This was installed as Appendix R on 7/28/134.
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My original Belden and Power notes are Version 1.
My original Appendix R is version 2.
This is version 3.

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**Appendix R: Belden 8281 Coaxial Cable, a Case Study ..... 1**
(a) Geometry of the cable ..... 1
(b) Capacitance C ..... 3
(c) Conductance G ..... 3
(d) External inductance $L_e$ ..... 4
(e) Total DC Inductance ..... 4
(f) High Frequency Inductance and Resistance ..... 6
(g) The Tinning Correction ..... 7
(h) Characteristic Impedance ..... 10
(i) Phase Velocity and Attenuation ..... 13

## Appendix R: Belden 8281 Coaxial Cable, a Case Study

Here we gradually work our way through Belden's data sheet for its "8281" coaxial cable, correlating the data presented there with the general theory of this document. The data sheet is available here

www.belden.com/techdatas/metric/8281.pdf

but we will be quoting most of it below. We note that over the last few decades, the parameters on this data sheet have changed slightly. At its market introduction (more than 20 years ago), this 75Ω coaxial cable was pretty much top of the line for general purpose RF use and is still available today. It is often used to carry uncompressed analog video signals. Today Belden offers even better coaxial cables for use with high bandwidth digital video signals. Such cables often have a foam dielectric to reduce attenuation, while the 8281 cable has a solid polyethylene dielectric.

Below we use various equations from our main document to construct a "model" of the Belden cable, but this model is limited to "high frequency" meaning here roughly $f > 1$ MHz. A more careful analysis would also produce a "low frequency" model for frequencies from DC to 1 MHz, and would then blend these two models at the boundary in some smooth manner.

### (a) Geometry of the cable

We start with this data from the Belden specification,

## Physical Characteristics (Overall)

### Conductor

**AWG:**

| # Coax | AWG | Stranding | Conductor Material | Dia. (mm) |
|---|---|---|---|---|
| 1 | 20 | Solid | BC - Bare Copper | 0.7874 |

**Total Number of Conductors:**

### Insulation

**Insulation Material:**

| Insulation Material | Dia. (mm) |
|---|---|
| PE - Polyethylene | 5.0292 |

### Outer Shield

**Outer Shield Material:**

| Layer # | Type | Outer Shield Material | Coverage (%) |
|---|---|---|---|
| 1 | Braid | TC - Tinned Copper | 95.000 |
| 2 | Braid | TC - Tinned Copper | 95.000 |



The center wire is solid copper with a specified diameter of 0.7874 mm, so $a_1$ = .7874/2 mm = 0.3937 mm = 393.7 μ. The diameter of the PE core is specified as 5.0202 mm so the radius is 5.0202/2 = 2.5101mm = 2510.1μ. This core is surrounded by a tinned double copper braid. One sometimes adds the radius (80 μ) of the fine braiding wire to the effective outer cable radius, but we shall add 11.5 μ to the radius since this makes C match the Belden data sheet value if $\varepsilon_d/\varepsilon_0$ = 2.3000. So $a_2$ = 2510.1+11.5 = 2521.6 μ. We then have,

| | |
|---|---|
| $a_1$ = 393.7 μ | inner wire radius |
| $a_2$ = 2521.6 μ | inside radius of the shield |

Obviously the 2.3000 number is not exact. We are just building a reasonable model here to try and replicate the Belden claimed cable parameters, and some parameters have to be tuned to get consistency. The double braid is not exactly the same as a solid cylindrical shell of copper, so things are approximate.

As we go along here, the corresponding Maple code will be displayed. So far then,

```
a1 :=  (1/2)*0.7874e-3;
```

$$a1 := .0003937000000$$

```
a2 := ((1/2)*5020.2 + 11.5) * 1e-6;
```

$$a2 := .002521600000$$

where all quantities are stated in the usual SI units (meters for $a_1$ and $a_2$). From these radii one computes K from (4.6.3),

$$K = 2 \ln(a_2/a_1) , \qquad (4.6.3)$$

to get K = 3.7141,

```
K := 2*ln(a2/a1);
```

$$K := 3.714119404$$

**(b) Capacitance C**

Assuming $\varepsilon_d/\varepsilon_0 = 2.3000$, one uses (4.4.17)

$C = 4\pi\varepsilon_d/K$ capacitance per unit length *(4.4.17)*

to get

```
e0 := 8.8541877e-12;
```

$$e0 := .88541877\ 10^{-11}$$

```
ed := 2.3*e0;
```

$$ed := .2036463171\ 10^{-10}$$

```
C := 4*pi*ed/K;
```

$$C := .6890179924\ 10^{-10}$$

The Belden data sheet quotes C = 68.901 pF/m.

**Nom. Capacitance Conductor to Shield:**

| Capacitance (pF/m) |
|---|
| 68.901 |

in agreement with our calculation. We regard C as a constant independent of ω.

