Study of 1991 equations in Coax Doc PhL 7.14.14

This was a frustrating situation, I could not get my 2014 model's attenuation to match either the Belden data or my 1991 calculation! This forced me to do a close review of the 1991 coax doc equations, done here. I found no errors in those equations, but attenuation curve did not match! I thought maybe it was the tinning factor, but that turned out not to be the case. I verified that my 1991 theory did in fact give the 1991 attenuation plot. One slowdown here was that the final Excel spreadsheet used to make this plot was missing, so I could not check it.

There were two errors in the end. One involved messy Maple algebra with various L expressions. The second was that plot was versus ω instead of f, a traditional error! In any event, the attenuation data now matches perfectly both with 1991 and Belden.

I am trying to find out why my attenuation calcs come out too small. One debug method is to do an equation-by-equation comparison of my 2014 work with my 1991 work in coax doc. So I will do each 1991 equation of interest right here.

Derive 1991 Chapter 2 Section 2.2 equation (10).

Eq (10) claims

$\alpha(\omega) \approx \beta_1(r+g)/2$ where earlier I state that $\beta_1 = \omega\sqrt{LC}$, see page 3.

where L is the total capacitance, not just the external.

How then does the above compare to my Appendix Q.2 claim that

$$- \text{Im}(k) \approx \frac{(RC + GL)}{2\sqrt{LC}} .$$

Well, use these definitions of the 1991 world,

$g \equiv G/\omega C$
$r \equiv R/\omega L$.

Then the 1991 claim is that

$\alpha(\omega) \approx \beta_1(r+g)/2 = \beta_1(R/\omega L + G/\omega C)/2$

$= (\beta_1/\omega)\,(R/L + G/C)/2 = (\beta_1/\omega)\,(RC+GL)/[2LC]$

$= \sqrt{LC}\,(RC+GL)/[2LC]$

$= (RC+GL)/[2\sqrt{LC}\,]$

Good, the α(ω) shown in 1991 Chapter 2.2 equation (10) agrees with a = -Im(k) as it appears in 2014 Appendix Q.2.

Derive 1991 Chapter 2 Section 2.3 equation (2).

The 1991 equation claims this,

$$R/(\omega L_e) = r_e = (1/2)\ 1\ /\ [\ln(b/a)]\ *\ 1/\sqrt{\pi f \mu_0 \sigma}\ *\ [1/a + 1/b]$$

Here are facts from my 2014 analysis:

$$L_e = \frac{\mu_0}{4\pi}\ K$$

$$R = 1/(2\pi\sigma\delta) * (1/a_1 + 1/a_2)$$

So in 2014 I would claim that $[\delta = \sqrt{2/\omega\mu_0\sigma}\ ]$

$$r_e \equiv R/(\omega L_e) = (1/\omega) * 1/(2\pi\sigma\delta) * (1/a_1 + 1/a_2)\ [\ \frac{\mu_0}{4\pi}\ K\ ]^{-1}$$

$$= \sqrt{\omega\mu_0\sigma/2} * (1/\omega) * 1/(2\pi\sigma) * (1/a_1 + 1/a_2)\ 4\pi/(\mu_0 K)$$

$$= \sqrt{\mu_0\sigma/2\omega} * 1/(2\pi\sigma) * (1/a_1 + 1/a_2)\ 4\pi/(\mu_0[2\ \ln(a_2/a_1)])$$

$$= \sqrt{\mu_0\sigma/2\omega} * 1/(\sigma) * (1/a_1 + 1/a_2)\ 1/(\mu_0[\ \ln(a_2/a_1)])$$

$$= \sqrt{\mu_0/2\omega\sigma}\ \ (1/a_1 + 1/a_2)\ 1/(\mu_0[\ \ln(a_2/a_1)])$$

$$= \sqrt{1/(2\omega\sigma\mu_0)}\ \ (1/a_1 + 1/a_2)\ 1/(\ \ln(a_2/a_1)])$$

$$= \sqrt{1/(4\pi f\sigma\mu_0)}\ \ (1/a_1 + 1/a_2)\ 1/(\ \ln(a_2/a_1)])$$

$$= (1/2)\sqrt{1/(\pi f\sigma\mu_0)}\ \ (1/a_1 + 1/a_2)\ 1/(\ \ln(a_2/a_1)])$$

and this DOES agree with the Coax doc equation (2). [yes]

Derive 1991 Chapter 2 Section 2.2 equation (18).

