mu\sigma}$ or $\delta^2 = 2/\omega\mu\sigma$ or $(1/\omega) = \delta^2\ \mu\sigma/2$ so that

$$(1/\omega)\ \frac{1}{\sigma(2\pi a_1)\delta} = \frac{1}{\sigma(2\pi a_1)\delta}\ \delta^2\ \mu\sigma/2 = \frac{1}{(2\pi a_1)}\ \delta\ \mu/2$$

so then for the center wire

$$L_i = (1/2)\ \frac{1}{(2\pi a_1)}\ \delta\ \mu = (1/4\pi)\ \mu\ (\delta/a_1) = \frac{\mu_i}{8\pi}\ (2\delta/a_1)$$

which compare to my little formula above

$$L_i \text{ (center)} = \frac{\mu_i}{8\pi}\ [\ (4/3)(\delta/a_1)\ ]$$

and they are not too far apart.

So here is my model for L_i :

Model for L_i for coax cable:

$L_{i} = 1.16\ L_{i.center} = 1.16\ (1/\omega)\ \mathrm{Im}(Z_{s1})$

$$Z_{s}(\omega) = \frac{e^{j3\pi/4}}{\sqrt{2}\,\pi a_1 \sigma\delta}\ \frac{J_0(\beta a_1)}{J_1(\beta a_1)}$$

Again, at high ω you get $Z_{s1}(\omega) \approx \frac{1}{\sigma(2\pi a_1)\delta}\ (1+j)$ and then

$$L_{i}\ (\text{high } \omega) = 1.16\ (1/\omega)\ \frac{1}{\sigma(2\pi a_1)\delta} = 1.16\ \frac{1}{(2\pi a_1)}\ \delta\ \mu/2$$

so it all fades away to nothing at very high ω. We know it also at DC.

Summary:

(1) At DC we estimate that including both center and shield:

$$L_{i} = 1.16\ \frac{\mu_i}{8\pi}$$

(2) At high ω we then estimate that for both we have

$$L_{i} = 1.16\ \frac{1}{(2\pi a_1)}\ \delta\ \mu/2 = 1.16\ \frac{\mu_i}{8\pi}\ 2\ (\delta/a_1)$$

How then do I piece these together. Glue together at $2\delta/a_1 = 1$ or $\delta = a_1/2$ or $(\delta/a_1) = 2$. But I want this all in terms of ω, not σ.

glue point: $\delta = a_1/2$ $(\delta/a_1) = 1/2$

// $\delta \equiv \sqrt{2/\omega\mu\sigma}$ or $\delta^2 = 2/\omega\mu\sigma$ or $(1/\omega) = \delta^2\ \mu\sigma/2$

$\sqrt{2/\omega\mu\sigma} = a_1/2$

$2/(\omega\mu\sigma) = (a_1/2)^2$

$2 = (a_1/2)^2\ \omega\mu\sigma = (a_1^2/4)\ \omega\mu\sigma$

$1 = (a_1/2)^2\ \omega\mu\sigma = (a_1^2/8)\ \omega\mu\sigma$

$\omega_t = (8/a_1^2)(1/\mu\sigma)$

```
wt := (8/a1^2)*(1/(mu0*sigmashield));
                    wt := 706924.7074
ft := evalf(wt/(2*Pi));
                    ft := 112510.5615
```

So the transition point is then [I don't use any such transition point in the final version]

$\omega_t = 706{,}924 = 7 \times 10^5$

$f_t = 112.510$ KHz

At this frequency my chart says

$\delta = 209\mu = (a_1/2) =$ correct

So we get L_i = flat out to frequency ω_t, and after that we have

$$1.16 \frac{\mu_i}{8\pi} 2 (\delta/a_1) = 1.16 \frac{\mu_i}{8\pi} 2 (\sqrt{2/\omega\mu\sigma}\,/a_1)$$

$$= 1.16 \frac{\mu_i}{8\pi} 2 \quad (\sqrt{2/\mu\sigma}\,/a_1)\, 1/\sqrt{\omega}$$