**(c) Conductance G**

The conductance G of the dielectric is related to the capacitance according to

$G = (\sigma_d/\varepsilon_d)\ C$ *(4.11.34)*

where $\sigma_d$ is the dielectric conductivity. Recall now (3.3.4),

$\sigma_{eff} = (\ \sigma_d + \omega\varepsilon'_d\ \tan_L)$ *(3.3.4)*

which gives the effective conductivity of the dielectric in terms of the DC conductivity $\sigma_d$, the real part of $\varepsilon_d$ called $\varepsilon'_d$ and the loss tangent factor. For PE we know that $\sigma_d \sim 10^{-15}$ so we neglect that term. We shall be using tanL = .0005 below, and since this is small, $\varepsilon'_d \approx \varepsilon_d$. Finally, to reduce symbol clutter we rename $\sigma_{eff}$ to be $\sigma_d$ so the above equation becomes

$\sigma_d = \varepsilon_d\ \tan_L\ \omega$ => $(\sigma_d/\varepsilon_d) = \tan_L\ \omega$

and then

$$G = \tan_L \omega C = \tan_L 2\pi f C . \qquad (R.1)$$

Although Fig 3.1 mentions $\tan_L = .0002$ for a high quality sample of polyethylene, our experience has shown that for the bulk low-cost PE product used in coaxial cables, $\tan_L \approx .0005$. The larger loss is due to many effects including milling, aging (oxidation), water absorption ("treeing") and additives intended to reduce these loss effects. Very poor quality PE can have a loss tangent ($\tan_L = \tan\delta$ = dissipation factor) of .0075. For more accuracy, one can develop frequency dependent models for $\tan_L$ .

The corresponding Maple expressions are duly entered,

```
tanL := .0005;
```

$$tanL := .0005$$

```
G := f -> tanL*2*pi*f*C;
```

$$G := f \rightarrow 2\, tanL\, \pi\, f\, C$$

where the last notation indicates that G(f) is a function of frequency f.

In either the $Z_0$ or attenuation calculations below, the quantity G + jωC ( = **y**) always appears as a grouping, and we have determined that

$$G + j\omega C = \tan_L \omega C + j\omega C = (-j \tan_L + 1)\, j\omega C = (1 - 0\ .0005j\ )\, j\omega C \quad . \qquad (R.2)$$

Based on this grouping, one sees that our model is not very sensitive to the value of $\tan_L$ as long as it is a relatively small value.

**(d) External inductance $L_e$**

Using the numbers developed so far, one computes $L_e$ from $L_e = \frac{\mu_0}{4\pi} K$ in (4.11.34) :

```
mu0 := 4*pi*1e-7;
```

$$\mu 0 := .1256637062\ 10^{-5}$$

```
K := 2*ln(a2/a1);
```

$$K := 3.714119404$$

```
Le := mu0*K/(4*pi);
```

$$Le := .3714119405\ 10^{-6}$$

which is $L_e$ = 371.41 nH/m.

**(e) Total DC Inductance**

From (C.3.10) we know that the center wire DC internal inductance is given by

$$L_i(\text{center}) = \frac{\mu_0}{8\pi} = 50 \text{ nH/m} \qquad \text{DC} \qquad \textit{(C.3.10)}$$

If we assume the shield has a thickness t, we know from (C.6.8) that

$$L_i(\text{shield}) = \frac{\mu_0}{8\pi}\,[\,(4/3)(t/a_2)\,] \qquad \text{DC} \qquad \text{thin shell,} \quad \text{valid for } t << a_2 \qquad \textit{(C.6.8)}$$

We can compute an effective shield thickness t by making use of its DC resistance $R_{2DC}$:

$$R_{2DC} = \rho/A = 1/(\sigma A) = 1/(\sigma\, 2\pi a_2 t)$$

$$\Rightarrow\ t = 1/(\sigma\, 2\pi a_2 R_{2DC}) \qquad \text{(R.3)}$$

Then

$$L_i(\text{shield}) = \frac{\mu_0}{8\pi}\,[\,(4/3)\,\frac{1}{\sigma\, 2\pi a_2^2 R_{2DC}}\,]\,. \qquad \text{(R.4)}$$

The value of $R_{2DC}$ is specified as 3.6091 ohms/km,

**Nominal Outer Shield DC Resistance:**

| DCR @ 20°C (Ohm/km) |
|---|
| 3.6091 |

Here then are some calculations leading to a total DC inductance for the cable,

```
mu0 := 4*pi*1e-7;
```

$$\mu 0 := .1256637062\ 10^{-5}$$

```
Le := mu0*K/(4*pi);
```

$$Le := .3714119405\ 10^{-6}$$

```
sigma := 5.81e7;
```

$$\sigma := .581\ 10^{8}$$

```
R2DC := 3.6091e-3;
```

$$R2DC := .0036091$$

```
t := 1/(sigma*2*pi*a2*R2DC);
```

$$t := .0003010016581$$

```
Li_shield_DC := (mu0/(8*pi))*(4/3)* 1/(sigma*2*pi*a2^2*R2DC);
```