We have to start with earlier equations. First, I agree with definition (11) for r_e which involves L_e. Note that L is *total* inductance. Now I make a claim that is not obvious to me right now:

$$\omega L = \omega L_e + R\ .$$

But now that I have rewritten Belden v3.doc some more, it has become obvious. If follows from

$$\omega L_i = R \qquad \text{at high } \omega$$

[here R is the high-ω real part of **z** for the cable, it is not the DC resistance]

and thus

$$\omega L = \omega(L_e + L_i) = \omega L_e + \omega L_i = \omega L_e + R$$

I now agree with (12) and all is well.

How about (13)? Follows from $G = \tan_L \omega C$ in belden v3, so fine.

(14) is OK and I distinguish the two cases. I agree also with (15).

Then (16) also looks fine so we get that β_1 replacement as shown bottom of page 4. At this point we have

$$\beta_1 = \beta_0 \sqrt{1+r_e} \quad .$$

But we already showed that

$$\alpha(\omega) \approx \beta_1 (r+g)/2$$

so we then get

$$\alpha(\omega) = \beta_0 \sqrt{1+r_e} \; * \; [\, \frac{r_e}{1+r_e} + \tan_L \,] /2$$

But $\beta_0 = (\omega/v_d) = (\omega/v_d)$ and $c/v_d = \sqrt{\varepsilon_{rel}}$ so then

$$\beta_0 = (\omega/v_d) = \omega\,(1/c)\,(c/v_d) = \omega\,(1/c)\sqrt{\varepsilon_{rel}}$$

and then we get

$$\alpha(\omega) = \omega\,(1/c)\sqrt{\varepsilon_{rel}}\;\sqrt{1+r_e} \; * \; [\, \frac{r_e}{1+r_e} + \tan_L \,] /2$$

$$= \pi f\,(1/c)\sqrt{\varepsilon_{rel}}\;\sqrt{1+r_e} \; * \; [\, \frac{r_e}{1+r_e} + \tan_L \,]$$

and this is equation (18) ! THAT didn't take long!

Derive 1991 Chapter 2 Section 2.5 equation (3).

This is just 869 α, where α comes from (18) which I derived above. So this equation is OK. I think this is the equation which should generate the Model column in Fig 3.1.2 which is a lost spreadsheet. So I will recreate this equation at the end of my Belden v3.mws right now. By the way, I agree with both Section 2.5 (3) and (4) and it is (3) that I just entered into Maple.

Where do the Fig 3.1.1 Belden attenuation numbers come from?

In my ancient binder having a coax tab section, I have some 1991 Belden data sheets. The data there is in column "dB/100m" and all numbers agree with the little coax doc spreadsheet (except 29.9 reads 30.0 in the Belden data). So there is no question: this data is atten per 100 m and that is where I got the data. So I don't think my error has to do with dB/1000 feet, say, thought that does produce a factor of 3.

1991 Spreadsheet Reproduced

This little spreadsheet appears as Fig 3.1.2 in coax doc. I hunted for the original 1991 XLS everywhere I could think of and could not find it, but luckily I was able to replicated the Model numbers with a mws file which is called coax 1991 spreadsheet.mws. I first got bad results but then understood the comment following this code and results came in right.

```
restart; alias(I=I,j=sqrt(-1));
```

$$j$$

```
mu0 := 4*pi*1e-7:       # from lines doc
pi := evalf(Pi):
a :=  (1/2)*0.7874e-3:  # from 1991 Ch 3 p 1
b := 2595e-6:           # from 1991 Ch 3 p 1
rho := 1.724e-8:       # from 1991 Ch 3 p 1
erel := 2.3:
c := 3e8:
tf := (f) -> 1.765+0.8*tanh(sqrt(f*1e-9)-1.9);  # from 2.6 summary box
```

$$tf := f \to 1.765 + .8 \tanh(\sqrt{.1\ 10^{-8} f} - 1.9)$$

```
re := (f) -> (1/2)*(1/ln(b/a))*sqrt(rho/(pi*f*mu0))*((1/a)+(1/b)*tf(f)); #2.6 summary box
```

$$re := f \to \frac{1}{2} \frac{\sqrt{\frac{\rho}{\pi f \mu 0}} \left(\frac{1}{a} + \frac{tf(f)}{b}\right)}{\ln\left(\frac{b}{a}\right)}$$

```
atten := (f) -> 9106*(f*1e-9)*sqrt(erel)*sqrt(1+re(f))*((re(f)/(1+re(f))) + .0005);
```

