New Sections 1.6 (e) and (f) PhL 10.3.14

(e) The Line Strength

In the discussion above we have already defined many kinds of electric fields:

$E_i(\mathbf{x},t)$	general complex electric field (component i)	(1.6.17)
$e_i(\mathbf{x},t)$	$\mathrm{Re}[E_i(\mathbf{x},t)]$ = candidate physical field	
$e'_i(\mathbf{x},t)$	$\mathrm{Im}[E_i(\mathbf{x},t)]$ = candidate physical field	
$\hat{E}_i(\mathbf{x},\omega)$	Fourier Integral Transform of $E_i(\mathbf{x},t)$	
$E_i(\mathbf{x},\omega)$	magnitude of a monochrome field with frequency ω	
$\varphi_i(\mathbf{x},\omega)$	phase of a monochrome field with frequency ω	
$\boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega)$	line strength of a monochromatic field with frequency ω	

The last item is new. It has been added to make the above list complete, and we define it right here. Recall the form of the Fourier Transform of a monochrome field given in (1.6.1),

$$\hat{E}_i(\mathbf{x},\omega) = [E_i(\mathbf{x},\omega_1)\, e^{j\varphi_i(\mathbf{x},\omega_1)}]\, 2\pi\delta(\omega-\omega_1) \quad . \qquad (1.6.10)$$

The factor [...] which multiplies $2\pi\delta(\omega-\omega_1)$ we shall refer to as the **line strength** of the monochromatic electric field component, and we shall use this notation,

$$\boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega_1) \equiv [E_i(\mathbf{x},\omega_1)\, e^{j\varphi_i(\mathbf{x},\omega_1)}] \qquad // E_i(\mathbf{x},\omega_1) = |\, \boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega_1)\,| \qquad (1.6.18)$$

and then

$$\hat{E}_i(\mathbf{x},\omega) = \boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega_1)\, 2\pi\delta(\omega-\omega_1) \quad . \qquad \boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega_1) = \text{line strength} \qquad (1.6.19)$$

Since we shall normally refer to a monochrome frequency as ω (rather than ω_1) with dependence $e^{j\omega t}$, we can rewrite the last three equations as

$$\hat{E}_i(\mathbf{x},\omega') = [E_i(\mathbf{x},\omega)\, e^{j\varphi_i(\mathbf{x},\omega)}]\, 2\pi\delta(\omega'-\omega) \qquad (1.6.20)$$

$$\boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega) \equiv [E_i(\mathbf{x},\omega)\, e^{j\varphi_i(\mathbf{x},\omega)}] \qquad // E_i(\mathbf{x},\omega) = |\, \boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega)\,| \qquad (1.6.21)$$

$$\hat{E}_i(\mathbf{x},\omega') = \boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega)\, 2\pi\delta(\omega'-\omega) \qquad \boldsymbol{\mathcal{E}}_i(\mathbf{x},\omega_1) = \text{line strength} \qquad (1.6.22)$$

(f) Overloaded Notation and Maxwell's Equations in ω space

A rigorous textbook probably should be careful about which of the above many field versions are the subject of any particular discussion. To the reader's possible dismay, we shall generally (but not always) refer to all these electric fields as E_i and shall depend on the reader to decipher which kind of field is implied in a given situation. The reason for this decision is that having a large number of notations for electric fields (and for magnetic fields, and various other derived quantities such as potential V and current i and current density J_i) adds a level of visual font complexity to equations which are already complex enough to begin with. It is a tradeoff between precision and font/decoration clutter.

In particular, earlier in this section we have carefully denoted the Fourier Transform of f(t) as f^(ω) which is a notation used by Stakgold and others (though Stakgold has our (1.6.8) phases negated as in his equation (5.32)). In the rest of this document, however, we represent the Fourier Transform of f(t) as f(ω) to avoid a proliferation of hat ^ symbols. Since the functions f(t) and f(ω) are completely different functions, the symbol f is "overloaded" (in the sense of overloaded variable names in computer languages) and we trust the reader to understand that f(ω) always means f^(ω). It is the presence of the argument ω that cues the reader to this fact. This overloaded notation has already been used in Section 1.5 and we continue it below. Similarly, a generic field $E_i(\omega)$ may refer to the full Fourier transform $E\hat{}_i$, or it may refer to the line strength $\mathcal{E}_i$ if monochromatic fields are being used. A field magnitude will always be properly indicated as a magnitude. The physical fields make only rare appearances.

In the Maxwell and related equations which include the ∂_t operator, if the fields are expanded onto their Fourier transformed components using (1.6.8b), then using the rule (1.6.9) one may instantly write the frequency-domain version of these equations, just as in the example of Section 1.5. For example,

$$\text{curl } \mathbf{H}(\mathbf{x},\omega) = j\omega\mathbf{D}(\mathbf{x},\omega) + \mathbf{J}(\mathbf{x},\omega) \tag{1.6.23}$$

$$\text{curl } \mathbf{E}(\mathbf{x},\omega) = -j\omega\mathbf{B}(\mathbf{x},\omega) \tag{1.6.24}$$

$$\text{div } \mathbf{J}(\mathbf{x},\omega) = -j\omega\rho(\mathbf{x},\omega) \quad . \tag{1.6.25}$$

Other equations in the Section 1.1 list have the same form but in terms of the frequency-domain functions. For example,

$$\mathbf{J}(\mathbf{x},\omega) = \sigma(\mathbf{x})\ \mathbf{E}(\mathbf{x},\omega) \tag{1.6.26}$$

where we momentarily allow σ(**x**) to have spatial dependence but not time dependence. All these ω dependent fields can be regarded either as full Fourier transforms, or as the line strengths corresponding to those transforms, where the 2πδ functions cancel on the two sides of the equation.