$$Li_shield_DC := .7957954160\ 10^{-8}$$

```
Li_center_DC := 50e-9;
```

$$Li_center_DC := .50\ 10^{-7}$$

```
Li_DC := Li_shield_DC + Li_center_DC;
```

$$Li_DC := .5795795416\ 10^{-7}$$

```
L_DC:= Li_DC + Le;
```

$$L_DC := .4293698947\ 10^{-6}$$

The center wire contributes 50.00 nH/m to the DC internal inductance, while the shield contributes another 7.96 nH/m giving a total of 57.96 nH/m for the total cable internal DC inductance. When this is added to the external inductance $L_e$ of 371.4 nH/m, the total is seen to be 429.37 nH/m. The Belden data sheet quotes 429.811 nH/m giving a small discrepancy of 1/10th of 1% compared to our calculation,

**Nom. Inductance:**

| Inductance (µH/m) |
|---|
| 0.429811 |

We see that Belden's "nominal inductance" is the total DC inductance of the cable $L_e + L_i$ .

**(f) High Frequency Inductance and Resistance**

At high ω where the skin effect dominates, one thinks of the current being restricted to a sheath of approximate thickness δ (skin depth). In Chapter 2 we showed that, for the center conductor of radius $a_1$, the high frequency resistance and internal inductance (per unit length) are given by,

$$R_1 = \frac{1}{\sigma(2\pi a_1)\delta} \qquad (2.4.18)$$

$$L_{1i} = (1/\omega)\, R_1 = \frac{1}{\omega\sigma(2\pi a_1)\delta} = \frac{1}{\omega\sigma(2\pi a_1)\sqrt{2/\omega\mu_0\sigma}} = \frac{1}{2\pi a_1}\sqrt{\frac{\mu_0}{2\sigma\omega}}\ . \qquad (2.4.19)$$

It was noted that $R_1$ has the simple interpretation of being the resistance of a shell of radius $a_1$ and thickness δ.

Since this same kind of thin skin-effect sheath also exists on the inner surface of the outer conductor, we shall assume that the corresponding parameters for the outer conductor are obtained by replacing $a_1$ by $a_2$ in the above, so

$$R_2 = \frac{1}{\sigma(2\pi a_1)\delta}$$

$$L_{2i} = \frac{1}{\omega\sigma(2\pi a_2)\delta} = \frac{1}{\omega\sigma(2\pi a_2)\sqrt{2/\omega\mu_0\sigma}} = \frac{1}{2\pi a_2}\sqrt{\frac{\mu_0}{2\sigma\omega}} \qquad (R.5)$$

We have assumed that the shield and center conductor are made of the same metal (copper) with σ and δ. For other cables, the shield might be aluminum foil, and one would then adjust the above equations.

Adding, we then arrive at these expressions for high frequency resistance and internal inductance:

$$R = \frac{1}{\sigma(2\pi)\delta}\left[\frac{1}{a_1} + \frac{1}{a_2}\right]$$

$$L_i = \frac{R}{\omega} = \frac{1}{\omega\sigma(2\pi)\delta}\left[\frac{1}{a_1} + \frac{1}{a_2}\right] = \frac{1}{f\sigma(2\pi)^2\delta}\left[\frac{1}{a_1} + \frac{1}{a_2}\right] \qquad (R.6)$$

At 1 MHz (2.3.9) says $\delta = 66\mu$ . Since the center conductor has $a_1$ = radius 394μ and the shield has thickness t = 301μ, we shall restrict our model to apply only to frequencies over 1 MHz (ballpark).

For the Belden 8281 cable, the first term in $\left[ \frac{1}{a_1} + \frac{1}{a_2} \right]$ is 6.4 times larger than the second term

```
(1/a1)/(1/a2);
```

```
6.404876810
```

so most of the R and $L_i$ at high frequency come from the inner conductor, not the shield.

**(g) The Tinning Correction**

A model complication is that the 160μ diameter copper braid wires (34 gauge) of the shield are coated with tin of thickness 1.3μ (50 micro-inches). This coating is added to prevent the copper shield from oxidizing. At 1 GHz (2.3.9) gives $\delta_{copper} = 2.09\mu$. Since tin has about 6.3 times more resistance than copper, and since $\delta = \sqrt{2/\omega\mu\sigma}$ , one finds that $\delta_{tin} = 5.24\mu$ at 1 GHz. As the frequency increases, one has to somehow gradually replace the copper δ with the tin δ in the second term of the R and $L_i$ expressions above. An analytic solution to this problem can be found by applying the Helmholtz equation $[\nabla^2+\beta^2]\mathbf{E} = 0$ to a simple one-dimensional model of the tin/copper interface. We have done this and then obtained the following "phenomenological" model to handle the tinning correction:

$$R = \frac{1}{\sigma(2\pi)\delta} \left[ \frac{1}{a_1} + \frac{1}{a_2} * tf \right]$$

$$L_i = \frac{1}{f\sigma(2\pi)^2\delta} \left[ \frac{1}{a_1} + \frac{1}{a_2} * tf \right] \qquad L_i = R/(2\pi f)$$

$$tf = 1.765 + 0.8 \tanh(\sqrt{f(GHz)} - 1.9) \qquad \text{" tinning factor"} \qquad \text{(R.7)}$$