$$atten := f \to .9106\ 10^{-5} f \sqrt{erel} \sqrt{1 + re(f)} \left(\frac{re(f)}{1 + re(f)} + .0005\right)$$

```
f := 1e9; atten(f); unassign('f');
```

$$f := .1\ 10^{10}$$

$$29.84207413$$

```
for f in [1,10,50,100,200,400,700,900,1000,2000]*1e6 do
  printf("f = %10.0f MHz  atten = %4.2f\n", f/1e6,atten(f))
;od;
```

```
=          1 MHz  atten =  .70
=         10 MHz  atten = 2.29
=         50 MHz  atten = 5.35
=        100 MHz  atten = 7.78
=        200 MHz  atten = 11.44
=        400 MHz  atten = 17.06
=        700 MHz  atten = 23.88
=        900 MHz  atten = 27.92
=       1000 MHz  atten = 29.84
=       2000 MHz  atten = 47.14
```

These numbers match the Model column of the Fig 3.1.2 spreadsheet.

Confusion in 1991 summary box 2.6

First of all, the tinning factor t(f) really is tuned specifically to copper and tin as the two metals. I refer to ρ_b as the resistivity of the sheath. This is ambiguous since the sheath has both tin and copper. I now think that the only correct way to interpret the r_e expression in the summary box is to assume $\rho_a = \rho_b$ so that the square root factor $\sqrt{\rho_b/\rho_a} = 1$. I was probably trying to generalize my result at the last minute to some other metals (like Al) for the wires, and in doing so, I added an error. If in the above code I set $\sqrt{\rho_b/\rho_a} \neq 1$, I get a result of atten = 35 which is wrong. So the above hand calculation clarifies that $\sqrt{\rho_b/\rho_a} = 1$ is required.

Junk from the main edit log before I found my error

The error was that I was plotting ω = 0 to 1e9 instead of f = 0 to 1e9. I will just save this junk here.

OK, here is my code to attempt to reproduce the 1991 Model data:

Here I attempt to reconstruct the Excel spreadsheet of Fig 3.1.2 (1991) at least for the Model column

```
> erel := ed/e0;
```

$$erel := 2.300000000$$

```
> atten := (869*pi*f*sqrt(erel)*sqrt(1+re)/c)*((re/(1+re))+tanL):
> c := 3e8;
```

$$c := .3\ 10^{9}$$

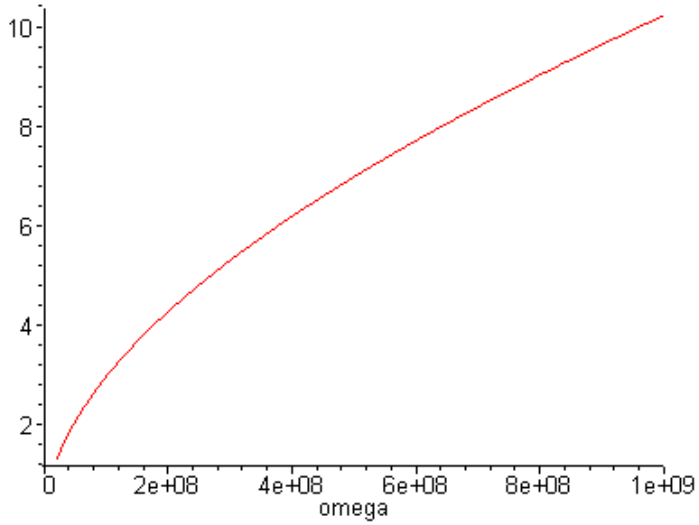
```
> re := R/(omega*Le);
```

$$re := .04456215020\ \frac{3239.957491 + 317.2588833\ \tanh(.00001261566261\ \sqrt{\omega} - 1.9)}{\omega\ \sqrt{\frac{1}{\omega}}}$$

```
> f := omega/(2*pi);
```

$$f := .1591549431\ \omega$$

```
> plot(atten,omega = 0..1e9);
```



Good news and bad news. The good news is that the general shape is right, and in fact that it seems identical to my 2014 plot. The bad news: So where is that 1991 factor of 10? Result not expected! I think the problem is with the quantity r_e since basically that is all that appears in 1991 Sec 2.5eq (3).

[Note that above plot is versus ω whereas the 1991 plot is versus f !]

