The Way Back Assumption PhL 6.29.14

This explanation of "the large σ_d" assumption is now installed. It is referred to just after it used in (1.3.20) and then at the end of that section I give a long explanation showing that it implies a lower limit for ω for which the **A** damped wave equation is valid. File this under Chapter 1.

WAY BACK just above (1.3.20), while deriving the wave equation for **A** in Region 2, I *snuck in* the following assumption which I have long kept well swept under the rug:

$\mathbf{E} << \partial_t \mathbf{A}$ in magnitude

where **E** and **A** are measured inside a conductor of a transmission line.

When you say the words "perfect conductor", you think $|\mathbf{E}| \approx 0$ and so you expect this to be true. But to test it, I have to know something about the vector potential inside the wire.

Plan A **no go**

Maybe Appendix G has something to say about this. For a round wire I compute that

$A_z(r) = -[I\mu_1/2\pi] \ln(r)$ $r > a$ region 1
$A_z(r) = -[I\mu_2/(4\pi a^2)](r^2-a^2) - [I\mu_1/2\pi] \ln(a)$ $r < a$ region 2 . (G.2.7)

where Region 2 is inside the wire. But this is all a DC calculation! And there is that DC offset that is arbitrary so how do you deal with that anyway?

Plan B **a hint**

Recall that

$$E_z(\mathbf{x}_1) - E_z(\mathbf{x}_2) = -\partial_z V(\mathbf{x}_1,\mathbf{x}_2) - j\omega W(\mathbf{x}_1,\mathbf{x}_2) \qquad (4.11.6a)'$$

I argue that the left side, although it varies, is very small and is the difference of the two large terns shown on the right. That at least HINTS that maybe $E_z << j\omega A_z$ somehow.

Plan C **best**

If I claim to have a reasonable solution of the transmission line system, I must have some estimate of the vector potential inside the wire in the time varying world. In Chapter 5 I claim that

$[\nabla_t^2 + (\beta_d^2-k^2)]\,\varphi_t(x,y) = 0$ $\varphi_t(C_1) = K_1$ $\varphi_t(C_2) = K_2$ $K_1 - K_2 = K$ (5.3.10)
$[\nabla_t^2 + (\beta_d^2-k^2)]\,A_{zt}(x,y) = 0$ $A_{zt}(C_1) = W_1$ $A_{zt}(C_2) = W_2$ $W_1 - W_2 = K$ (5.3.11)

where $(\beta_d^2 - k^2) = \dfrac{j\omega Z_s 4\pi\xi_d}{K}$.

so at least I am saying something about A_{zt} on the outer surface of the conductors. Maybe it is like φ and is continuous through the surface (possible slope jump). Then we sort of have

$$A_{zt}(x,y) \sim K$$

and then

$$A_z(x,y,z) = \frac{\mu_d}{4\pi} i(z) A_{zt}(x,y)$$

so we then have

$$A_z(x,y,z) \sim \frac{\mu_d}{4\pi} i(z) K$$

and lo!, we have some kind of crude estimate for the size of A_z on the conductors, and perhaps inside the conductors as well. So in this case I need to show that

$$E_z << \partial_t A_z$$

$$E_z << j\omega A_z$$

$$E_z << \omega \frac{\mu_d}{4\pi} i(z) K \quad ?$$

Then fiddle some more

$$J_z << \omega \frac{\mu_d}{4\pi} I \sigma K \quad ?$$

But roughly

$$I = \pi a^2 J_z \qquad J_z = I/(\pi a^2)$$

so I then want to show that

$$I/(\pi a^2) << \omega \frac{\mu_d}{4\pi} I \sigma K \quad ?$$

$$1/(\pi a^2) << \omega \frac{\mu_d}{4\pi} \sigma K \quad ?$$

$$\omega >> 1/(\pi a^2) / \left[\frac{\mu_d}{4\pi} \sigma K\right] \quad ?$$

$$\omega >> \frac{1}{\pi a^2}\,\frac{4\pi}{\sigma\mu K} \quad ?$$

$$\omega >> \frac{1}{a^2}\,\frac{4}{\sigma\mu K} \quad ?$$

Dimensions check:

RHS = m^{-2} m/henry * ohm-m = 1/henry * ohm = ohm / [ohm-sec] = sec^{-1} correct

Recall that $\mu\sigma = 73$ roughly, so

$$\omega >> \frac{1}{a^2}\,\frac{4}{73K} \approx \frac{1}{a^2}\,\frac{4}{73K} = .055\ (1/a^2)\ (1/K)$$

Suppose a = 1 cm = 10^{-2} m. Suppose K = 17.5. Then we need

$\omega >> 10^4 * .055 / 17.5 = 100 * 5.5 / 17.5 = 31.4$ radians/sec

$f = \omega/2\pi = 5$ Hz

That is something I can live with. Now do the Belden:

$.055\ (1/a^2)\ (1/K)$

Let's just guess that maybe K = 5. We have $a = 394\mu \approx 400e\text{-}6 = 4e\text{-}4$. Then

$.055\ (1/a^2)\ (1/K) = .055$

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a := 4e-4;
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$$a := .0004$$

```
K := 5;
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$$K := 5$$

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evalf(.055*(1/a^2)*(1/K));
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$$68750.00000$$

```
evalf(.055*(1/a^2)*(1/K)/(2*Pi));
```

$$10941.90233$$

So this says roughly that we need $\omega > 10$ KHz for the Belden cable, again that is OK.