Here is a plot of this tinning factor for f ranging from 10 KHz to 1 GHz,

```
semilogplot(tf(f),f=1e4..1e9,numpoints = 500);
```

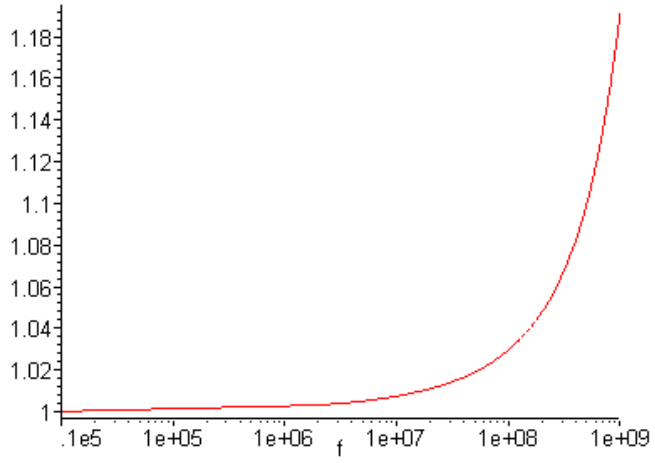


Fig R.1

Tinning Factor versus Frequency

It was shown above that $\frac{1}{a_1}$ is 6.4 times larger than $\frac{1}{a_2}$, so the tinning factor correction is fairly small at frequencies below 1 GHz (our region of interest). Even at 1 GHz we have

$$\frac{R_{corrected}}{R_{uncorrected}} = \frac{\frac{1}{a_1} + \frac{1}{a_2} * tf}{\frac{1}{a_1} + \frac{1}{a_2}} = 1.026$$

```
((1/a1)+(1/a2)* tf(1e9))/((1/a1)+(1/a2));
```

1.025923686

so the tinning factor increases R and $L_i$ by about 2.6% at 1 GHz, and less below 1 GHz. Although small, we shall include this tinning correction in our calculations below.

Here then are the Maple entries for high-frequency R and $L_i$, where $\delta = \sqrt{2/(\omega\mu\sigma)} = \sqrt{1/(\pi f\mu\sigma)}$ :

```
delta := f ->sqrt(1/(pi*f*mu0*sigma));
```

$$\delta := f \rightarrow \sqrt{\frac{1}{\pi f \mu 0 \sigma}}$$

```
R := f -> 1/(2*pi*sigma*delta(f)) * ( (1/a1)+(1/a2)*tf(f) );
```

$$R := f \to \frac{1}{2}\,\frac{\frac{1}{a1}+\frac{tf(f)}{a2}}{\pi\,\sigma\,\delta(f)}$$

```
Li := f -> R(f)/(2*pi*f);
```

$$Li := f \to \frac{1}{2}\,\frac{R(f)}{\pi\,f}$$

```
L := f -> Li(f) + Le;
```

$$L := f \to Li(f) + Le$$

Since $\delta \sim 1/\sqrt{f}$ and $L_i \sim 1/(\delta f) \sim 1/\sqrt{f}$, the internal inductance $L_i$ drops off rapidly at high frequencies and is in general much smaller than $L_e$. This is due to the fact shown in (C.6.8) that the internal inductance of an annular shell goes to zero as that shell (thickness δ) becomes thinner. Here is a plot of $L_i$ (red), $L_e$ = 371.4 nH (black), and $L = L_i + L_e$ (green) for f in the range 1 MHz to 1 GHz,

```
semilogplot([Li(f),Le,L(f)], f=1e6..1e9, color = [red,black,green]);
```

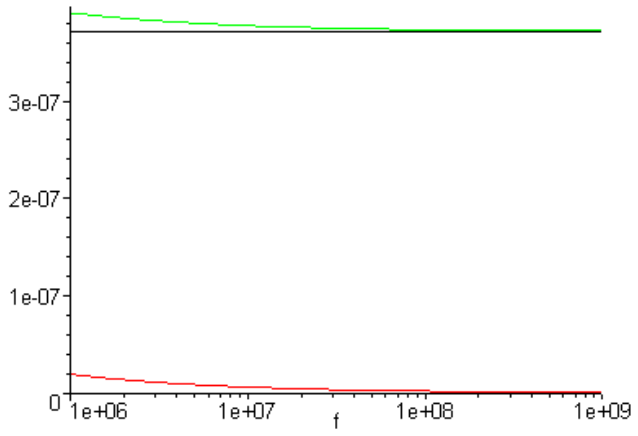


Fig R.2

Thus, for our range of interest, L is dominated by $L_e$.