Recall that

$\sqrt{2/\omega_t\mu\sigma} = a_1/2$
$\sqrt{2/\omega\mu\sigma} = \delta$

Then

$$(\delta/a_1) = \sqrt{2/\omega\mu\sigma} \;/\; [2\sqrt{2/\omega_t\mu\sigma}\;] = (1/2)\,(\omega_t/\omega)^{1/2}$$

$$2\,(\delta/a_1) = (\omega_t/\omega)^{1/2}$$

Check: transition point is $\omega = \omega_t$ so we get

$2\,(\delta/a_1) = 1$

$(\delta/a_1) = 1/2$

Correct. Then here is the model in the non-flat part of the curve

$$L_i = 1.16 \frac{\mu_i}{8\pi} 2\,(\delta/a_1) = 1.16 \frac{\mu_i}{8\pi} (\omega_t/\omega)^{1/2}$$

Finally, here is the model

$$L_i = 1.16\ \frac{\mu_i}{8\pi} \qquad \text{for } \omega < \omega_t$$

$$L_i = 1.16\ \frac{\mu_i}{8\pi}\ (\omega_t/\omega)^{1/2} \qquad \text{for } \omega > \omega_t$$

and finally after many hours of goofing around I have some kind of model for L_i for the coax cable.

$$L_i = 1.16\ \frac{\mu_i}{8\pi}\ [\ \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2}\ \theta(\omega>\omega_t)\ ]$$

[In my final Appendix R, I don't state L_i in this way because I don't care there about low L_i range]

And then our model for total inductance is

$$L(\omega) = L_e + 1.16\ \frac{\mu_i}{8\pi}\ [\ \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2}\ \theta(\omega>\omega_t)\ ] \qquad \omega_t = (8/a_1^2)(1/\mu\sigma)$$

where $(\delta_t/a_1) = 1/2$. $\qquad f_t = 112.510$ KHz

Let's put this into numbers:

```
Le := evalf((mu0/(4*Pi))* K);
```

$$Le := .3708559652\ 10^{-6}$$

So now we need to learn at what ω the Belden people measured their "L" value.
Perhaps it is 1 MHz ? How large is this extra term?

$$1.16\ \frac{\mu_i}{8\pi} \approx 58\ \text{nH/m} =$$

```
evalf( 1.16*(mu0/(8*Pi)));
```

$$.5800000001\ 10^{-7}$$

which is .058 μH/m. So my model is now giving, in μH ,

$$L(\omega) = .3709 + .058\ [\ \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2}\ \theta(\omega>\omega_t)\ ]$$

At the transition point I then have (my model predicts L_)

$$L(\omega) = .3709 + .058 = 0.4289\ \mu\text{H/m}$$

```
L_ := Le + 58e-9;
```

$$L_ := .4288559652\ 10^{-6}$$

and at any other point the result will be LOWER. Recall now that their **quoted value** was

L = 0.429811 x 10^{-6} μH/m

which is VERY close to my number at the transition point, but their number is larger than my transition point number so I cannot model their number no mater what. But we are very close:

Thus, my model has no way to fit their number since all my numbers at high ω will be lower

That suggests that they have measured their number at some higher ω! But what is the ratio at the transition point?

their quote / my model value = L / L_ = .4298/.4288 = 1.002

```
L/L_;
```

$$1.002226936$$

What does the direct C calculation say?

C = $4\pi\varepsilon_d/K$

```
C1 : evalf(4*Pi*ed/K);
```

$$.6900509458\ 10^{-10}$$

This assumes ε_{rel} = 2.30000 exactly. They give this number instead,

68.901 pF/m

which, although very close, is off a bit. Ratio is

1.001510785

so that is 1/10th of 1 %.

Could I adjust ε_d to make things work even better? I would need to lower my ε_d. Leave as is!

Question: What happens with the speed of light v_d business? I know that

(1) in the high ω limit, $L = L_e$ [correct, and this gives the 66%]

(2) in that same limit, I know that $v_d = 1/\sqrt{LC}$

(3) so what I really know is that

$v_d = 1/\sqrt{L_eC}$

```
vd2 := 1/sqrt(Le*C);
```

$vd2 := .1978265617$

```
vd2/c;
```

.6594218723

so this problem is rescued! [yup]

Question: What about the 75 ohm impedance? Again I can go *at high ω* where $L = L_e$

$Z_0 = \sqrt{L_e/C}$

```
sqrt(Le/C);
```

73.36516049

This seems farther from the nominal 75Ω than it should be.

What "L" should go into the full Z_0 formula? It is the full L, not L_e. So finally I get to combine the two models.

First, we know that

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

I have expressions for everything here! My model for L gives L_.

$$L(\omega) = L_e + 1.16 \frac{\mu_i}{8\pi} [\, \theta(\omega<\omega_t) + (\omega_t/\omega)^{1/2}\, \theta(\omega>\omega_t) \,]$$