Examining r_e. In my little Maple deal above I am using $r_e = R/(\omega L_e)$, same as 1991. My 2014 R is coming from this:

```
tin := 1.765 + 0.8*tanh(sqrt(fGHz)-1.9);
```

$$tin := 1.765 + .8\ \tanh(\sqrt{fGHz} - 1.9)$$

```
Li_ := (1/(omega*sigma*2*pi*delta))* ( (1/a1)+(1/a2)*tin ):
R := omega*Li_:
```

I have spotted something! Look at the tinning correction in the Section 2.6 summary page. I have omitted the ratio of conductivities! I am not clear about the meaning of ρ_b! Is this for tin or for copper? I think I intended it to be tin and that would increase R for the sheath. I will add a few lines and see if this fixes things up. // Done, this change raised both from 10 to 12, but still far from 30!

Verify my 2014 version of the tinning correction.

The 1991 version says (summary or Appendix 2.1 equation (20))

$$t(f) = 1.765 + 0.8 \tanh[\sqrt{f(GHz)} - 1.9]$$

So I will process this equation right here.

$$f(GHz) = 10^{-9} f$$

$$\sqrt{f(GHz)} = \sqrt{10^{-9} f} = \sqrt{10^{-10}\, 10f} = 10^{-5}\sqrt{10f} = 10^{-5}\sqrt{10\omega/2\pi}$$

$$= 10^{-5}\sqrt{10/2\pi}\ \sqrt{\omega} = 10^{-5}\sqrt{5/\pi}\ \sqrt{\omega} = 10^{-5}\ 1.262\ \sqrt{\omega} = 1.262\ \text{x}\ 10^{-5}\ \sqrt{\omega}$$

So we then have

$$f(f) = 1.765 + 0.8 \tanh[\sqrt{f(GHz)} - 1.9]$$

$$= 1.765 + 0.8 \tanh[\ 1.262\ \text{x}\ 10^{-5}\ \sqrt{\omega} - 1.9\ ]$$

$$= 1.765 + 0.8 \tanh[\ .00001262\ \sqrt{\omega} - 1.9\ ]$$

This agrees with what I have already in Maple

```
tin := 1.765 + 0.8*tanh(sqrt(5/pi)*(1e-5)*sqrt(omega)-1.9);
```

$$tin := 1.765 + .8\tanh(.00001261566261\sqrt{\omega} - 1.9)$$

I do have this thing correctly installed, but still getting 12 both ways instead of 30!

More comparison. I agree with the 1991 expression for r_e in the 2.6 summary box, I derivecd this in checking 1991 and I checked it just now. So let us not examine the 2014 Maple version of this object. I basically have this:

$$R = \omega L_{i_}$$

$$L_{i_} = 1/[2\pi\omega\sigma\delta] * (1/a_1 + 1/a_2 * tin * \sqrt{\sigma/\sigma_{tin}}\ )$$

Therefore my Maple code is generating R this way:

$$R = 1/[2\pi\sigma\delta] * (1/a1 + 1/a2 * tin * \sqrt{\sigma/\sigma_{tin}})$$

and I use this same R for both 1991 and 2014 plots. From this R, we may compute

$$r_e \equiv R/(\omega L_e) = 1/[2\pi\sigma\delta] * (1/a1 + 1/a2 * tin * \sqrt{\sigma/\sigma_{tin}}) * (\omega L_e)^{-1}.$$

and Maple has

```
K := 2*ln(a2/a1);                    Le := mu0*K/(4*pi);
```

where this is the 2014 version of K. We then have

$L_e = \mu_0 * 2 \ln(a_2/a_1) / [4\pi] = (1/2\pi)\, \mu_0 \ln(a_2/a_1)$ = agrees with 1991 Section 1.6 box page 8

So I then have (ignore tin stuff for now)

$$r_e \equiv R/(\omega L_e) = (1/\omega) * R/L_e = (1/\omega) * 1/(2\pi\sigma\delta) * (1/a_1 + 1/a_2) \left[\frac{\mu_0}{4\pi} K_{2014} \right]^{-1}$$

I do this algebra and checked it in checking 1991 and it comes out

$$r_e \equiv (1/2) \sqrt{1/(\pi f \sigma \mu_0)}\ (1/a_1 + 1/a_2)\ 1/(\ln(a_2/a_1))]$$

which agrees with the 2.6 summary box. So here is what I do:

(1) I compute R at high ω for both conductors in a way that agrees with the 1991 doc
(2) from that I get r_e also same way.
(3) then both plots are low by a factor of 3. [this was just the ω versus f issue]