The corresponding plot of resistance R is the following,

```
semilogplot(R(f), f=1e6..1e9,numpoints = 400);
```

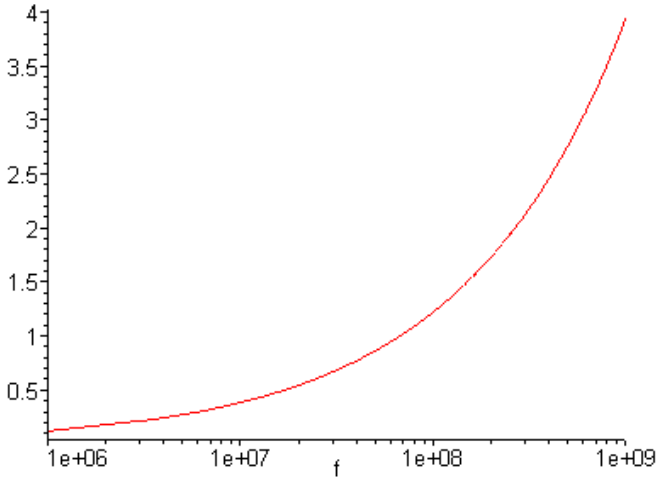


Fig R.3

Notice that this R is in ohms/m, whereas the DC resistances of the center wire and shield are stated in ohms/km,

**Nom. Conductor DC Resistance:**

| DCR @ 20°C (Ohm/km) |
|---|
| 32.4819 |

**Nominal Outer Shield DC Resistance:**

| DCR @ 20°C (Ohm/km) |
|---|
| 3.6091 |

Compared to these DC resistances, resistance R is quite large, and of course this is due to the skin effect.

**(h) Characteristic Impedance**

Although the cable has a nominal $Z_0$ of 75Ω,

**Nom. Characteristic Impedance:**

| Impedance (Ohm) |
|---|
| 75 |

the actual $Z_0$ is a slow function of frequency and can vary slightly (~1.5Ω) from the advertised nominal value. Recall from Chapter 4 that

$$Z_0 \equiv V(z)/i(z) = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (4.11.16)$$

which is in general complex. We enter this into Maple,

```
Z0 := f -> sqrt((R(f)+j*2*pi*f*L(f))/(G(f) + j*2*pi*f*C));
```

$$Z0 := f \to \sqrt{\frac{R(f) + 2\,j\,\pi\,f\,L(f)}{G(f) + 2\,j\,\pi\,f\,C}}$$

where the functions R(f), L(f) and G(f) have been stated above. We then plot Re{$Z_0$} for f ranging from 1MHz to 10 GHz,

```
semilogplot([Re(Z0(f)),75], f = 1e6..1e10, color=[red,black],numpoints = 1000);
```

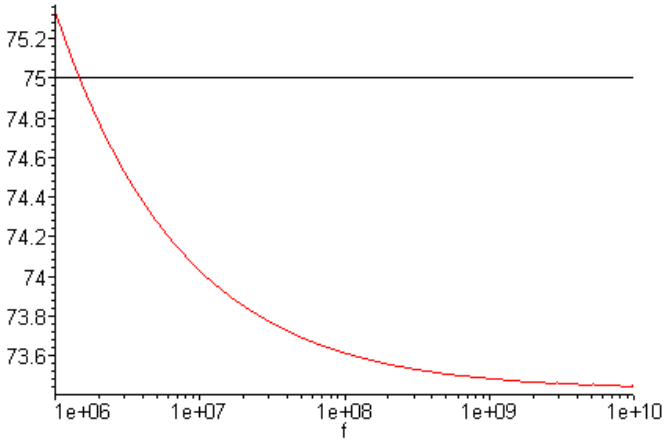


Fig R.4

Recall that our cable model using high frequency expressions for $L_i$ and R is only valid above 1 MHz more or less. The plot shows that the cable has $Z_0$ = 75Ω near f = 2 MHz, but drops to 73.48 Ω at 1 GHz, and is a little larger than 75Ω below 2 MHz.

The imaginary part of $Z_0$ over this same frequency range is on the order of - 1 Ω :

```
semilogplot(Im(Z0(f)), f = 1e6..1e10,numpoints = 1000);
```

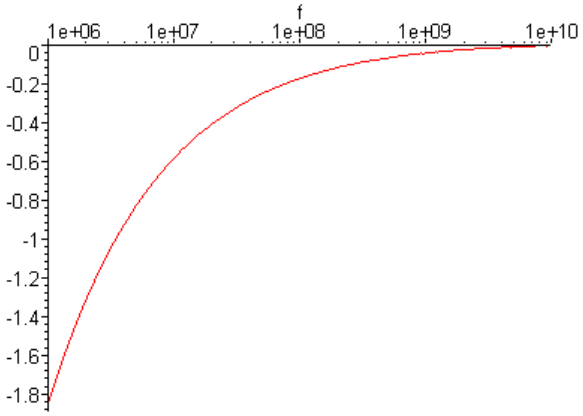


Fig R.5

A proper no-reflections termination of the cable thus requires both a resistance on the order of 75Ω and a small reactive component.

