Old retarded potential section 1.4 PhL 9.19.13

This is just storage of the original section 1.4, can be thrown eventually.

1.4 The Retarded Potentials: a path not taken

In simple situations it is possible to solve the above wave equations for φ and **A** directly in the time-domain. For example, if σ = 0 (so $\mathbf{J}_a = \mathbf{J}_T$) and μ and ε are not time dependent one can define a Green's Function G by

$$(\nabla^2 - \mu\varepsilon \partial_t^2) G(\mathbf{x},t;\mathbf{x}',t') = - 4\pi \delta^{(3)}(\mathbf{x}-\mathbf{x}') \delta(t-t') \qquad (1.4.1)$$

[In light of our previous note, this is $\partial^2 G(x^\mu; x'^\mu) = - 4\pi \delta^{(4)}(x^\mu - x'^\mu)$.] The solution of (1.4.1) is

$$G(\mathbf{x},t;\mathbf{x}',t') = (1/R) \delta(t' + (R/v) - t) \qquad (1.4.2)$$

where R = |**x**-**x**'|, and v = $c/\sqrt{\mu\varepsilon}$ = the phase velocity in the medium . This lets one write down the solution to either potential equation as follows:

$$\varphi(\mathbf{x},t) = \frac{1}{4\pi\varepsilon} \int \frac{\rho_T(\mathbf{x}',t-R/v)}{R} dV' \qquad (1.4.3)$$

$$\mathbf{A}(\mathbf{x},t) = \frac{\mu}{4\pi} \int \frac{\mathbf{J}_T(\mathbf{x}',t-R/v)}{R} dV' \qquad (1.4.4)$$

These expressions for the potentials are typically used in radiation problems to find the fields of accelerating particles or antennas. Notice that the time in the sources is "retarded" by R/v.

With the medium σ loss term present, and with frequency dependent (hence time dependent) μ and ε, it is better to work in the frequency domain instead of the time domain. Thus, although they are interesting, we shall not use these retarded potentials.