Since the imaginary part is so small, there is little distinction between Re($Z_0$) and |$Z_0$|. This is illustrated in the following plot,

```
magZ0 := f -> sqrt(Re(Z0(f))^2 + Im(Z0(f))^2);
```

$$magZ0 := f \rightarrow \sqrt{\Re(Z0(f))^2 + \Im(Z0(f))^2}$$

```
semilogplot([Re(Z0(f)),magZ0(f),75],f = 1e6..1e10,numpoints = 1000, color =
[red, green, black]);
```

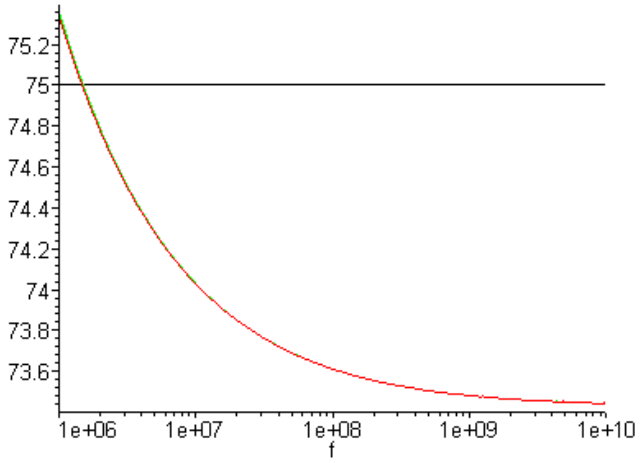


Fig R.6

where the red ( $\mathrm{Re}(Z_0)$ ) and green ( $|Z_0|$ ) curves lie right on top of each other.

At large ω, we expect $Z_0$ to approach a limiting value of 73.42Ω ,

$$Z_0 = \sqrt{L/C} \rightarrow \sqrt{L_e/C}$$

```
Z0_limit := sqrt(Le/C);
```

$$Z0_limit := 73.41970742$$

```
Z0_limit := (1/(4*pi))*K*sqrt(mu0/ed);    # see (4.4.14)
```

$$Z0_limit := 73.41970738$$

in agreement with Fig R.4 above.

At very low ω, we have instead that $Z_0 = \sqrt{R/G}$ . One can use R = .036 Ω/m by adding the DC resistances of the shield and center conductor. However, G is miniscule at DC since polyethylene is such a good insulator, so $Z_0$ is in the 3 MΩ range,

```
R2DC := 3.6091e-3;
```

$$R2DC := .0036091$$

```
R1DC := 32.4819e-3;
```

$$R1DC := .0324819$$

```
RDC := R2DC + R1DC;
```

$$RDC := .0360910$$

```
sigmad  := 1e-15;   #DC conductivity of PE
```

$$sigmad := .1\ 10^{-14}$$

```
GDC := (sigmad/ed) * C;  # see in box (4.11.34)
```

$$GDC := .3383405123\ 10^{-14}$$

```
Z0DC := sqrt(RDC/GDC);
```

$$Z0DC := .3266047198\ 10^{7}$$

Here we have assumed $\sigma_d \sim 10^{-15}$ mho/m, though this could be much larger for the kind of PE that is used in Belden cables, resulting in a somewhat smaller Z0DC.

Our model does not account for micro detail involving the mesh shield and manufacturing variations, and one finds with a network analyzer (and an actual piece of Belden 8281 cable) that there is "noise" superimposed on our idealized plot of $Z_0$ versus f which has an RMS value on the order of 1 ohm, see the work of Van Der Burgt. He argues that due to this "noise", it makes little sense to try to pin down a $Z_0$ tolerance beyond current values, although cable makers still try to do it as part of their marketing specmanship wars.

**(i) Phase Velocity and Attenuation**

Recall (5.3.6) which we apply to the voltage on a transmission line whose left end is at z = 0:

$$V(z) = V(0)\, e^{-jkz} = V(0)\, e^{-az}\, e^{-jbz} \qquad jk = a + jb = \sqrt{\mathbf{zy}} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$a \equiv \mathrm{Re}(\sqrt{\mathbf{zy}}) = \mathrm{Re}[\sqrt{(R+j\omega L)(G+j\omega C)}] = -\,\mathrm{Im}(k) \qquad \text{// attenuation per distance of F(z)}$$
$$b = \mathrm{Im}(\sqrt{\mathbf{zy}}) = \mathrm{Im}[\sqrt{(R+j\omega L)(G+j\omega C)}] = \mathrm{Re}(k)\,. \qquad \text{// phase of F(z)} \qquad (5.3.6)$$

Conventional symbols for the attenuation and phase constants are α and β, but here we call them a and b.

Phase Velocity

One can see that, for large ω,

$$b = \mathrm{Im}[\sqrt{(R+j\omega L)(G+j\omega C)}] = \mathrm{Im}[\sqrt{(j\omega L)(j\omega C)}] = \omega\, \mathrm{Im}[\sqrt{-LC}] = \omega \sqrt{LC}$$

and therefore the cable phase velocity is given by

$$v_{\mathbf{phase}} = \omega/\mathrm{Re}(k) = \omega/b = 1/\sqrt{LC}\ .$$

Since $L \approx L_e$ in our frequency range of interest, and since the speed of light in the dielectric is determined by $v_d = 1/\sqrt{L_e C} = 1/\sqrt{\mu_d \varepsilon_d}$ [see (4.11.28) ], we conclude that

$$v_{phase} = v_d = 1/\sqrt{L_e C} = 1/\sqrt{\mu_d \varepsilon_d}$$

and we can compute this two different ways, knowing that the result must be the same,

```
vphase_over_c := (1/sqrt(Le*C))/c;
```

*vphase* := .6589243140

```
vd_over_c := (1/sqrt(mu0*ed))/c;
```

*vd_over_c* := .6589243140

This is in agreement with the Belden claim,

**Nominal Velocity of Propagation:**

| VP (%) |
|---|
| 66 |

The time for a phase front to move 1 meter is given by $1/v_d$ ,

```
delay := 1/(c * vd_over_c);
```

*delay* := .5058749939 $10^{-8}$

which is 5.059 nsec. Belden gives

**Nominal Delay:**

| Delay (ns/m) |
|---|
| 5.05274 |

which is within 1/10th a 1% of our computed value.

**Reader Exercise:** Derive an expression for group velocity $v_g$ in the presence of attenuation (k is complex). How does your result compare with the classical expression $1/v_g = \partial k/\partial \omega$ or $v_g = \partial \omega/\partial k$ ? Using expressions of the model above, compute $v_g$ as a function of frequency. Since $v_g$ varies with f, the cable exhibits dispersion -- pulses spread out as they are attenuated. Determine the group delay for a narrow pulse to travel 1 m down the Belden cable. How does this delay compare with the phase delay noted above? What is the effect of the tinning correction on group delay?

Attenuation

It is traditional to express attenuation in "voltage decibels" defined in this manner,

$$dB(z) = -20 \log_{10}(\text{voltage attenuation over distance z}) \quad \text{" decibels"}$$

$$= -20 \log_{10}(e^{-az}) = 20\, az \log_{10}(e) = [\,20 \log_{10}(e)]\, az = 8.686\, az \qquad (R.8)$$

```
evalf(20*log10(exp(1)));
```

$$8.685889638$$

Belden provides attenuation data for z = 100 m of cable, so we just write

$$dB = 868.6\ a = 868.6\ [\ -\ \mathrm{Im}(k)\ ]\ >\ 0 \qquad \text{(R.9)}$$

Here then is a plot of attenuation for frequency f in the range 1 MHz to 1 GHz :

```
k := f -> -j*sqrt((R(f)+j*2*pi*f*L(f))*(G(f)+j*2*pi*f*C));
```

$$k := f \to -j\sqrt{(\mathrm{R}(f) + 2j\pi f \mathrm{L}(f))\,(\mathrm{G}(f) + 2j\pi f C)}$$

```
a := f -> -Im(k(f));
```

$$a := f \to -\Im(\mathrm{k}(f))$$

```
dB := f -> 868.5*a(f);
```

$$dB := f \to 868.5\ \mathrm{a}(f)$$

```
semilogplot(dB(f),f = 1e6..1e9,numpoints=200);
```

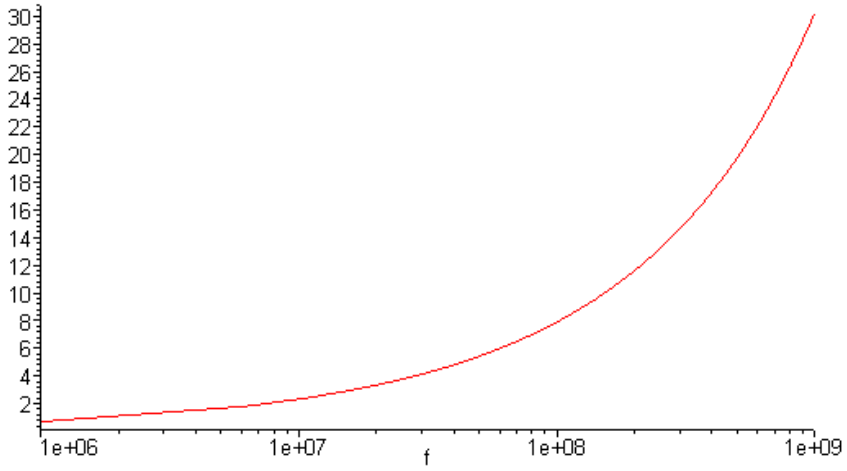


Fig R.7

In order to compare this theoretical attenuation prediction with Belden's provided data, we first evaluate our attenuation at the frequencies listed on the Belden data sheet,

```
for f in [1,3.6,10,71.5,135,270,360,540,720,750,1000,2000]*1e6 do
  printf("     f = %8.1f MHz  dB(f) = %4.2f  \n", f/1e6,dB(f));
od;
```

```
f =      1.0 MHz   dB(f) =   .71
f =      3.6 MHz   dB(f) = 1.37
f =     10.0 MHz   dB(f) = 2.33
f =     71.5 MHz   dB(f) = 6.59
f =    135.0 MHz   dB(f) = 9.33
f =    270.0 MHz   dB(f) = 13.78
f =    360.0 MHz   dB(f) = 16.28
f =    540.0 MHz   dB(f) = 20.70
f =    720.0 MHz   dB(f) = 24.65
f =    750.0 MHz   dB(f) = 25.28
f =   1000.0 MHz   dB(f) = 30.26
f =   2000.0 MHz   dB(f) = 47.77
```

Model Calculation of Attenuation

**Nom. Attenuation:**

| Freq. (MHz) | Attenuation (dB/100m) |
|---|---|
| 1 | 0.9843 |
| 3.6 | 1.6405 |
| 10.0 | 2.6248 |
| 71.5 | 6.8901 |
| 135 | 9.843 |
| 270 | 14.1083 |
| 360 | 16.7331 |
| 540 | 20.6703 |
| 720 | 24.2794 |
| 750 | 24.9356 |
| 1000 | 30.1852 |

Belden's Specified Attenuation

The following spreadsheet then compares Belden's data with our model prediction,

| MHz | Belden | Model | Model no Tin |
|---|---|---|---|
| 1 | 0.9843 | 0.71 | 0.71 |
| 3.6 | 1.6405 | 1.37 | 1.37 |
| 10 | 2.6248 | 2.33 | 2.33 |
| 71.5 | 6.8901 | 6.59 | 6.57 |
| 135 | 9.843 | 9.33 | 9.29 |
| 270 | 14.1083 | 13.78 | 13.69 |
| 360 | 16.7331 | 16.28 | 16.14 |
| 540 | 20.6703 | 20.7 | 20.46 |
| 720 | 24.2795 | 24.65 | 24.29 |
| 750 | 24.9365 | 25.28 | 24.9 |
| 1000 | 30.1852 | 30.26 | 29.67 |
| 2000 | | 47.77 | 46.02 |

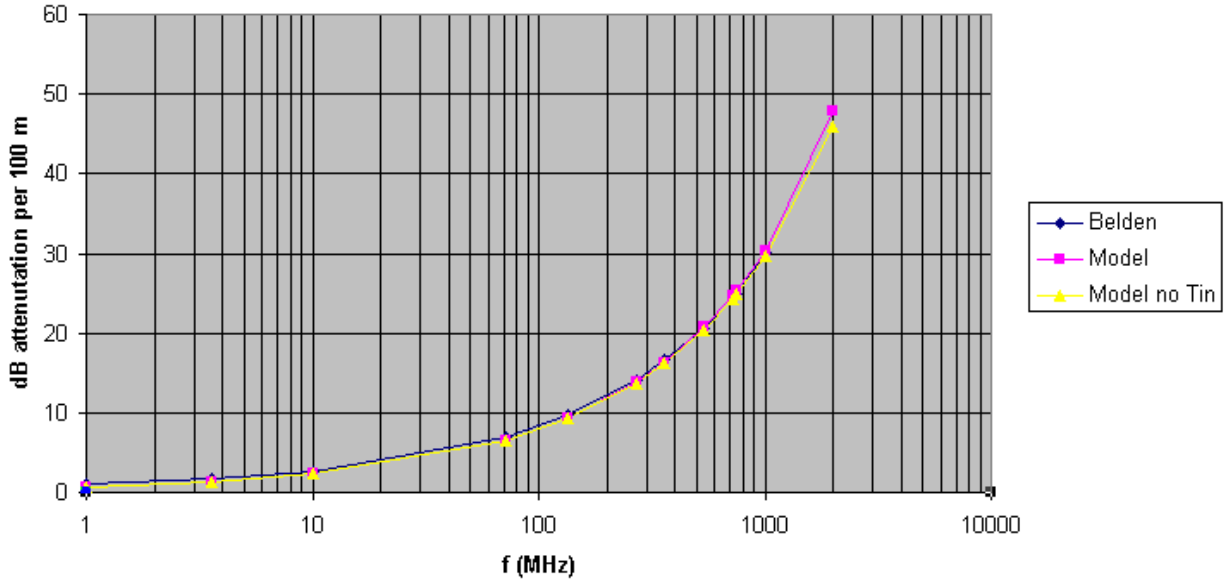


Fig R.8

Since the black diamond Belden points are pretty much covered over by the Model's purple squares, we conclude that the Model is pretty good, although there is a 30% error at 1 MHz which is at the limit of our model validity.

We have included in the spreadsheet a model column which ignores the tinning correction (yellow triangles). This column was obtained by setting tf(f) = 1 in the Maple code. We added a point at 2 GHz for which Belden gives no data, and at that point one sees that the tinning correction starts to become a little more visible. Tinning increases attenuation.

At one time we measured the attenuation of 100 m of Belden 8281 cable using a network analyzer and found that the above model (with tinning correction) accurately represents the cable up to 100 GHz.